CHAPTER 6

PASSBAND DATA TRANSMISSION

This chapter builds on the material developed in Chapter 5 on signal-space analysis. It discusses the subject of digital data transmission over a band-pass channel that can be linear or nonlinear. As with analog communications, this mode of data transmission relies on the use of a sinusoidal carrier wave modulated by the data stream.

Specifically, the following topics are covered:

- *Different methods of digital modulation, namely, phase-shift keying, quadrature-amplitude modulation, and frequency-shift keying, and their individual variants.*
- *Coherent detection of modulated signals in additive white Gaussian noise, which requires the receiver to be synchronized to the transmitter with respect to both carrier phase and bit timing.*
- *Noncoherent detection of modulated signals in additive white Gaussian noise, disregarding phase information in the received signal.*
- *Modems for the transmission and reception of digital data over the public switched telephone network.*
- *Sophisticated modulation techniques, namely, carrierless amplitude/phase modulation and discrete multitone, for data transmission over a wideband channel with medium to severe intersymbol interference.*
- *Techniques for synchronizing the receiver to the transmitter.*

6.1 *Introduction*

In *baseband pulse transmission*, which we studied in Chapter 4, a data stream represented in the form of a discrete pulse-amplitude modulated (PAM) signal is transmitted directly over a low-pass channel. In *digital passband transmission*, on the other hand, the incoming data stream is modulated onto a carrier (usually sinusoidal) with fixed frequency limits imposed by a band-pass channel of interest; passband data transmission is studied in this chapter.

The communication channel used for passband data transmission may be a microwave radio link, a satellite channel, or the like. Yet other applications of passband data transmission are in the design of passband line codes for use on digital subscriber loops and orthogonal frequency-division multiplexing techniques for broadcasting. In any event, the modulation process making the transmission possible involves switching (keying) the amplitude, frequency, or phase of a sinusoidal carrier in some fashion in accordance with the incoming data. Thus there are three basic signaling schemes, and they are known as

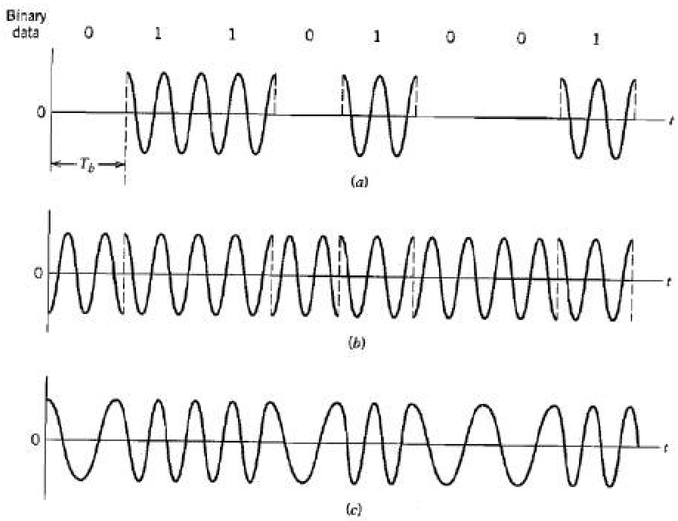

FIGURE 6.1 Illustrative waveforms for the three basic forms of signaling binary information. (*a*) Amplitude-shift keying. (*b*) Phase-shift keying. (*c*) Frequency-shift keying with continuous phase.

amplitude-shift keying (ASK), *frequency-shift keying* (FSK), and *phase-shift keying* (PSK). They may be viewed as special cases of amplitude modulation, frequency modulation, and phase modulation, respectively.

Figure 6.1 illustrates these three methods of modulation for the case of a source supplying binary data. The following points are noteworthy from Figure 6.1:

- Although in continuous-wave modulation it is usually difficult to distinguish between phase-modulated and frequency-modulated signals by merely looking at their waveforms, this is not true for PSK and FSK signals.
- Unlike ASK signals, both PSK and FSK signals have a constant envelope.

This latter property makes PSK and FSK signals impervious to amplitude nonlinearities, commonly encountered in microwave radio and satellite channels. It is for this reason, in practice, we find that PSK and FSK signals are preferred to ASK signals for passband data transmission over nonlinear channels.

HIERARCHY OF DIGITAL MODULATION TECHNIQUES[1]

Digital modulation techniques may be classified into *coherent* and *noncoherent* techniques, depending on whether the receiver is equipped with a phase-recovery circuit or not. The phase-recovery circuit ensures that the oscillator supplying the locally generated carrier wave in the receiver is synchronized (in both frequency and phase) to the oscillator supplying the carrier wave used to originally modulate the incoming data stream in the transmitter.

As discussed in Chapter 4, in an *M-ary signaling scheme*, we may send any one of M possible signals $s_1(t), s_2(t), \ldots, s_M(t)$, during each signaling interval of duration T. For almost all applications, the number of possible signals $M = 2^n$, where n is an integer. The symbol duration $T = nT_b$, where T_b is the bit duration. In passband data transmission, these signals are generated by changing the amplitude, phase, or frequency of a sinusoidal carrier in M discrete steps. Thus we have *M-ary ASK*, *M-ary PSK*, and *M-ary FSK* digital modulation schemes. Another way of generating M-ary signals is to combine different methods of modulation into a hybrid form. For example, we may combine discrete changes in both the amplitude and phase of a carrier to produce *M-ary amplitude-phase keying* (APK). A special form of this hybrid modulation is *M-ary quadrature-amplitude modulation* (QAM), which has some attractive properties. M-ary ASK is a special case of M-ary QAM.

