7.3 A Notion of Spread Spectrum

An important attribute of spread-spectrum modulation is that it can provide protection against externally generated interfering (jamming) signals with finite power. The jamming signal may consist of a fairly powerful broadband noise or multitone waveform that is directed at the receiver for the purpose of disrupting communications. Protection against jamming waveforms is provided by purposely making the information-bearing signal occupy a bandwidth far in excess of the minimum bandwidth necessary to transmit it. This has the effect of making the transmitted signal assume a noiselike appearance so as to blend into the background. The transmitted signal is thus enabled to propagate through the channel undetected by anyone who may be listening. We may therefore think of spread spectrum as a method of "camouflaging" the information-bearing signal.

One method of widening the bandwidth of an information-bearing (data) sequence involves the use of *modulation*. Let $\{b_k\}$ denote a binary data sequence, and $\{c_k\}$ denote a pseudo-noise (PN) sequence. Let the waveforms $b(t)$ and $c(t)$ denote their respective polar nonreturn-to-zero representations in terms of two levels equal in amplitude and opposite in polarity, namely, ± 1. We will refer to $b(t)$ as the information-bearing (data) signal, and to $c(t)$ as the PN signal. The desired modulation is achieved by applying the data signal $b(t)$ and the PN signal $c(t)$ to a product modulator or multiplier, as in Figure 7.5*a*. We know from Fourier transform theory that multiplication of two signals produces a signal whose spectrum equals the convolution of the spectra of the two component signals. Thus, if the message signal $b(t)$ is narrowband and the PN signal $c(t)$ is wideband, *the product (modulated) signal $m(t)$ will have a spectrum that is nearly the same as the wideband PN signal*. In other words, in the context of our present application, the PN sequence performs the role of a *spreading code*.

By multiplying the information-bearing signal $b(t)$ by the PN signal $c(t)$, each information bit is "chopped" up into a number of small time increments, as illustrated in the waveforms of Figure 7.6. These small time increments are commonly referred to as *chips*.

For *baseband* transmission, the product signal $m(t)$ represents the *transmitted signal*. We may thus express the transmitted signal as

$$m(t) = c(t)b(t) \tag{7.7}$$

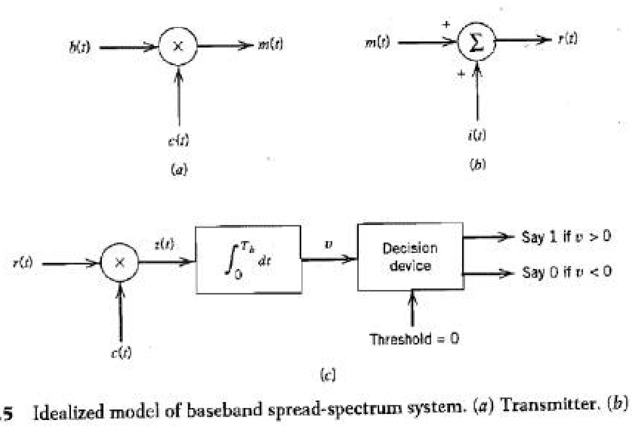
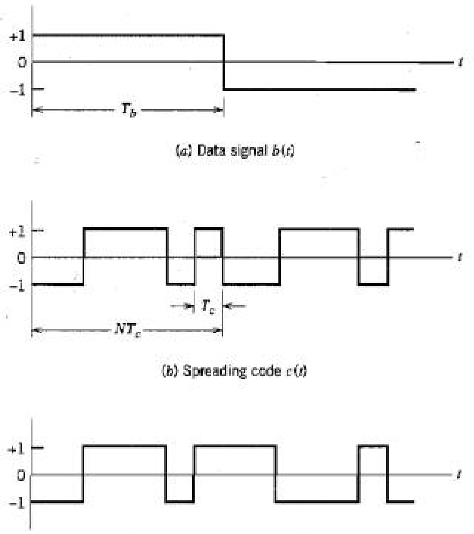


FIGURE 7.5 Idealized model of baseband spread-spectrum system. (*a*) Transmitter. (*b*) Channel. (*c*) Receiver.



(a) Data signal $b(t)$

(b) Spreading code $c(t)$

(c) Product signal $m(t)$

FIGURE 7.6 Illustrating the waveforms in the transmitter of Figure 7.5*a*.

