(v) $\left(\frac{1}{4}\right)^{n-1} u[n-1]$

Time shifting property.

$$\mathcal{F}\{x[n-m]\} \longrightarrow e^{-j\omega m} X(e^{+j\omega})$$

$$\mathcal{F}\left\{\left(\frac{1}{4}\right)^{n} u[n]\right\} \longrightarrow \frac{-4}{e^{-j\omega}-4}$$

$m=1$

$$\mathcal{F}\left\{\left(\frac{1}{4}\right)^{n-1} u[n-1]\right\} \longrightarrow e^{-j\omega}\left[\frac{-4}{e^{-j\omega}-4}\right].$$

7/05/2020. ANALYSIS OF LTI system using DTFT:

Transfer function or frequency response.

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$ from $y[n] = x[n] * h[n]$ & transform is applied.

$H(e^{j\omega})$ gives the frequency response of discrete time system which is periodic in nature. with a period of 2π [period from $-\pi$ to π].

The frequency response $H(e^{j\omega})$ is a complex func: of frequency and it can be expressed as

$$H(e^{j\omega}) = H_R(e^{j\omega}) + H_I(e^{j\omega})$$

This can be split into two: Magnitude and phase ~~response~~ spectrum.

$$|H e^{j\omega}| = \sqrt{[H_R(e^{j\omega})]^2 + [H_I e^{j\omega}]^2}$$

[Magnitude Spectrum].

$$\angle H(e^{j\omega}) = \tan^{-1}\left[\frac{H_I(e^{j\omega})}{H_R(e^{j\omega})}\right]$$

Qn. Determine and sketch magnitude and phase response of the following system.

$$y[n] = \frac{1}{3}[x[n] + x[n-1] + x[n-2]].$$

→ Plot magnitude & phase spectrum.

First, we have to find the DTFT of above ~~signal~~ difference eq:

→ DTFT of difference equation is found by using time shifting property.

$$\cancel{F[x[n-m]]} \quad \mathcal{F}(x[n-m]) = e^{-j\omega m} X(e^{j\omega})$$

→ After finding $H(e^{j\omega})$, find $|H(e^{j\omega})|$ & $\angle H e^{j\omega}$.

ω takes values from $-\pi$ to π

bec the spectrum is continuous & periodic with period 2π.

Ans)

$$y[n] = \frac{1}{3}\left[x[n] + x[n-1] + x[n-2]\right]$$

$$Y(e^{jw}) = \frac{1}{3}\left[X(e^{jw}) + \cancel{\frac{e^{-jw}(-1)}{e^{jw}-1}} + \cancel{\frac{e^{-2jw}(-2)}{(e^{-jw}-2)}}\right.$$

$$Y(e^{jw}) = \frac{1}{3}\left[X(e^{jw}) + e^{-jw}X(e^{jw}) + e^{-2jw}(X(e^{jw}))\right]$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{1}{3}\left[1 + e^{-jw} + e^{-2jw}\right)$$

