find its value

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -b^{n} \cdot z^{-n}$$

$$= -\sum_{n=1}^{\infty} b^{-n} z^{n}$$

$$= -\sum_{n=1}^{\infty} (b^{-1}z)^{n}$$

$$= -\left[b^{-1}z + (b^{-1}z)^2 + (b^{-1}z)^3 + \cdots + \infty \right]$$

$$= -b^{-1}z \left[1 + b^{-1}z + (b^{-1}z)^2 + \cdots + \infty \right]$$

$$= -b^{-1}z \left[\frac{1}{1 - b^{-1}z} \right]$$

$$= -z/b \left[\frac{1}{1 - z/b} \right]$$

$$= \frac{-z}{b}\left[\frac{1}{\frac{b-z}{b}}\right)$$

$$= \frac{-z}{\cancel{b}} \times \frac{\cancel{b}}{b-z}$$

$$= \frac{\cancel{-}z}{\cancel{-}(z-b)} = \frac{z}{z-b}.$$

$$z[-b^n u(-n-1)] = \frac{z}{z-b}. \quad ROC\ |z|<b$$

The Right sided signal $a^n u(n)$ and the left sided signal $-a^n u(-n-1)$ will have the same transform value. But their ROC's are different

Z transform value $= z/z-a$

ROC: $a^n u(n) = |z|>a$, $-a^n u(-n-1) = |z|<a$

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Q. Determine the z transform of $x(n) = a^n u(n) - b^n u(-n-1)$ and find the ROC

z transform of $a^n u(n) = z/z-a$

ROC : $|z| > a$

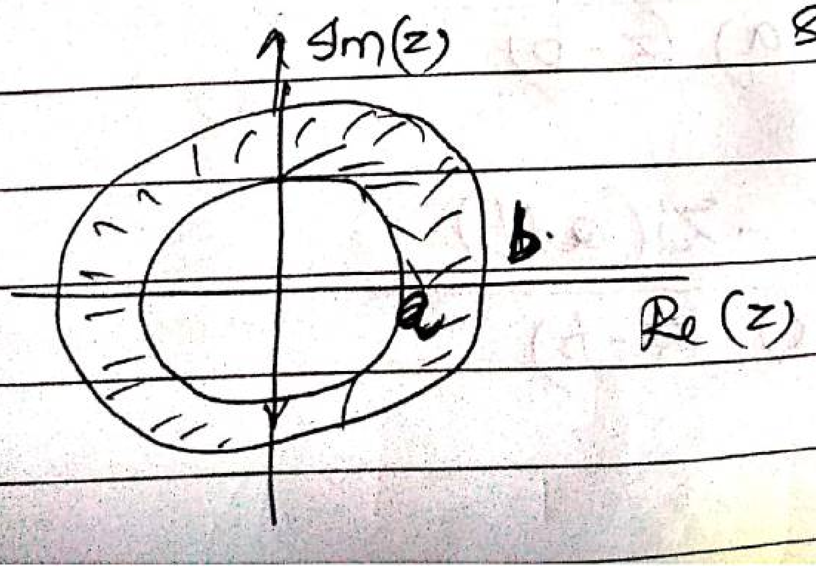
z transform of $-b^n u(-n-1)$

$= z/z-b$. ROC: $|z| < b$

ROC of $x(n) = |z| > a \cap |z| < b$

$a < |z| < b$

Suppose $b > a$



If $x(n)$ is expressed as a sum of two or more signal, then their ROC is a combination of their individual ROC's

Z transform Value

$$= \frac{z}{z-a} + \frac{z}{z-b}$$

$$= \frac{z(z-b) + z(z-a)}{(z-a)(z-b)}$$

$$= \frac{z^2 - bz + z^2 - az}{(z-a)(z-b)}$$

$$= \frac{2z^2 - z(a+b)}{(z-a)(z-b)}.$$

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Q) Determine the z transform & R.O.C of $X(z)$