The received signal $r(t)$ consists of the transmitted signal $m(t)$ plus an additive *interference* denoted by $i(t)$, as shown in the channel model of Figure 7.5*b*. Hence,

$$\begin{aligned} r(t) &= m(t) + i(t) \\ &= c(t)b(t) + i(t) \end{aligned} \tag{7.8}$$

To recover the original message signal $b(t)$, the received signal $r(t)$ is applied to a *demodulator* that consists of a multiplier followed by an integrator, and a decision device, as in Figure 7.5*c*. The multiplier is supplied with a locally generated PN sequence that is an exact *replica* of that used in the transmitter. Moreover, we assume that the receiver operates in perfect *synchronism* with the transmitter, which means that the PN sequence in the receiver is lined up exactly with that in the transmitter. The multiplier output in the receiver is therefore given by

$$\begin{aligned} z(t) &= c(t)r(t) \\ &= c^2(t)b(t) + c(t)i(t) \end{aligned} \tag{7.9}$$

Equation (7.9) shows that the data signal $b(t)$ is multiplied *twice* by the PN signal $c(t)$, whereas the unwanted signal $i(t)$ is multiplied only *once*. The PN signal $c(t)$ alternates between the levels -1 and $+1$, and the alternation is destroyed when it is squared; hence,

$$c^2(t) = 1 \qquad \text{for all } t \tag{7.10}$$

Accordingly, we may simplify Equation (7.9) as

$$z(t) = b(t) + c(t)i(t) \tag{7.11}$$

We thus see from Equation (7.11) that the data signal $b(t)$ is reproduced at the multiplier output in the receiver, except for the effect of the interference represented by the additive term $c(t)i(t)$. Multiplication of the interference $i(t)$ by the locally generated PN signal $c(t)$ means that the spreading code will affect the interference just as it did the original signal at the transmitter. We now observe that the data component $b(t)$ is narrowband, whereas the spurious component $c(t)i(t)$ is wideband. Hence, by applying the multiplier output to a baseband (low-pass) filter with a bandwidth just large enough to accommodate the recovery of the data signal $b(t)$, most of the power in the spurious component $c(t)i(t)$ is filtered out. The effect of the interference $i(t)$ is thus significantly reduced at the receiver output.

In the receiver shown in Figure 7.5*c*, the low-pass filtering action is actually performed by the integrator that evaluates the area under the signal produced at the multiplier output. The integration is carried out for the bit interval $0 \le t \le T_b$, providing the sample value v. Finally, a decision is made by the receiver: If v is greater than the threshold of zero, the receiver says that binary symbol 1 of the original data sequence was sent in the interval $0 \le t \le T_b$, and if v is less than zero, the receiver says that symbol 0 was sent; if v is exactly zero the receiver makes a random guess in favor of 1 or 0.

In summary, the use of a spreading code (with pseudo-random properties) in the transmitter produces a wideband transmitted signal that appears *noiselike* to a receiver that has *no* knowledge of the spreading code. From the discussion presented in Section 7.2, we recall that (for a prescribed data rate) the longer we make the period of the spreading code, the closer will the transmitted signal be to a truly random binary wave, and the harder it is to detect. Naturally, the price we have to pay for the improved protection against interference is increased transmission bandwidth, system complexity, and processing delay. However, when our primary concern is the security of transmission, these are not unreasonable costs to pay.

7.4 Direct-Sequence Spread Spectrum with Coherent Binary Phase-Shift Keying

The spread-spectrum technique described in the previous section is referred to as *direct-sequence spread spectrum*. The discussion presented there was in the context of baseband transmission. To provide for the use of this technique in passband transmission over a satellite channel, for example, we may incorporate *coherent binary phase-shift keying* (PSK) into the transmitter and receiver, as shown in Figure 7.7. The transmitter of Figure 7.7*a* first converts the incoming binary data sequence $\{b_k\}$ into a polar NRZ waveform $b(t)$, which is followed by two stages of modulation. The first stage consists of a product modulator or multiplier with the data signal $b(t)$ (representing a data sequence) and the PN signal $c(t)$ (representing the PN sequence) as inputs. The second stage consists of a binary PSK modulator. The transmitted signal $x(t)$ is thus a *direct-sequence spread binary phase-shift-keyed* (DS/BPSK) *signal*. The phase modulation $\theta(t)$ of $x(t)$ has one of two values, 0 and π, depending on the polarities of the message signal $b(t)$ and PN signal $c(t)$ at time t in accordance with the truth table of Table 7.3.

Figure 7.8 illustrates the waveforms for the second stage of modulation. Part of the modulated waveform shown in Figure 7.6*c* is reproduced in Figure 7.8*a*; the waveform shown here corresponds to one period of the PN sequence. Figure 7.8*b* shows the waveform of a sinusoidal carrier, and Figure 7.8*c* shows the DS/BPSK waveform that results from the second stage of modulation.