$$H(e^{jw}) = \frac{1}{3}\left[1 + e^{-jw} + e^{-2jw}\right]$$

$$e^{-2jw} = e^{-jw} \cdot e^{-jw}$$

$$= e^{-jw}\frac{\left[e^{jw} + 1 + e^{-jw}\right]}{3.}$$

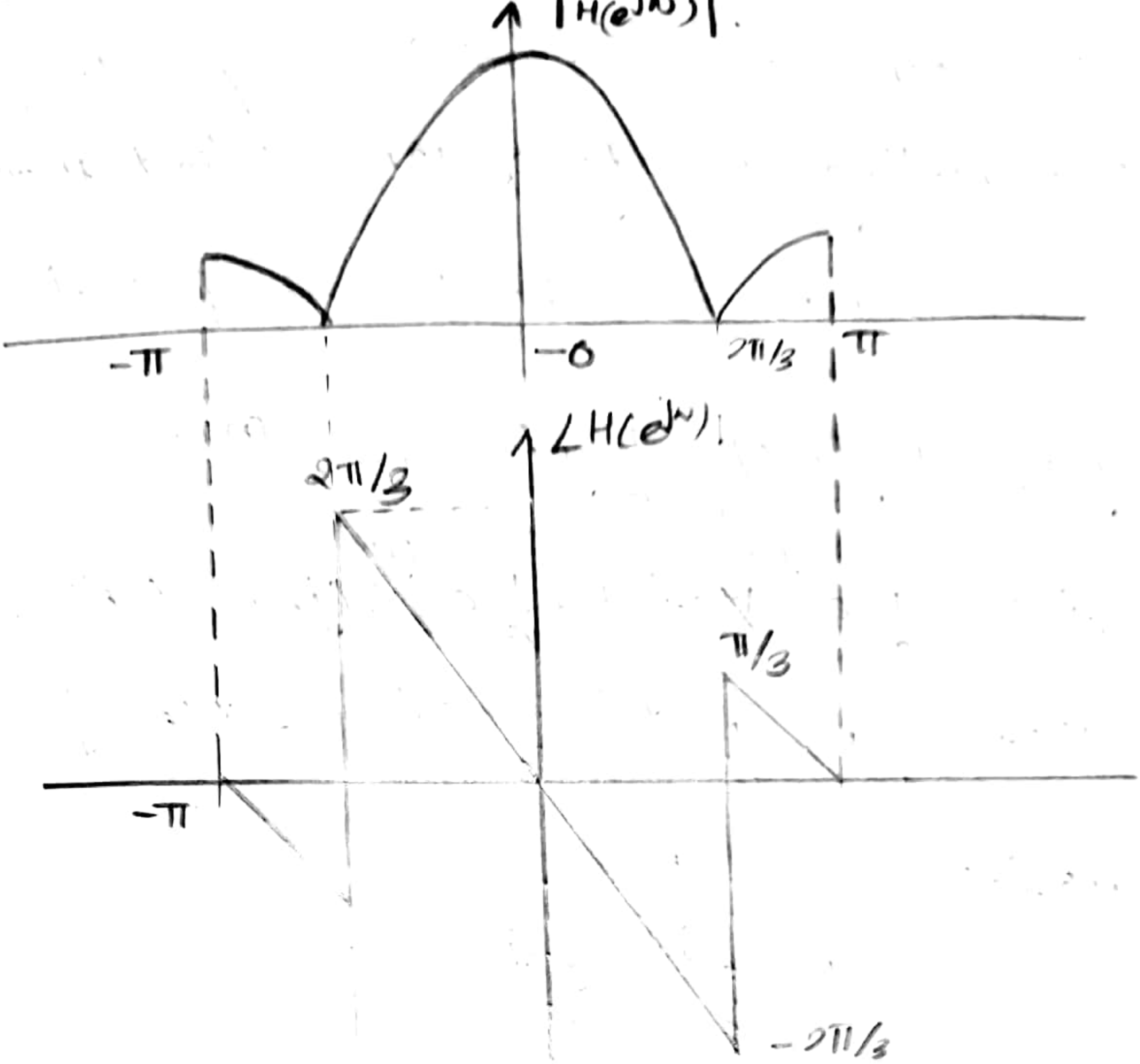
$$= \frac{e^{-jw}}{3}\left[1 + 2\cos w\right]$$

$$|H(e^{jw})| = \frac{|e^{-jw}|}{3}\,|1 + 2\cos w|$$

$$= \frac{1}{3}\,|1 + 2\cos w|.$$

$$\angle H(e^{jw}) = \begin{cases} -w & \text{for } H(e^{jw}) > 0 \\ -w+\pi & \text{for } H(e^{jw}) < 0. \end{cases}$$

w	0	$\pi/4$	$\pi/3$	$\pi/2$.	$2\pi/3$	$3\pi/4$	π
$H(e^{jw})$	1	0.8047	0.661	0.333	0	−0.138	−0.333
$\lvert H(e^{jw})\rvert$	1	0.8047	0.661	0.333	0	−0.1.38	−0.333
$\angle H(e^{jw})$	0	$\frac{-\pi}{7}$	$-\pi/3$.	$-\pi/2$.	$-2\pi/3$	$\pi/4$	0.



Qn). Consider a causal and stable LTI system whose input $x[n]$ and o/p $y[n]$ are related through the second order diff: eq:

$$y[n] - \frac{1}{6} y[n-1] - \frac{1}{6} [y[n-2]] = x[n]$$

a) Determine the frequency response $H(e^{j\omega})$ for the system.

b) Determine the impulse response $h[n]$ for the system.

Ans: Apply DTFT on given difference eq: Apply time shifting property of DTFT & find $H(e^{j\omega})$.

To obtain $h[n]$, take the inverse of $H(e^{j\omega})$.