M-ary signaling schemes are preferred over binary signaling schemes for transmitting digital information over band-pass channels when the requirement is to conserve bandwidth at the expense of increased power. In practice, we rarely find a communication channel that has the exact bandwidth required for transmitting the output of an information source by means of binary signaling schemes. Thus when the bandwidth of the channel is less than the required value, we may use M-ary signaling schemes for maximum efficiency. To illustrate the bandwidth-conservation capability of M-ary signaling schemes, consider the transmission of information consisting of a binary sequence with bit duration T_b. If we were to transmit this information by means of binary PSK, for example, we would require a bandwidth that is inversely proportional to T_b. However, if we take blocks of n bits and use an M-ary PSK scheme with $M = 2^n$ and symbol duration $T = nT_b$, the bandwidth required is proportional to $1/nT_b$. This shows that the use of M-ary PSK enables a reduction in transmission bandwidth by the factor $n = \log_2 M$ over binary PSK.

M-ary PSK and M-ary QAM are examples of *linear modulation*. However, they differ from each other in one important respect: An M-ary PSK signal has a constant envelope, whereas an M-ary QAM signal involves changes in the carrier amplitude. Accordingly, M-ary PSK can be used to transmit digital data over a nonlinear band-pass channel, whereas M-ary QAM requires the use of a linear channel.

M-ary PSK, M-ary QAM, and M-ary FSK are commonly used in coherent systems. Amplitude-shift keying and frequency-shift keying lend themselves naturally to use in noncoherent systems whenever it is impractical to maintain carrier phase synchronization. But in the case of phase-shift keying, we cannot have "noncoherent PSK" because the term noncoherent means doing without carrier phase information. Instead, we employ a "pseudo PSK" technique known as *differential phase-shift keying* (DPSK), which (in a loose sense) may be viewed as the noncoherent form of PSK. In practice, M-ary FSK and M-ary DPSK are the commonly used forms of digital modulation in noncoherent systems.

■ PROBABILITY OF ERROR

A major goal of passband data transmission systems is the optimum design of the receiver so as to minimize the average probability of symbol error in the presence of *additive white Gaussian noise* (AWGN). With this goal in mind, much of the material presented in this chapter builds on the signal-space analysis tools presented in Chapter 5. Specifically, in the study of each system we begin with the formulation of a signal constellation and the construction of decision regions in accordance with maximum likelihood signal detection over an AWGN channel. These formulations set the stage for evaluating the probability of symbol error P_e. Depending on the method of digital modulation under study, the evaluation of P_e proceeds in one of two ways:

- In the case of certain simple methods such as coherent binary PSK and coherent binary FSK, exact formulas are derived for P_e.

Returning to the functional model of Figure 6.2, the bandpass communication channel, coupling the transmitter to the receiver, is assumed to have two characteristics:

1. The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of the modulated signal $s_i(t)$ with negligible or no distortion.
2. The channel noise $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$.

The assumptions made herein are basically the same as those invoked in Chapter 5 dealing with signal-space analysis.

The receiver, which consists of a *detector* followed by a *signal transmission decoder*, performs two functions:

1. It reverses the operations performed in the transmitter.
2. It minimizes the effect of channel noise on the estimate $\hat{m}$ computed for the transmitted symbol m_i.

6.3 *Coherent Phase-Shift Keying*

With the background material on the coherent detection of signals in additive white Gaussian noise that was presented in Chapter 5 at our disposal, we are now ready to study specific passband data transmission systems. In this section we focus on coherent phase-shift keying (PSK) by considering binary PSK, QPSK and its variants, and finish up with *M*-ary PSK.

Binary Phase-Shift Keying

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \tag{6.8}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \tag{6.9}$$

where $0 \le t \le T_b$, and E_b is the *transmitted signal energy per bit*. To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c. A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined in Eqautions (6.8) and (6.9), are referred to as *antipodal signals*.

From this pair of equations it is clear that, in the case of binary PSK, there is only one basis function of unit energy, namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \qquad 0 \le t < T_b \tag{6.10}$$

Then we may express the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as follows:

$$s_1(t) = \sqrt{E_b}\,\phi_1(t), \qquad 0 \le t < T_b \tag{6.11}$$

and

$$s_2(t) = -\sqrt{E_b}\,\phi_1(t), \qquad 0 \le t < T_b \tag{6.12}$$

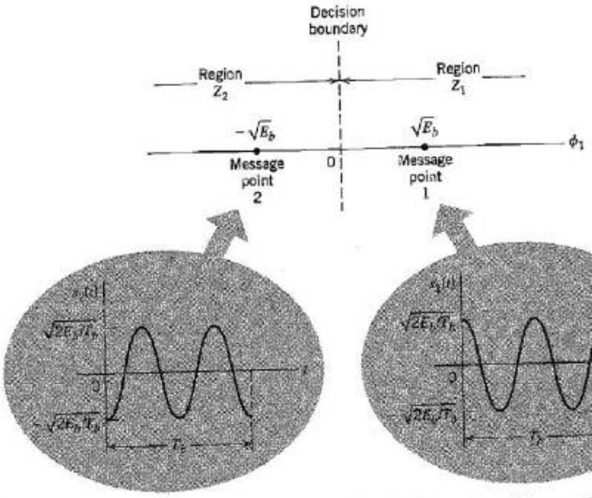

FIGURE 6.3 Signal-space diagram for coherent binary PSK system. The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$, displayed in the inserts, assume $n_c = 2$.

A coherent binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e., $N = 1$), with a signal constellation consisting of two message points (i.e., $M = 2$). The coordinates of the message points are

$$\begin{aligned} s_{11} &= \int_0^{T_b} s_1(t)\phi_1(t)\, dt \\ &= +\sqrt{E_b} \end{aligned} \tag{6.13}$$

and

$$\begin{aligned} s_{21} &= \int_0^{T_b} s_2(t)\phi_1(t)\, dt \\ &= -\sqrt{E_b} \end{aligned} \tag{6.14}$$

The message point corresponding to $s_1(t)$ is located at $s_{11} = +\sqrt{E_b}$, and the message point corresponding to $s_2(t)$ is located at $s_{21} = -\sqrt{E_b}$. Figure 6.3 displays the signal-space diagram for binary PSK. This figure also includes two inserts, showing example waveforms of antipodal signals representing $s_1(t)$ and $s_2(t)$. Note that the constellation of Figure 6.3 has minimum average energy.