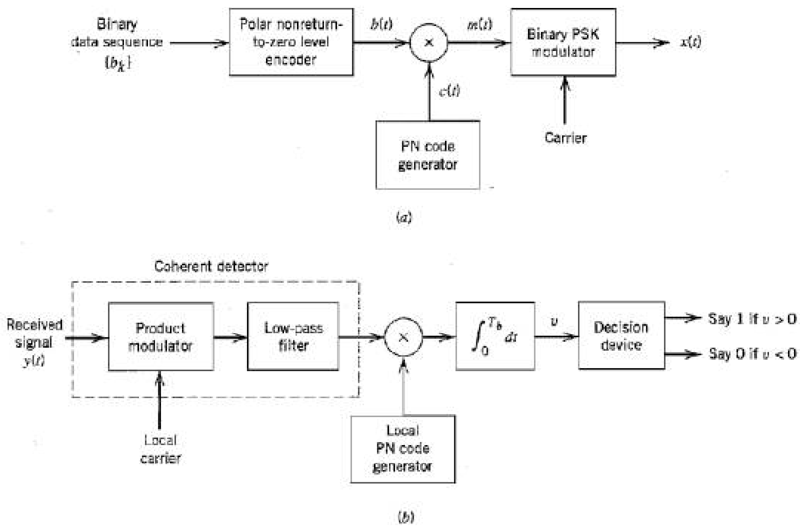


FIGURE 7.7 Direct-sequence spread coherent phase-shift keying. (*a*) Transmitter. (*b*) Receiver.

The receiver, shown in Figure 7.7*b*, consists of two stages of demodulation. In the first stage, the received signal $y(t)$ and a locally generated carrier are applied to a product modulator followed by a low-pass filter whose bandwidth is equal to that of the original message signal $m(t)$. This stage of the demodulation process reverses the phase-shift keying applied to the transmitted signal. The second stage of demodulation performs spectrum despreading by multiplying the low-pass filter output by a locally generated replica of the PN signal $c(t)$, followed by integration over a bit interval $0 \leq t \leq T_b$, and finally decision-making in the manner described in Section 7.3.

MODEL FOR ANALYSIS

In the normal form of the transmitter, shown in Figure 7.7*a*, the spectrum spreading is performed prior to phase modulation. For the purpose of analysis, however, we find it more convenient to interchange the order of these operations, as shown in the model of

TABLE 7.3 *Truth table for phase modulation* $\theta(t)$, *radians*

		Polarity of Data Sequence b(t) *at Time* t	
		+	−
Polarity of PN sequence c(t) *at time* t	+	0	π
	−	π	0

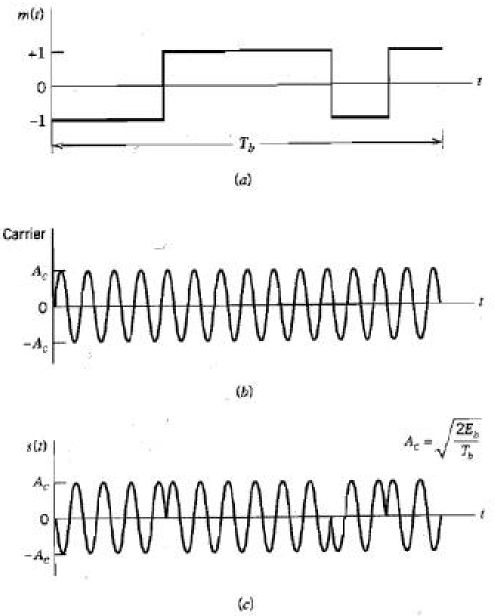


FIGURE 7.8 (*a*) Product signal $m(t) = c(t)b(t)$. (*b*) Sinusoidal carrier. (*c*) DS/BPSK signal.

Figure 7.9. We are permitted to do this because the spectrum spreading and the binary phase-shift keying are both linear operations; likewise for the phase demodulation and spectrum despreading. But for the interchange of operations to be feasible, it is important to synchronize the incoming data sequence and the PN sequence. The model of Figure 7.9 also includes representations of the channel and the receiver. In this model, it is assumed that the interference $j(t)$ limits performance, so that the effect of channel noise may be ignored. Accordingly, the channel output is given by

$$\begin{aligned} y(t) &= x(t) + j(t) \\ &= c(t)s(t) + j(t) \end{aligned} \tag{7.12}$$