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n].$$

$$Y(e^{j\omega}) - \frac{1}{6}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{6}e^{-2j\omega}Y(j\omega) = X(e^{j\omega}).$$

$$Y(e^{j\omega})\left[1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}\right] = X(e^{j\omega}]$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{e^{-j\omega}}{6} - \frac{e^{-2j\omega}}{6}}$$

$$= \frac{1}{\left(1 - \frac{e^{-j\omega}}{2}\right)\left(1 + \frac{e^{-j\omega}}{3}\right)}.$$

$$= \frac{A}{\left(1 - \frac{e^{-j\omega}}{2}\right)} + \frac{B}{\left(1 + \frac{e^{-j\omega}}{B}\right)}$$

$$A = \cancel{\left(1 - \frac{e^{-j\omega}}{2}\right)} \frac{1}{\cancel{\left(1 - \frac{e^{-j\omega}}{2}\right)}\left(1 + \frac{e^{-j\omega}}{3}\right)}\Bigg|_{e^{-j\omega} = 2}$$

$$= \frac{1}{1 + \frac{2}{3}}.$$

$$= \frac{3}{5}$$

$$B = \left(1+\cancel{\frac{e^{j\omega}}{3}}\right) \frac{1}{\cancel{\left(1+\frac{e^{-j\omega}}{3}\right)}\left(1-\frac{e^{-j\omega}}{2}\right)}\Bigg|_{e^{-j\omega}=-3.}$$

$$= \frac{1}{1+\frac{3}{2}}$$

$$= \frac{2}{5}$$

$$H(e^{j\omega}) = \frac{3}{5}\left[\frac{1}{1-\frac{e^{-j\omega}}{2}}\right] + \frac{2}{5}\left[\frac{1}{1+\frac{e^{j\omega}}{3}}\right]$$

$$\mathcal{F}\left\{\frac{1}{1-ae^{-j\omega}}\right\} = a^n u[n]$$

$$\mathcal{F}\left\{\frac{1}{1+ae^{-j\omega}}\right\} = (-a^n)u[n]$$

$$h[n] = \frac{3}{5} \times \left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n]$$

Find the frequency response of following causal systems.

1. $y[n] = \frac{1}{2}x[n] + x[n-1] + \frac{1}{2}x[n-2]$

$$Y(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) + \frac{e^{-2j\omega}}{2}X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \left(\frac{1}{2} + e^{-j\omega} + \frac{e^{-2j\omega}}{2}\right).$$

$$H(e^{j\omega}) = \frac{1}{2} + e^{-j\omega} + \frac{e^{-2j\omega}}{2}.$$

$$= e^{-j\omega}\left[\frac{e^{j\omega}}{2} + 1\ \frac{e^{-j\omega}}{2}\right]$$

$$= e^{-j\omega}\left[\frac{e^{+j\omega} + e^{-j\omega}}{2} + 1\right]$$

$$= e^{-j\omega}\left[\cos\omega + 1\right].$$

2\. $y[n] - \frac{1}{4} y[n-1] - \frac{3}{8} y[n-2] = x[n-1]. + x[n]$

Applying DTFT.

$$Y(e^{j\omega}) - \frac{1}{4} e^{-j\omega} Y(e^{j\omega}) - \frac{3}{8} e^{-2j\omega} Y(e^{j\omega}) = X(e^{j\omega}) + e^{-j\omega} X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + e^{-j\omega}}{\left(1 - \frac{1}{4} e^{-j\omega} - \frac{3}{8} e^{-2j\omega}\right)}$$

$$\therefore H(e^{j\omega}) = \frac{1 + e^{-j\omega}}{\left(1 - \frac{1}{4} e^{-j\omega} - \frac{3}{8} e^{-2j\omega}\right)}$$

Qn) Find the DTFT of :

$x[n] = \left(\frac{1}{2}\right)^n u[n]$ & plot its spectrum

$x[n] = \left(\frac{1}{2}\right)^n u[n]$ $\qquad e^{-j\omega} = \cos\omega - j\sin\omega$.