Error Probability of Binary PSK

To realize a *rule for making a decision* in favor of symbol 1 or symbol 0, we apply Equation (5.59) of Chapter 5. Specifically, we partition the signal space of Figure 6.3 into two regions:

- The set of points closest to message point 1 at $+\sqrt{E_b}$.
- The set of points closest to message point 2 at $-\sqrt{E_b}$.

This is accomplished by constructing the midpoint of the line joining these two message points, and then marking off the appropriate decision regions. In Figure 6.3 these decision regions are marked Z_1 and Z_2, according to the message point around which they are constructed.

The decision rule is now simply to decide that signal $s_1(t)$ (i.e., binary symbol 1) was transmitted if the received signal point falls in region Z_1, and decide that signal $s_2(t)$ (i.e., binary symbol 0) was transmitted if the received signal point falls in region Z_2. Two kinds of erroneous decisions may, however, be made. Signal $s_2(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_1 and so the receiver decides in favor of signal $s_1(t)$. Alternatively, signal $s_1(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_2 and so the receiver decides in favor of signal $s_2(t)$.

To calculate the probability of making an error of the first kind, we note from Figure 6.3 that the decision region associated with symbol 1 or signal $s_1(t)$ is described by

$$Z_1\colon 0 < x_1 < \infty$$

where the observable element x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt \tag{6.15}$$

The conditional probability density function of random variable X_1, given that symbol 0 [i.e., signal $s_2(t)$] was transmitted, is defined by

$$\begin{aligned} f_{X_1}(x_1|0) &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right] \\ &= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] \end{aligned} \tag{6.16}$$

The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore

$$\begin{aligned} p_{10} &= \int_0^{\infty} f_{X_1}(x_1|0)\,dx_1 \\ &= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 \end{aligned} \tag{6.17}$$

Putting

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b}) \tag{6.18}$$

and changing the variable of integration from x_1 to z, we may rewrite Equation (6.17) in the compact form

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\,dz \\ &= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \end{aligned} \tag{6.19}$$

where erfc(·) is the complementary error function.

Consider next an error of the second kind. We note that the signal space of Figure 6.3 is symmetric with respect to the origin. It follows therefore that p_{01}, the conditional probability of the receiver deciding in favor of symbol 0, given that symbol 1 was transmitted, also has the same value as in Equation (6.19).

Thus, averaging the conditional error probabilities p_{10} and p_{01}, we find that the *average probability of symbol error* or, equivalently, the *bit error rate for coherent binary PSK* is (assuming equiprobable symbols)

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{6.20}$$

As we increase the transmitted signal energy per bit, E_b, for a specified noise spectral density N_0, the message points corresponding to symbols 1 and 0 move further apart, and the average probability of error P_e is correspondingly reduced in accordance with Equation (6.20), which is intuitively satisfying.

Generation and Detection of Coherent Binary PSK Signals

To generate a binary PSK signal, we see from Equations (6.8)–(6.10) that we have to represent the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively. This signal transmission encoding is performed by a polar nonreturn-to-zero (NRZ) level encoder. The resulting binary wave and a sinusoidal carrier $\phi_1(t)$, whose frequency $f_c = (n_c/T_b)$ for some fixed integer n_c, are applied to a product modulator, as in Figure 6.4*a*. The carrier and the timing pulses used to generate the binary wave are usually extracted from a common master clock. The desired PSK wave is obtained at the modulator output.

To detect the original binary sequence of 1s and 0s, we apply the noisy PSK signal $x(t)$ (at the channel output) to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$, as in Figure 6.4*b*. The correlator output, x_1, is compared with a threshold of zero volts. If $x_1 > 0$, the receiver decides in favor of symbol 1. On the

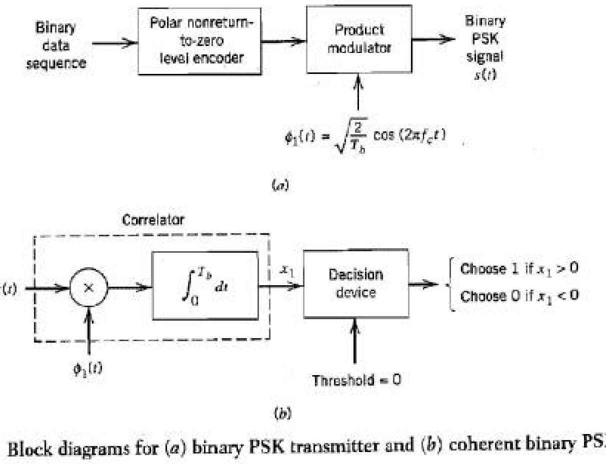

FIGURE 6.4 Block diagrams for (*a*) binary PSK transmitter and (*b*) coherent binary PSK receiver.

■ QUADRIPHASE-SHIFT KEYING

The provision of reliable performance, exemplified by a very low probability of error, is one important goal in the design of a digital communication system. Another important goal is the efficient utilization of channel bandwidth. In this subsection, we study a bandwidth-conserving modulation scheme known as coherent quadriphase-shift keying, which is an example of *quadrature-carrier multiplexing*.

In *quadriphase-shift keying* (QPSK), as with binary PSK, information carried by the transmitted signal is contained in the phase. In particular, the phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$. For this set of values we may define the transmitted signal as

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\dfrac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{6.23}$$

where $i = 1, 2, 3, 4$; E is the transmitted signal energy per symbol, and T is rhe symbol duration. The carrier frequency f_c equals n_c/T for some fixed integer n_c. Each possible value of the phase corresponds to a unique dibit. Thus, for example, we may choose the foregoing set of phase values to represent the *Gray-encoded* set of dibits: 10, 00, 01, and 11, where only a single bit is changed from one dibit to the next.