$$x(n) = (2/3)^n u(n) + (-1/2)^n \cdot u(n)$$

$$Z[a^n \cdot u(n)] = \frac{z}{z-a}$$

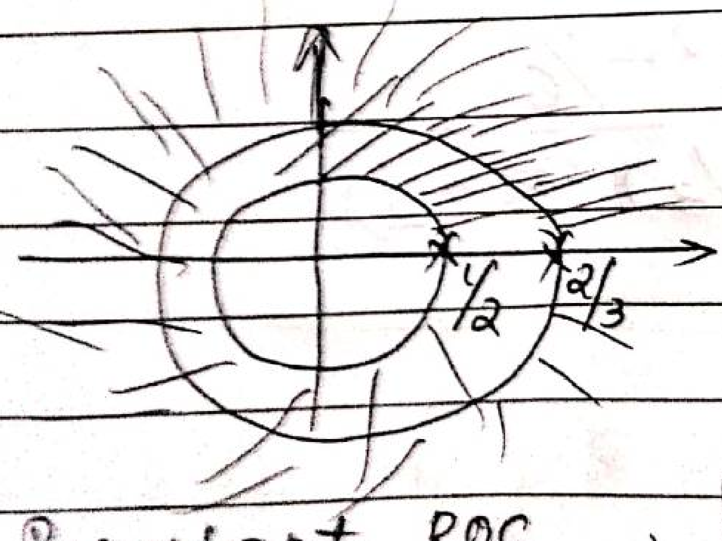
$$Z[(2/3)^n \cdot u(n)] = \frac{z}{z-2/3} \quad \therefore ROC: |z| > a$$

$$|z| > 2/3$$

$$Z[(-1/2)^n \cdot u(n)] = \frac{z}{z+1/2} \quad ; \quad |z| > a$$

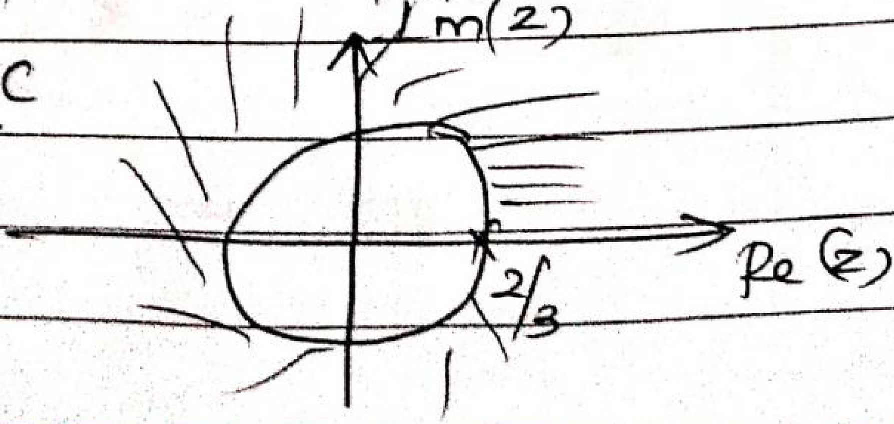
$$|z| > -1/2$$

$$z > 1/2$$



1/2 & 2/3 are the poles of X(z)

Resultant ROC



z transform value

$$X(z) = \frac{z}{z-2/3} + \frac{z}{z+1/2}$$

$$= \frac{z(z+1/2) + z(z-2/3)}{(z-2/3)(z+1/2)}$$

$$= \frac{z^2 + \frac{1}{2}z + z^2 - \frac{2}{3}z}{(z-2/3)(z+1/2)}$$

$$\frac{1}{2} - \frac{2}{3}$$

$$\frac{3-4}{6}$$

$$= \frac{2z^2 - 1/6}{(z-2/3)(z+1/2)}$$

Q. Find the z transform and R.O.C of $x[n] = a^{|n|}$ $|a|<1$

$a^{|n|}$ is defened as $=$ a^n for $n>0$

a^{-n} for $n<0$

$a^{|n|} = a^n u(n) + a^{-n} \cdot u(-n-1)$

$z[a^n u(n)] = \frac{z}{z-a}$ ROC : $|z|>a$

$z((a^{-1})^n \cdot u(-n-1)] = \frac{-z}{z-a^{-1}}$ ROC : $|z|<a^{-1}$

$= \frac{-z}{z-1/a}$ $\underline{\underline{|z|<1/a}}$

ROC : $ROC_1 \cap ROC_2 = a<|z|<1/a$

$$X(z) = \frac{z}{z-a} - \frac{z}{z-1/a}$$

$$= \frac{z(z-1/a) - z(z-a)}{(z-a)\cdot(z-1/a)}$$

$$= \frac{\cancel{z^2} - z/a \cancel{- z^2} + az}{(z-a)(z-1/a)}$$

$$= \frac{az - z/a}{(z-a)(z-1/a)}$$

Find the z transform and ROC of finite duration Sequence

1. Right Sided sequence

If x(n) is given in the form of a sequence and is also a right sided signal ie the signal exists only for n>0, (for positive values of n)

$x[n]$ ~~x(n)~~ $= \{ \underset{\uparrow}{1}, 2, 3, 4 \}$

here the signals x(n) is represented in the form of a sequence & its starting value is at n=0 & ends at n = 3. Here x(n) exists for +ve values of n. So definitely the signal is a right sided signal

$x(0) = 1, \quad x(1) = 2, \quad x(2) = 3, \quad x(3) = 4$

z transform of a signal $x(n)$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

Here $x(n) \rightarrow n$ value changes from $n = 0$ to 3

$$X(z) = \sum_{n=0}^{3} x(n) \cdot z^{-n}$$

$$= x(0) \cdot z^{0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$$

$$X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

This is the z transform of signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

ROC! The existance value of the above z transform based on the values of $|z|$ or r

$X(z)$ becomes finite when $z = \infty$ when $z = \infty$, $z^{-1} = 1/z = 1/\infty = 0$

$X(z) = 1 + 2 \times 1/z + 3\, 1/z^2 + 4\, 1/z^3$

$= 1 + 2/\infty + 3/\infty + 4/\infty$

$= 1$

when $z = 0$.