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

$$= \frac{1}{1 - \frac{1}{2}[\cos w - j \sin w]}$$

$$= \frac{1}{\left(1 - \frac{1}{2}\cos w\right) - j\left(\frac{\sin w}{2}\right)}$$

$$|X(e^{jw})| = \frac{1}{\sqrt{\left(1 - \frac{1}{2}\cos w\right)^2 - \left(\frac{\sin w}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{1 - \cos w + \frac{1}{4}}} = \frac{1}{\sqrt{5/4 - \cos w}} = \frac{1}{\sqrt{1.25 - \cos w}}$$

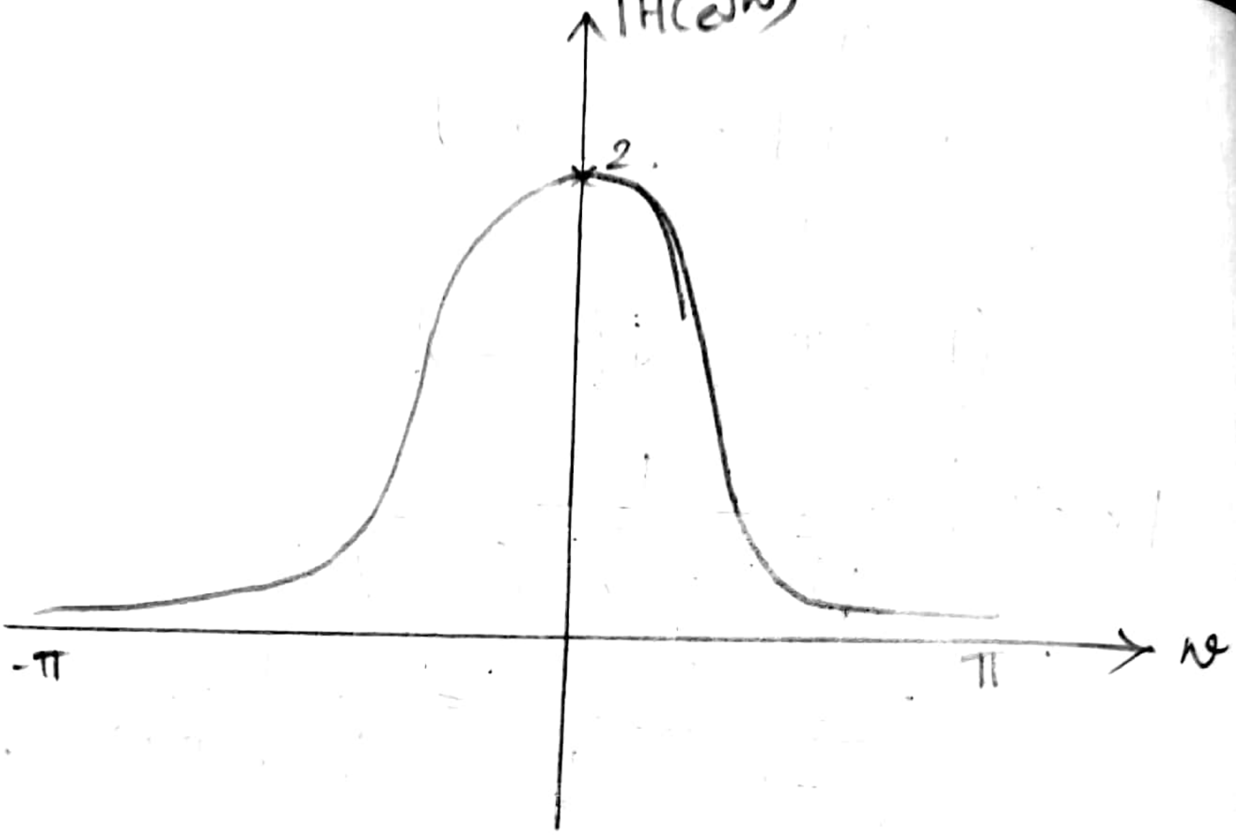
$$\angle X(e^{jw}) = \tan^{-1}\left[\frac{X_I(e^{jw})}{X_R(e^{jw})}\right]$$