Signal-Space Diagram of QPSK

Using a well-known trigonometric identity, we may use Equation (6.23) to redefine the transmitted signal $s_i(t)$ for the interval $0 \le t \le T$ in the equivalent form:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t) \tag{6.24}$$

where $i = 1, 2, 3, 4$. Based on this representation, we can make the following observations:

- There are two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$, contained in the expansion of $s_i(t)$. Specifically, $\phi_1(t)$ and $\phi_2(t)$ are defined by a pair of *quadrature carriers*:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T \tag{6.25}$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \le t \le T \tag{6.26}$$

TABLE 6.1 ***Signal-space characterization of QPSK***

Gray-encoded Input Dibit	*Phase of QPSK Signal (radians)*	*Coordinates of Message Points*	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

other hand, if $x_1 < 0$, it decides in favor of symbol 0. If x_1 is exactly zero, the receiver makes a random guess in favor of 0 or 1.

Power Spectra of Binary PSK Signals

From the modulator of Figure 6.4*a*, we see that the complex envelope of a binary PSK wave consists of an in-phase component only. Furthermore, depending on whether we have symbol 1 or symbol 0 at the modulator input during the signaling interval $0 \le t \le T_b$, we find that this in-phase component equals $+g(t)$ or $-g(t)$, respectively, where $g(t)$ is the *symbol shaping function* defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases} \tag{6.21}$$

We assume that the input binary wave is random, with symbols 1 and 0 equally likely and the symbols transmitted during the different time slots being statistically independent. In Example 1.6 of Chapter 1 it is shown that the power spectral density of a random binary wave so described is equal to the energy spectral density of the symbol shaping function divided by the symbol duration. The energy spectral density of a Fourier transformable signal $g(t)$ is defined as the squared magnitude of the signal's Fourier transform. Hence, the baseband power spectral density of a binary PSK signal equals

$$\begin{aligned} S_B(f) &= \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} \\ &= 2E_b \operatorname{sinc}^2(T_b f) \end{aligned} \tag{6.22}$$

This power spectrum falls off as the inverse square of frequency, as shown in Figure 6.5.

Figure 6.5 also includes a plot of the baseband power spectral density of a binary FSK signal, details of which are presented in Section 6.5. Comparison of these two spectra is deferred to that section.

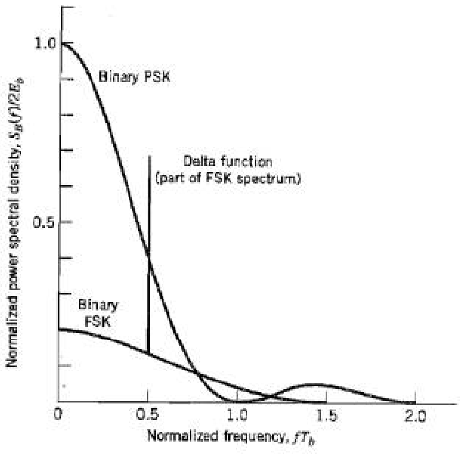

FIGURE 6.5 Power spectra of binary PSK and FSK signals.

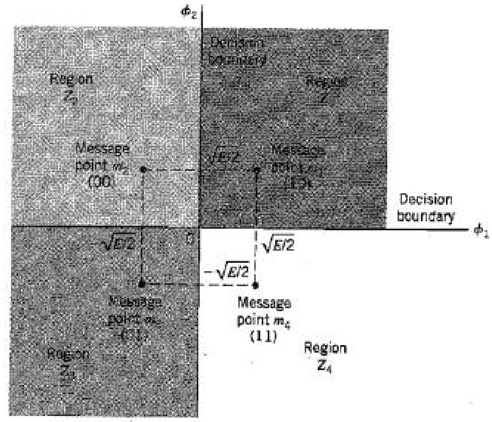

FIGURE 6.6 Signal-space diagram of coherent QPSK system.

- There are four message points, and the associated signal vectors are defined by

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos\left((2i-1)\dfrac{\pi}{4}\right) \\ -\sqrt{E}\sin\left((2i-1)\dfrac{\pi}{4}\right) \end{bmatrix}, \qquad i = 1, 2, 3, 4 \tag{6.27}$$

  The elements of the signal vectors, namely, s_{i1} and s_{i2}, have their values summarized in Table 6.1. The first two columns of this table give the associated dibit and phase of the QPSK signal.

Accordingly, a QPSK signal has a two-dimensional signal constellation (i.e., $N = 2$) and four message points (i.e., $M = 4$) whose phase angles increase in a counterclockwise direction, as illustrated in Figure 6.6. As with binary PSK, the QPSK signal has minimum average energy.

EXAMPLE 6.1

Figure 6.7 illustrates the sequences and waveforms involved in the generation of a QPSK signal. The input binary sequence 01101000 is shown in Figure 6.7*a*. This sequence is divided into two other sequences, consisting of odd- and even-numbered bits of the input sequence. These two sequences are shown in the top lines of Figures 6.7*b* and 6.7*c*. The waveforms representing the two components of the QPSK signal, namely, $s_{i1}\phi_1(t)$ and $s_{i2}\phi_2(t)$, are also shown in Figures 6.7*b* and 6.7*c*, respectively. These two waveforms may individually be viewed as examples of a binary PSK signal. Adding them, we get the QPSK waveform shown in Figure 6.7*d*.