$X(z) = 1 + 2 \times 1/0 + 3\, 1/0^2 + 4/0^3$

$= \infty$.

$\therefore$ We can say that the Roc values of $X(z)$ exists for all values of z except at $z = 0$

2. left sided sequence:

Here also the signal $x(n)$ is given in the form of a sequence and is also a left sided signal.

left sided signals are those signal which exists for negative values of n

Eg: $x(n) = \{2, 3, 1, \underset{\uparrow}{2}\}$

Here the signal whose n values changes from -3 to 0

Z transform of a signal $x(n)$ is $X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$X(z) = \sum_{n=-3}^{0} x(n) z^{-n}$$

$$= x(-3) z^{3} + x(-2) z^{2} + x(-1) z + x(0)$$

$$= 2z^{3} + 3z^{2} + z + 2$$

ROC : when $z = \infty$, $X(z) = \infty$

when $z = 0$, $X(z) = 2$, finite

In case of left sided sequence signals, ROC values of $X(z)$ exists for all values of z except at $z = \infty$

3 Two sided sequence

If $x(n)$ is given in the form of a sequence and is two sided also

$$x(n) = \{1, 2, 3, \underset{\uparrow}{2}, -1, 2, 3\}$$

The value of $x(n)$, extends from $n = -3$ to 3

$$X(z) = x(-3)z^{3} + x(-2)z^{2} + x(-1)z + x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$

$$= z^{3} + 2z^{2} + 3z + 2 - z^{-1} + 2z^{-2} + 3z^{-3}$$

ROC of two sided signal is a combn of ROC of left sided and right sided signal

The ~~Val~~ ROC value of $X(z)$ exists for all values of z except at $z=0$ & $z=\infty$

PROPERTIES OF ROC'S OF Z TRANSFORM

1. The ROC is a Ring or circle in the z plane centered at the origin

2 The ROC cannot contain any poles

3 If $x(n)$ is a causal sequence, then its ROC exists in the entire z plane except at $z=0$

4 If $x(n)$ is an anticausal sequence, then its ROC exists in the entire z plane except at $z=\infty$

5 If $x(n)$ is a finite duration two sided sequence, then its ROC exists in the z plane except at $z=0$ and at $z=\infty$

6 If $x(n)$ is an infinite duration two sided ~~sequence~~ signal, then its ROC will consists of a circle in the z plane, bounded by their poles and there is no pole exists inside the ROC.

Z transform of standard signals

	Signal	Z transform	ROC
1	$x(n) = \delta[n]$.	1.	Everywhere in the z plane
2	$x[n] = u[n]$	$\frac{1}{1-z^{-1}}$ or $\frac{z}{z-1}$	$\|z\| > 1$
3	$x(n) = +u[-n-1]$	$\frac{-1}{1-z^{-1}}$ or $\frac{-z}{z-1}$	$\|z\| < 1$
4	$x[n] = a^n u[n]$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\|z\| > a$
5	$x(n) = (-a)^n u[n]$	$\frac{1}{1+az^{-1}}$ or $\frac{z}{z+a}$	$\|z\| > -a$
6	$x(n) = -a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\|z\| < a$
7.	$x(n) = n a^{n-1} u(n)$	$z/(z-a)^2$	$\|z\| > a$
8	$x(n) = -n a^{n-1} u(-n-1)$	$z/(z-a)^2$	$\|z\| < a$

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Signal	Z transform	ROC
9 $\frac{n(n-1)}{2!} a^{n-2} u(n)$	$\frac{z}{(z-a)^3}$	$z > a$
10 $\frac{-n(n-1)}{2!} a^{n-2} \cdot u(-n-1)$	$\frac{z}{(z-a)^3}$	$\|z\| < a$
11. $\frac{n(n-1)(n-2)}{3!} a^{n-3} \cdot u(n)$	$\frac{z}{(z-a)^4}$	$\|z\| > a$
12 $\frac{-n(n-1)(n-2) a^{n-3} \cdot u(-n-1)}{3!}$	$\frac{z}{(z-a)^4}$	$\|z\| < a$

INVERSE Z TRANSFORM

1. Long division Method.

Two cases. ① Causal $|z| > a$ ② Non Causal $|z| < a$

Inverse z transform is used to find the value of $x(n)$ if $X(z)$ is given.