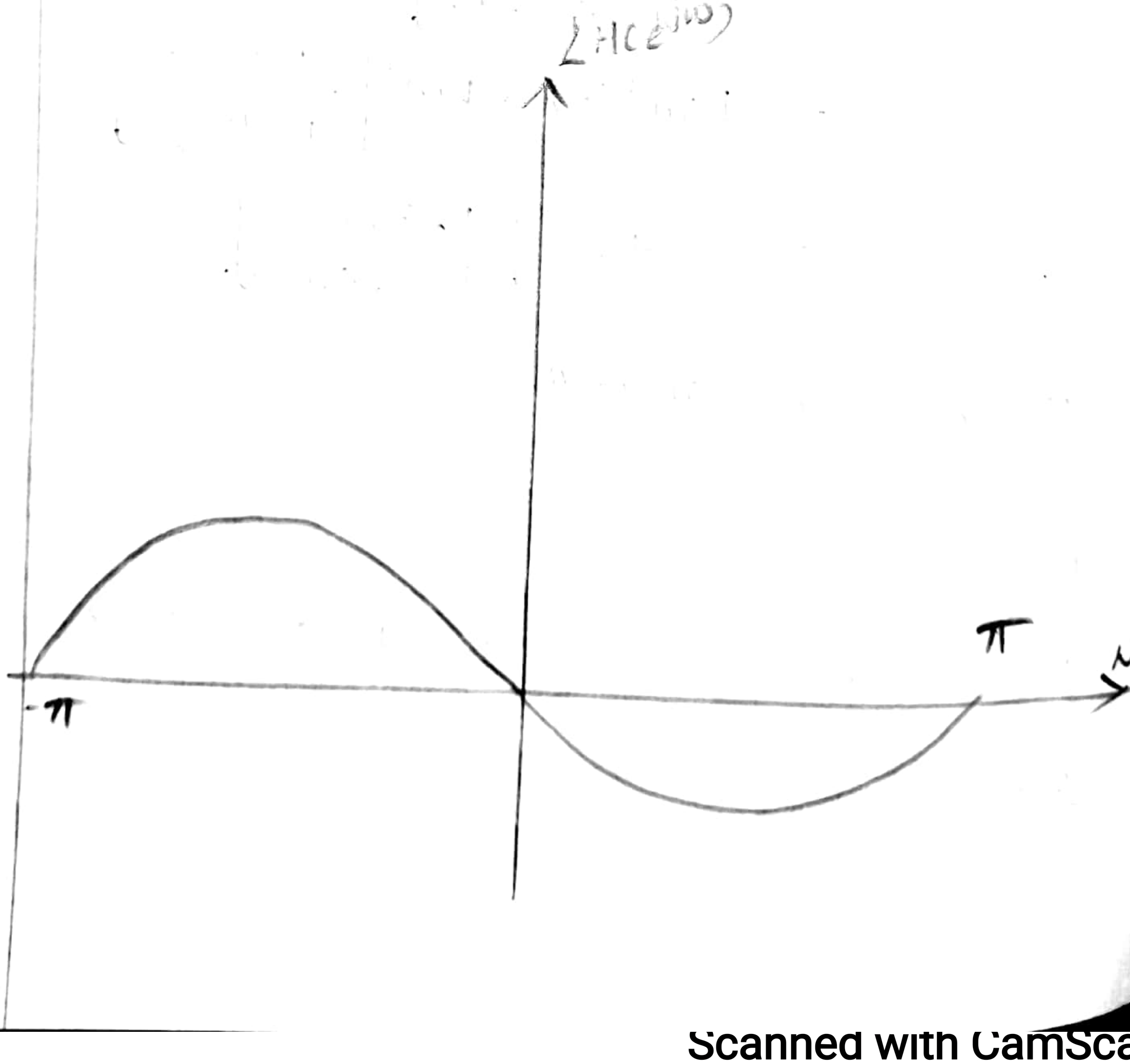
$$= \tan^{-1}(0) - \tan^{-1}\left[\frac{\sin(w)/2}{1 - \cos w/2}\right]$$

$$= -\tan^{-1}\left[\frac{1/2(\sin w)}{1 - 1/2(\cos w)}\right]$$

w varies b/w $-\pi$ to π.

w	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\lvert H(e^{jw})\rvert$	2	1.6138	1.3512	1.1547	0.8944	0.7559	0.7148	0.6874	0.6664
$\angle H(e^{jw})$	0°	−0.3217	−0.5005	−0.5235	−0.4636	−.3334	−0.2555	−0.1721	0


|H(e^jw)
2
-π
π
w


∠H(e^jw)
π
-π
w

Qn) Consider a discrete time LTI system with impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Use fourier transform to determine the response of the following signals..

1) $x[n] = \left(\frac{1}{3}\right)^n u[n]$.

Ans. here $x[n]$ and $h[n]$ are given

$x[n] = \left(\frac{1}{3}\right)^n u[n]$ | $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$X(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{3}e^{-j\omega}}$ | $H(e^{j\omega}) = \dfrac{1}{1 - \frac{e^{-j\omega}}{2}}$.