To define the decision rule for the detection of the transmitted data sequence, we partition the signal space into four regions, in accordance with Equation (5.59) of Chapter 5. The individual regions are defined by the set of points closest to the message point represented by signal vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, and $\mathbf{s}_4$. This is readily accomplished by constructing the perpendicular bisectors of the square formed by joining the four message points and then marking

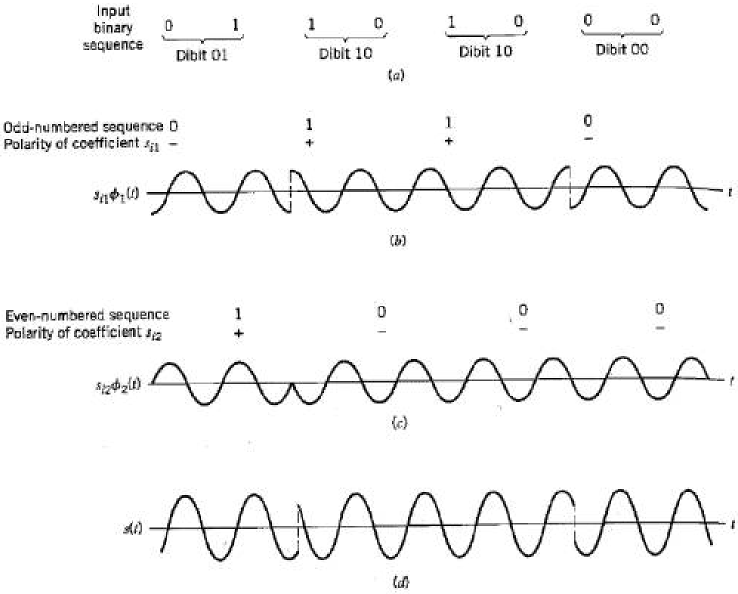

FIGURE 6.7 (*a*) Input binary sequence. (*b*) Odd-numbered bits of input sequence and associated binary PSK wave. (*c*) Even-numbered bits of input sequence and associated binary PSK wave. (*d*) QPSK waveform defined as $s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$.

off the appropriate regions. We thus find that the decision regions are quadrants whose vertices coincide with the origin. These regions are marked Z_1, Z_2, Z_3, and Z_4, in Figure 6.6, according to the message point around which they are constructed. ◀

Error Probability of QPSK

In a coherent QPSK system, the received signal $x(t)$ is defined by

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, 3, 4 \end{cases} \tag{6.28}$$

where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. Correspondingly, the observation vector **x** has two elements, x_1 and x_2, defined by

$$\begin{aligned} x_1 &= \int_0^T x(t)\phi_1(t)\,dt \\ &= \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] + w_1 \\ &= \pm\sqrt{\frac{E}{2}} + w_1 \end{aligned} \tag{6.29}$$

and

$$\begin{aligned} x_2 &= \int_0^T x(t)\phi_2(t)\, dt \\ &= -\sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] + w_2 \\ &= \mp\sqrt{\frac{E}{2}} + w_2 \end{aligned} \tag{6.30}$$

Thus the observable elements x_1 and x_2 are sample values of independent Gaussian random variables with mean values equal to $\pm\sqrt{E/2}$ and $\mp\sqrt{E/2}$, respectively, and with a common variance equal to $N_0/2$.

The decision rule is now simply to decide that $s_1(t)$ was transmitted if the received signal point associated with the observation vector **x** falls inside region Z_1, decide that $s_2(t)$ was transmitted if the received signal point falls inside region Z_2, and so on. An erroneous decision will be made if, for example, signal $s_4(t)$ is transmitted but the noise $w(t)$ is such that the received signal point falls outside region Z_4.

To calculate the average probability of symbol error, we note from Equation (6.24) that a coherent QPSK system is in fact equivalent to two coherent binary PSK systems working in parallel and using two carriers that are in phase quadrature; this is merely a statement of the quadrature-carrier multiplexing property of coherent QPSK. The in-phase channel output x_1 and the quadrature channel output x_2 (i.e., the two elements of the observation vector **x**) may be viewed as the individual outputs of the two coherent binary PSK systems. Thus, according to Equations (6.29) and (6.30), these two binary PSK systems may be characterized as follows:

- The signal energy per bit is $E/2$.
- The noise spectral density is $N_0/2$.

Hence, using Equation (6.20) for the average probability of bit error of a coherent binary PSK system, we may now state that the average probability of bit error in *each* channel of the coherent QPSK system is

$$\begin{aligned} P' &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E/2}{N_0}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned} \tag{6.31}$$

Another important point to note is that the bit errors in the in-phase and quadrature channels of the coherent QPSK system are statistically independent. The in-phase channel makes a decision on one of the two bits constituting a symbol (dibit) of the QPSK signal, and the quadrature channel takes care of the other bit. Accordingly, the *average probability of a correct decision* resulting from the combined action of the two channels working together is

$$\begin{aligned} P_c &= (1-P')^2 \\ &= \left[1 - \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2 \\ &= 1 - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) \end{aligned} \tag{6.32}$$

bandwidth. For a prescribed performance, QPSK uses channel bandwidth better than binary PSK, which explains the preferred use of QPSK over binary PSK in practice.

Generation and Detection of Coherent QPSK Signals

Consider next the generation and detection of QPSK signals. Figure 6.8*a* shows a block diagram of a typical QPSK transmitter. The incoming binary data sequence is first transformed into polar form by a *nonreturn-to-zero level* encoder. Thus, symbols 1 and 0 are represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively. This binary wave is next divided by means of a *demultiplexer* into two separate binary waves consisting of the odd- and even-numbered input bits. These two binary waves are denoted by $a_1(t)$ and $a_2(t)$. We note that in any signaling interval, the amplitudes of $a_1(t)$ and $a_2(t)$ equal s_{i1} and s_{i2}, respectively, depending on the particular dibit that is being transmitted. The two binary waves $a_1(t)$ and $a_2(t)$ are used to modulate a pair of quadrature carriers or orthonormal basis functions: $\phi_1(t)$ equal to $\sqrt{2/T}\cos(2\pi f_c t)$ and $\phi_2(t)$ equal to $\sqrt{2/T}\sin(2\pi f_c t)$. The result is a pair of