①. How the long division method is applicable to causal signal.

- In this case, $X(z)$ is given & condition of signal is specified ie $|z| > a$, so we have to extract the causal signal from the given z transform expression. In this method, the signal $x(n)$ is expressed in the form of a causal sequence (After long division Method)

- The z transform of a causal signal is $X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$

i.e $X(z) = x(0) + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + \text{---}$ —①

Here we find the values of $x(0), x(1), x(2)$, -- & expressed in the form of a sequence

$$x(n) = \{x(0), x(1), x(2) \text{ --- }\}$$

- Before long division Method, we need to arrange the Divisor & divident terms.

For Eg: $X(z) = \dfrac{z+1}{z^2 - 2z + 1}$

The given signal is a causal sequence, So we have to arrange the Nr & Dr in terms of Descending powers of z.

In this Eg. $z^2 - 2z + 1 \overline{)\, z+1}$

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$$\begin{array}{r|l} & \frac{1}{z} + \frac{3}{z^2} + \frac{5}{z^3} \\ z^2-2z+1 & z+1 \\ & \overset{(-)}{z} \overset{(+)}{-2} \mp \frac{1}{z} \\ \hline & 3 - \frac{1}{z} \\ & \overset{(-)}{3} \overset{(+)}{-} \frac{6}{z} \overset{(-)}{+} \frac{3}{z^2} \\ \hline & \frac{5}{z} - \frac{3}{z^2} \\ & \overset{(-)}{\frac{5}{z}} \overset{(+)}{-} \frac{10}{z^2} + \frac{5}{z^3} \\ \hline & \frac{7}{z^2} - \frac{5}{z^3} \end{array}$$

This process goes on continously.

We get $X(z)$ in the following form

$$X(z) = \frac{1}{z} + \frac{3}{z^2} + \frac{5}{z^3} + \cdots$$

or

$$= z^{-1} + 3z^{-2} + 5z^{-3} + \cdots$$

Compare this with (1)

$x(0) = 0$, $x(1) = 1$, $x(2) = 3$, $x(3) = 5$

(1) $z/z^2 = 1/z$

$1/z \times z^2 = z$

$1/z \times -2z = -2$

$1/z \times 1 = 1/z$

then change the sign & add them

(2) $3/z^2$

$3/z^2 \times z^2 = 3$

$3/z^2 \times -2z = \frac{-6}{z}$

$3/z^2 \times 1 = 3/z^2$

then change sign and add them

(3) $5/z \times \frac{1}{z^2} = 5/z^3$

$5/z^3 \times z^2 = 5/z$

$5/z^3 \times -2z = -10/z^2$

$5/z^3 \times 1 = 5/z^3$

then change the sign & add

$x(n) = \{0, 1, 3, 5, - - -\}$

② If $x(n)$ is a non-causal sequence

① Arrange the Nr & Dr of $X(z)$ in terms of ascending powers of z

z^0, z^1, z^2 like that

$$X(z) = \frac{z+1}{z^2-2z+1} \quad |z| < 1$$

$$
\begin{array}{r|l}
 & 1 + 3z + 5z^2 \\
\hline
1-2z+z^2 & 1+z \\
 & \underline{{}^{(-)}1 \; {}^{(+)}{-2z} \; {}^{(-)}{+z^2}} \\
 & 3z - z^2 \\
 & \underline{{}^{(-)}3z \; {}^{(+)}{-6z^2} \; {}^{(-)}{+3z^3}} \\
 & 5z^2 - 3z^3 \\
 & \underline{{}^{(-)}5z^2 \; {}^{(+)}{-10z^3} \; {}^{(-)}{+5z^4}} \\
 & 7z^3 - 5z^4
\end{array}
$$

z transform of a NC sequence is

$$X(z) = \sum_{n=-\infty}^{0} x(n) z^{-n}$$

$$= x(-\infty) z^{-\infty} + \cdots x(-4) \cdot z^{4} + x(-3) z^{3}$$

$$+ x(-2) z^{2} + x(-1) z^{1} + x(0)$$

By comparing the Quotient with the above expression, we get

$x(-2) = 5$, $x(-1) = 3$, $x(0) = 1$

So, $x(n) = \{ \cdots 5, 3, \underset{\uparrow}{1} \}$

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Qn. using long division Method, determine the inverse z-transform of

$$X(z) = \frac{z+1}{z^2-3z+2}$$

(a) $x(n)$ is causal

(b) $x(n)$ is anticausal

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