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}.$$

$$Y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega})$$

$$= \left(\frac{1}{1 - \frac{e^{-j\omega}}{3}}\right)\left(\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right)$$

$$= \frac{1}{\left(1 - \frac{e^{-j\omega}}{3}\right)\left(1 - \frac{e^{-j\omega}}{2}\right)}$$

$$= \frac{A}{1 - \frac{e^{-j\omega}}{3}} + \frac{B}{1 - \frac{e^{-j\omega}}{2}}.$$

$$A = \left.\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right|_{e^{j\omega} = 3}.$$

$$= \frac{1}{1 - \frac{2}{3}} = \underline{\underline{+3}}.$$

$$B = \left.\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right|_{e^{-j\omega} = 3}$$

$$= -2$$

$$Y(e^{j\omega}) = \frac{3}{1 - \frac{e^{-j\omega}}{2}} - \frac{2}{1 - \frac{e^{-j\omega}}{3}}$$

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

H.W. 2.

$x[n] = (-1)^n u[n]$ $\qquad$ $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(e^{j\omega}) = \frac{1}{1 + e^{-j\omega}} \qquad H(e^{j\omega}) = \frac{1}{1 - \frac{e^{-j\omega}}{2}}$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$= \left(\frac{1}{1 + e^{-j\omega}}\right) \cdot \left(\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right)$$

$$= \frac{1}{(1 + e^{-j\omega})\left(1 - \frac{e^{-j\omega}}{2}\right)}$$

$$= \frac{A}{1 + e^{-j\omega}} + \frac{B}{1 - \frac{e^{-j\omega}}{2}}$$

$$A = \left.\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right|_{e^{-j\omega} = -1}$$

$A = 2$

$B = \left. \dfrac{1}{1 - \frac{e^{-jw}}{2}} \right|_{e^{-jw} = 2}$

$= \dfrac{1}{1-2}$

$= -1$

$H(e^{jw}) = 2\left[\dfrac{1}{1+e^{-jw}}\right] + \dfrac{-1}{\left(1 - \frac{e^{-jw}}{2}\right)}$

$h[n] = 2(-1)^n u[n] - \left(+\frac{1}{2}\right)^n u[n]$

Qn). Causal and stable LTI system has the following property.

$$\left(\frac{4}{5}\right)^n u[n] \rightarrow n\left(\frac{4}{5}\right)^n u[n].$$

1) Find the frequency response?
2) Determine the difference eq: relating to i/p $x[n]$ and corresponding o/p $y[n]$.

Ans: $x[n] = \left(\frac{4}{5}\right)^n u[n]$

$y[n] = n\left(\frac{4}{5}\right)^n u[n].$

$X(e^{jw}) = \dfrac{1}{1 - \frac{4}{5}e^{-jw}}$

$Y(e^{jw}) = j\dfrac{d}{dw}\left[\dfrac{1}{1 - \frac{4}{5}e^{-jw}}\right]$

[multiplication by n property]

$$Y(e^{j\omega}) = j\left[\frac{-1}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2} \times -\frac{4}{5}e^{-j\omega} \times -j\right]$$

$$= +\frac{4}{5}\left[\frac{e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}\right].$$

(OR)

$$Y(e^{j\omega}) = j\left[\frac{\left(1-\frac{4}{5}e^{-j\omega}\right)\times 0 - 1\left(-\frac{4}{5}e^{-j\omega}\times -j\right)}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}\right]$$

$$= \frac{j\left[\frac{4}{5}e^{-j\omega}\times -j\right]}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2} = \frac{4}{5}\left[\frac{e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}\right]$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}.$$

$$= \frac{\frac{4}{5}\left[\frac{e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}\right]}{\left(1-\frac{4}{5}e^{-j\omega}\right)}$$

$$= \frac{4}{5}\left[\frac{e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^{\cancel{2}}}\right] \times \cancel{\left(1-\frac{4}{5}e^{-j\omega}\right)}$$

$$= \frac{4}{5}\left[\frac{e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}\right]$$

b) difference eq:

$$H(e^{jw}) = \frac{4}{5}\left[\frac{e^{-jw}}{1-\frac{4}{5}e^{-jw}}\right].$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{4}{5}\left[\frac{e^{-jw}}{1-\frac{4}{5}e^{-jw}}\right].$$

$$Y(e^{jw})\left[1-\frac{4}{5}e^{-jw}\right] = \frac{4}{5}e^{-jw}X(e^{jw})$$

Taking Inverse fourier transform.

$$y[n] - \frac{4}{5}y[n-1] = \frac{4}{5}x[n-1]$$