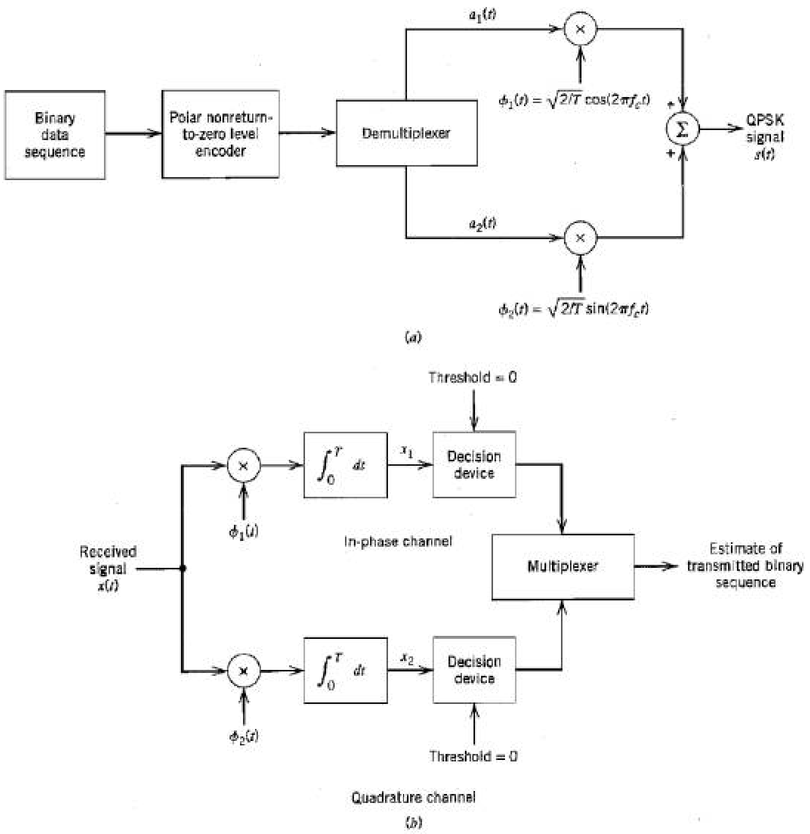

FIGURE 6.8 Block diagrams of (*a*) QPSK transmitter and (*b*) coherent QPSK receiver.

binary PSK signals, which may be detected independently due to the orthogonality of $\phi_1(t)$ and $\phi_2(t)$. Finally, the two binary PSK signals are added to produce the desired QPSK signal.

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$, as in Figure 6.8*b*. The correlator outputs x_1 and x_2, produced in response to the received signal $x(t)$, are each compared with a threshold of zero. If $x_1 > 0$, a decision is made in favor of symbol 1 for the in-phase channel output, but if $x_1 < 0$, a decision is made in favor of symbol 0. Similarly, if $x_2 > 0$, a decision is made in favor of symbol 1 for the quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0. Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in a ***multiplexer*** to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error in an AWGN channel.

Power Spectra of QPSK Signals

Assume that the binary wave at the modulator input is random, with symbols 1 and 0 being equally likely, and with the symbols transmitted during adjacent time slots being statistically independent. We make the following observations pertaining to the in-phase and quadrature components of a QPSK signal:

1. Depending on the dibit sent during the signaling interval $-T_b \le t \le T_b$, the in-phase component equals $+g(t)$ or $-g(t)$, and similarly for the quadrature component. The $g(t)$ denotes the symbol shaping function, defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{E}{T}}, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases} \tag{6.39}$$

Hence, the in-phase and quadrature components have a common power spectral density, namely, $E\,\text{sinc}^2(Tf)$.

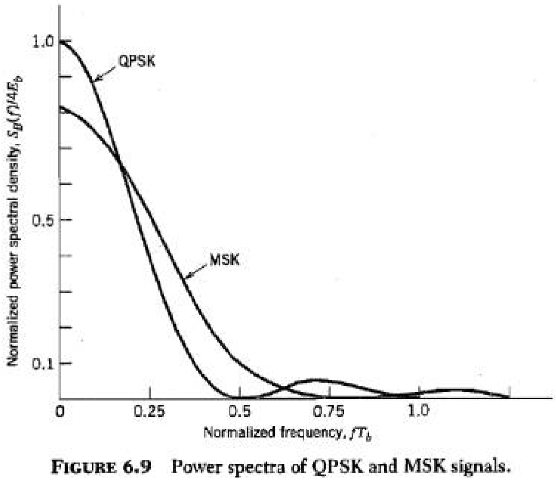

FIGURE 6.9 Power spectra of QPSK and MSK signals.

BINARY FSK

In a *binary FSK system*, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \tag{6.86}$$

where $i = 1, 2$, and E_b is the transmitted signal energy per bit; the transmitted frequency is

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and } i = 1, 2 \tag{6.87}$$

Thus symbol 1 is represented by $s_1(t)$, and symbol 0 by $s_2(t)$. The FSK signal described here is known as *Sunde's FSK*. It is a *continuous-phase signal* in the sense that phase continuity is always maintained, including the inter-bit switching times. This form of digital modulation is an example of *continuous-phase frequency-shift keying* (CPFSK), on which we have more to say later on in the section.

From Equations (6.86) and (6.87), we observe directly that the signals $s_1(t)$ and $s_2(t)$ are orthogonal, but not normalized to have unit energy. We therefore deduce that the most useful form for the set of orthonormal basis functions is

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \tag{6.88}$$

where $i = 1, 2$. Correspondingly, the coefficient s_{ij} for $i = 1, 2$, and $j = 1, 2$ is defined by

$$\begin{aligned} s_{ij} &= \int_0^{T_b} s_i(t)\phi_j(t)\, dt \\ &= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt \\ &= \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \ne j \end{cases} \end{aligned} \tag{6.89}$$

Thus, unlike coherent binary PSK, a coherent binary FSK system is characterized by having a signal space that is two-dimensional (i.e., $N = 2$) with two message points (i.e., $M = 2$), as shown in Figure 6.25. The two message points are defined by the

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \tag{6.90}$$

and

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \tag{6.91}$$

with the Euclidean distance between them equal to $\sqrt{2E_b}$. Figure 6.25 also includes a couple of inserts, which show waveforms representative of signals $s_1(t)$ and $s_2(t)$.

