5/5/20 Home-work

Find DTFT of the following signals?

(i) $x[n] = 10\left(\frac{1}{6}\right)^{-n} u[-n]$

(ii) $x[n] = n\left(\frac{1}{2}\right)^{n} u[n]$

(iii) $y[n] = \left(\frac{1}{2}\right)^{n} u[n] * \left(\frac{1}{3}\right)^{n} u[n]$

(iv) $x[n] = (n+1) \cdot a^{n} u[n]$

(v) $x[n] = \left(\frac{1}{4}\right)^{n-1} \cdot u(n-1)$

Answers:

(i) $x[n] = 10\left(\frac{1}{6}\right)^{-n} u[-n]$.

Time Reversal property

$$\mathcal{F}\{x[-n]\} \longrightarrow X(e^{-j\omega}).$$

$$\mathcal{F}\left\{10\left(\tfrac{1}{6}\right)^{n} u[n]\right\} = 10 \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{6}\right)^{n} u[n] e^{-j\omega n}$$

$$= 10 \sum_{n=0}^{\infty} \left(\frac{1}{6} e^{-j\omega}\right)^{n}$$

$$= 10 \left[\frac{1}{1 - \frac{1}{6 e^{j\omega}}}\right]$$

$$\mathcal{F}\left\{10\left(\tfrac{1}{6}\right)^{-n} u[-n]\right\} = 10 \left[\frac{1}{1 - \frac{e^{j\omega}}{6}}\right] = \frac{60}{6 - e^{j\omega}}.$$

$$= \frac{-60}{(e^{j\omega} - 6)}$$

(ii) $x[n] = n\left(\frac{1}{2}\right)^n u[n]$.

multiplication by n property.

$$\mathcal{F}\{n\,x[n]\} \longrightarrow j\frac{d}{d\omega}[X(e^{j\omega})].$$

$$\mathcal{F}\left\{n\left(\frac{1}{2}\right)^n u[n]\right\} \longrightarrow ?.$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n u[n]e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}e^{-j\omega}\right)^n$$

$$= \frac{1}{1-\frac{e^{-j\omega}}{2}}.$$

$$= \frac{-2}{e^{-j\omega}-2}.$$

$$\mathcal{F}\left\{n\left(\frac{1}{2}\right)^n u[n]\right\} \to j\frac{d}{d\omega}\left[\frac{-2}{e^{-j\omega}-2}\right]$$

$$-j2 \times \left[\frac{-1}{(e^{-j\omega}-2)^2}\right] \times -j e^{-j\omega}.$$

$$\frac{-2}{(e^{-j\omega}-2)^2}e^{-j\omega}$$

(iii) $y[n] = \left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n].$

$$\mathcal{F}\{x[n] * h[n]\} \longrightarrow X(e^{j\omega}).H(e^{j\omega})$$

[convolution property].

$$\left(\frac{1}{2}\right)^n u[n] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - \frac{e^{-j\omega}}{2}}.$$

$$= \frac{-2}{e^{-j\omega} - 2}.$$

$$\left(\frac{1}{3}\right)^n u[n] \xrightarrow{DTFT} \sum_{n=0}^{\infty} \left(\frac{1}{3} e^{-j\omega}\right)^n$$

$$= \cancel{\sum_{n=0}} \frac{1}{1 - \frac{e^{-j\omega}}{3}}$$

$$= \frac{-3}{e^{-j\omega} - 3}.$$

$$\mathcal{F}\left\{\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n]\right\} \longrightarrow \left(-\frac{2}{e^{-j\omega} - 2}\right)\left(-\frac{3}{e^{-j\omega} - 3}\right)$$

$$\rightarrow \frac{6}{(e^{-j\omega} - 2)(e^{-j\omega} - 3)}$$

(iv) $x[n] = (n+1)\, a^n u[n]$.

$x[n] = n a^n u[n] + a^n u[n]$

$x[n] \longrightarrow X(e^{j\omega})$

$a^n u[n] \longrightarrow \sum_{n=-\infty}^{\infty} a^n u[n] e^{-j\omega n}$

$\rightarrow \sum_{n=0}^{\infty} (a e^{-j\omega})^n$

$$\frac{1}{1 - ae^{-j\omega}}$$

$n\, x[n] \xrightarrow{DTFT} j \frac{d}{d\omega} [X(e^{j\omega})]$

$\rightarrow j \frac{d}{d\omega} \left[\frac{1}{1 - ae^{-j\omega}} \right]$

$\rightarrow j \times \frac{-1}{(1 - ae^{-j\omega})^2} \times -ae^{-j\omega} \times -j$

$\rightarrow \frac{a e^{-j\omega}}{(1 - ae^{-j\omega})^2}$.

$\therefore$ DTFT $\left\{ (n+1) a^n u[n] \right\} = \frac{ae^{-j\omega}}{(1 - ae^{-j\omega})^2} + \frac{1}{1 - ae^{-j\omega}}$

(v) $\left(\frac{1}{4}\right)^{n-1} u[n-1]$

Time shifting property.

$$\mathcal{F}\{x[n-m]\} \longrightarrow e^{-j\omega m} X(e^{+j\omega})$$

$$\mathcal{F}\left\{\left(\frac{1}{4}\right)^{n} u[n]\right\} \longrightarrow \frac{-4}{e^{-j\omega}-4}$$

$m=1$

$$\mathcal{F}\left\{\left(\frac{1}{4}\right)^{n-1} u[n-1]\right\} \longrightarrow e^{-j\omega}\left[\frac{-4}{e^{-j\omega}-4}\right].$$

7/05/2020 ANALYSIS OF LTI system using DTFT:

Transfer function or frequency response.

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$ from $y[n] = x[n] * h[n]$ & transform is applied.

$H(e^{j\omega})$ gives the frequency response of discrete time system which is periodic in nature. with a period of 2π [period from $-\pi$ to π].

The frequency response $H(e^{j\omega})$ is a complex func: of frequency and it can be expressed as

$$H(e^{j\omega}) = H_R(e^{j\omega}) + H_I(e^{j\omega})$$