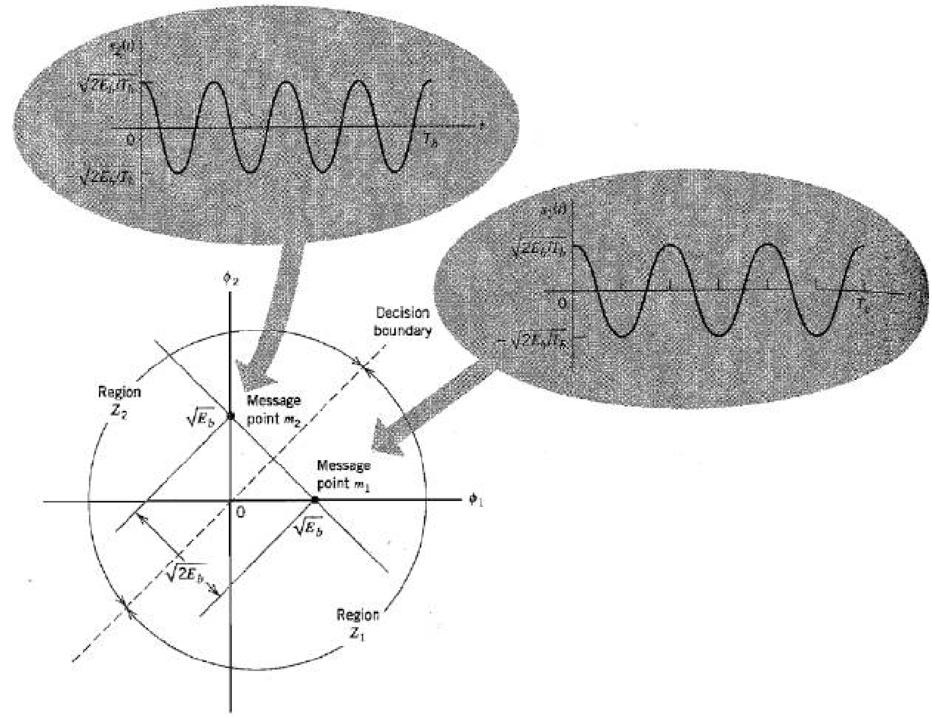

FIGURE 6.25 Signal-space diagram for binary FSK system. The diagram also includes two inserts showing example waveforms of the two modulated signals $s_1(t)$ and $s_2(t)$.

Error Probability of Binary FSK

The observation vector **x** has two elements x_1 and x_2 that are defined by, respectively,

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt \tag{6.92}$$

and

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)\,dt \tag{6.93}$$

where $x(t)$ is the received signal, the form of which depends on which symbol was transmitted. Given that symbol 1 was transmitted, $x(t)$ equals $s_1(t) + w(t)$, where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. If, on the other hand, symbol 0 was transmitted, $x(t)$ equals $s_2(t) + w(t)$.

Now, applying the decision rule of Equation (5.59), we find that the observation space is partitioned into two decision regions, labeled Z_1 and Z_2 in Figure 6.25. The decision boundary, separating region Z_1 from region Z_2 is the perpendicular bisector of the line joining the two message points. The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector $\mathbf{x}$ falls inside region Z_1. This occurs when $x_1 > x_2$. If, on the other hand, we have $x_1 < x_2$, the received signal point falls inside region Z_2, and the receiver decides in favor of symbol 0. On the decision boundary, we have $x_1 = x_2$, in which case the receiver makes a random guess in favor of symbol 1 or 0.

Define a new Gaussian random variable Y whose sample value y is equal to the difference between x_1 and x_2; that is,

$$y = x_1 - x_2 \tag{6.94}$$

The mean value of the random variable Y depends on which binary symbol was transmitted. Given that symbol 1 was transmitted, the Gaussian random variables X_1 and X_2, whose sample values are denoted by x_1 and x_2, have mean values equal to $\sqrt{E_b}$ and zero, respectively. Correspondingly, the conditional mean of the random variable Y, given that symbol 1 was transmitted, is

$$\begin{aligned} E[Y|1] &= E[X_1|1] - E[X_2|1] \\ &= +\sqrt{E_b} \end{aligned} \tag{6.95}$$

On the other hand, given that symbol 0 was transmitted, the random variables X_1 and X_2 have mean values equal to zero and $\sqrt{E_b}$, respectively. Correspondingly, the conditional mean of the random variable Y, given that symbol 0 was transmitted, is

$$\begin{aligned} E[Y|0] &= E[X_1|0] - E[X_2|0] \\ &= -\sqrt{E_b} \end{aligned} \tag{6.96}$$

The variance of the random variable Y is independent of which binary symbol was transmitted. Since the random variables X_1 and X_2 are statistically independent, each with a variance equal to $N_0/2$, it follows that

$$\begin{aligned} \text{var}[Y] &= \text{var}[X_1] + \text{var}[X_2] \\ &= N_0 \end{aligned} \tag{6.97}$$

Suppose we know that symbol 0 was transmitted. The conditional probability density function of the random variable Y is then given by

$$f_Y(y|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(y + \sqrt{E_b})^2}{2N_0}\right] \tag{6.98}$$

Since the condition $x_1 > x_2$, or equivalently, $y > 0$, corresponds to the receiver making a decision in favor of symbol 1, we deduce that the conditional probability of error, given that symbol 0 was transmitted, is

$$\begin{aligned} p_{10} &= P(y > 0 \,|\, \text{symbol 0 was sent}) \\ &= \int_0^\infty f_Y(y|0)\, dy \\ &= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(y + \sqrt{E_b})^2}{2N_0}\right] dy \end{aligned} \tag{6.99}$$

Put

$$\frac{y + \sqrt{E_b}}{\sqrt{2N_0}} = z \tag{6.100}$$

Then, changing the variable of integration from y to z, we may rewrite Equation (6.99) as follows:

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_0}}^{\infty} \exp(-z^2)\, dz \\ &= \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \end{aligned} \tag{6.101}$$

Similarly, we may show the p_{01}, the conditional probability of error given that symbol 1 was transmitted, has the same value as in Equation (6.101). Accordingly, averaging p_{10} and p_{01}, we find that the *average probability of bit error* or, equivalently, the *bit error rate for coherent binary FSK* is (assuming equiprobable symbols)

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \tag{6.102}$$

Comparing Equations (6.20) and (6.102), we see that, in a coherent binary FSK system, we have to double the *bit energy-to-noise density ratio*, E_b/N_0, to maintain the same bit error rate as in a coherent binary PSK system. This result is in perfect accord with the signal-space diagrams of Figures 6.3 and 6.25, where we see that in a binary PSK system the Euclidean distance between the two message points is equal to $2\sqrt{E_b}$, whereas in a binary FSK system the corresponding distance is $\sqrt{2E_b}$. For a prescribed E_b, the minimum distance d_{min} in binary PSK is therefore $\sqrt{2}$ times that in binary FSK. Recall from Chapter 5 that the probability of error decreases exponentially as d_{min}^2, hence the difference between the formulas of Equations (6.20) and (6.102).

Generation and Detection of Coherent Binary FSK Signals

To generate a binary FSK signal, we may use the scheme shown in Figure 6.26*a*. The incoming binary data sequence is first applied to an *on–off level encoder*, at the output of which symbol 1 is represented by a constant amplitude of $\sqrt{E_b}$ volts and symbol 0 is represented by zero volts. By using an *inverter* in the lower channel in Figure 6.26*a*, we in effect make sure that when we have symbol 1 at the input, the oscillator with frequency f_1 in the upper channel is switched on while the oscillator with frequency f_2 in the lower channel is switched off, with the result that frequency f_1 is transmitted. Conversely, when we have symbol 0 at the input, the oscillator in the upper channel is switched off and the oscillator in the lower channel is switched on, with the result that frequency f_2 is transmitted. The two frequencies f_1 and f_2 are chosen to equal different integer multiples of the bit rate $1/T_b$, as in Equation (6.87).

In the transmitter of Figure 6.26*a*, we assume that the two oscillators are synchronized, so that their outputs satisfy the requirements of the two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$, as in Equation (6.88). Alternatively, we may use a single keyed (voltage-controlled) oscillator. In either case, the frequency of the modulated wave is shifted with a continuous phase, in accordance with the input binary wave.

To detect the original binary sequence given the noisy received signal $x(t)$, we may use the receiver shown in Figure 6.26*b*. It consists of two correlators with a common input, which are supplied with locally generated coherent reference signals $\phi_1(t)$ and $\phi_2(t)$. The correlator outputs are then subtracted, one from the other, and the resulting difference, y, is compared with a threshold of zero volts. If $y > 0$, the receiver decides in favor of 1. On the other hand, if $y < 0$, it decides in favor of 0. If y is exactly zero, the receiver makes a random guess in favor of 1 or 0.

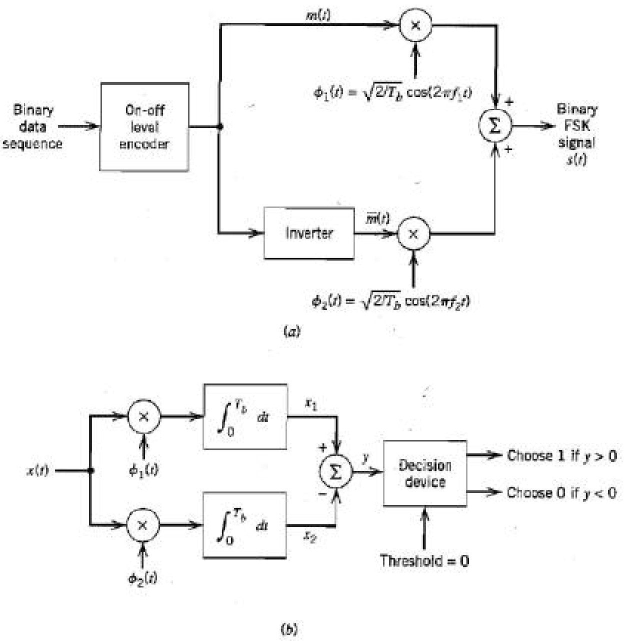

FIGURE 6.26 Block diagrams for (*a*) binary FSK transmitter and (*b*) coherent binary FSK receiver.

Power Spectra of Binary FSK Signals

Consider the case of Sunde's FSK, for which the two transmitted frequencies f_1 and f_2 differ by an amount equal to the bit rate $1/T_b$, and their arithmetic mean equals the nominal carrier frequency f_c; phase continuity is always maintained, including inter-bit switching times. We may express this special binary FSK signal as follows:

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right), \qquad 0 \le t \le T_b \tag{6.103}$$

and using a well-known trigonometric identity, we get

$$\begin{aligned} s(t) &= \sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm\frac{\pi t}{T_b}\right) \sin(2\pi f_c t) \\ &= \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right) \sin(2\pi f_c t) \end{aligned} \tag{6.104}$$

In the last line of Equation (6.104), the plus sign corresponds to transmitting symbol 0, and the minus sign corresponds to transmitting symbol 1. As before, we assume that the symbols 1 and 0 in the random binary wave at the modulator input are equally likely, and that the symbols transmitted in adjacent time slots are statistically independent. Then,