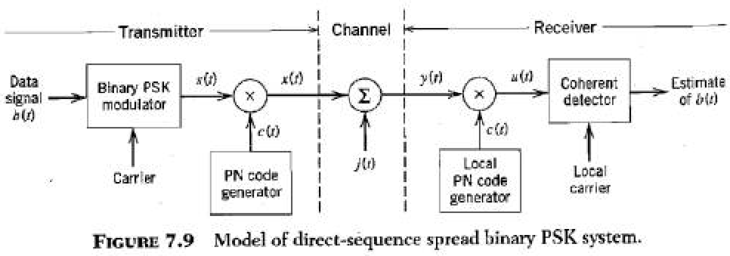


FIGURE 7.9 Model of direct-sequence spread binary PSK system.

where $s(t)$ is the binary PSK signal, and $c(t)$ is the PN signal. In the channel model included in Figure 7.9, the interfering signal is denoted by $j(t)$. This notation is chosen purposely to be different from that used for the interference in Figure 7.5*b*. The channel model in Figure 7.9 is passband in spectral content, whereas that in Figure 7.5*b* is in baseband form.

In the receiver, the received signal $y(t)$ is first multiplied by the PN signal $c(t)$ yielding an output that equals the coherent detector input $u(t)$. Thus,

$$\begin{aligned} u(t) &= c(t)y(t) \\ &= c^2(t)s(t) + c(t)j(t) \\ &= s(t) + c(t)j(t) \end{aligned} \tag{7.13}$$

In the last line of Equation (7.13), we have noted that, by design, the PN signal $c(t)$ satisfies the property described in Equation (7.10), reproduced here for convenience:

$$c^2(t) = 1 \qquad \text{for all } t$$

Equation (7.13) shows that the coherent detector input $u(t)$ consists of a binary PSK signal $s(t)$ embedded in additive code-modulated interference denoted by $c(t)j(t)$. The modulated nature of the latter component forces the interference signal (jammer) to spread its spectrum such that the detection of information bits at the receiver output is afforded increased reliability.

■ SYNCHRONIZATION

For its proper operation, a spread-spectrum communication system requires that the locally generated PN sequence used in the receiver to despread the received signal be *synchronized* to the PN sequence used to spread the transmitted signal in the transmitter.[4] A solution to the synchronization problem consists of two parts: *acquisition* and *tracking*. In acquisition, or *coarse* synchronization, the two PN codes are aligned to within a fraction of the chip in as short a time as possible. Once the incoming PN code has been acquired, tracking, or *fine* synchronization, takes place. Typically, PN acquisition proceeds in two steps. First, the received signal is multiplied by a locally generated PN code to produce a measure of *correlation* between it and the PN code used in the transmitter. Next, an appropriate *decision-rule and search strategy* is used to process the measure of correlation so obtained to determine whether the two codes are in synchronism and what to do if they are not. As for tracking, it is accomplished using phase-lock techniques very similar to those used for the local generation of coherent carrier references. The principal difference between them lies in the way in which phase discrimination is implemented.

7.5 *Signal-Space Dimensionality and Processing Gain*

Having developed a conceptual understanding of spread-spectrum modulation and a method for its implementation, we are ready to undertake a detailed mathematical analysis of the technique. The approach we have in mind is based on the signal-space theoretic ideas of Chapter 5. In particular, we develop signal-space representations of the transmitted signal and the interfering signal (jammer).

tation given in Equation (7.17), we may express the average power of the interference $j(t)$ as follows:

$$\begin{aligned} J &= \frac{1}{T_b}\int_0^{T_b} j^2(t)\,dt \\ &= \frac{1}{T_b}\sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b}\sum_{k=0}^{N-1} \tilde{j}_k^2 \end{aligned} \tag{7.20}$$

Moreover, due to lack of knowledge of signal phase, the best strategy a jammer can apply is to place equal energy in the cosine and sine coordinates defined in Equations (7.18) and (7.19); hence, we may safely assume

$$\sum_{k=0}^{N-1} j_k^2 = \sum_{k=0}^{N-1} \tilde{j}_k^2 \tag{7.21}$$

Correspondingly, we may simplify Equation (7.20) as

$$J = \frac{2}{T_b}\sum_{k=0}^{N-1} j_k^2 \tag{7.22}$$

Our aim is to tie these results together by finding the signal-to-noise ratios measured at the input and output of the DS/BPSK receiver in Figure 7.9. To that end, we use Equation (7.13) to express the coherent detector output as

$$\begin{aligned} v &= \sqrt{\frac{2}{T_b}}\int_0^{T_b} u(t)\cos(2\pi f_c t)\,dt \\ &= v_s + v_{cj} \end{aligned} \tag{7.23}$$

where the components v_s and v_{cj} are due to the despread binary PSK signal, $s(t)$, and the spread interference, $c(t)j(t)$, respectively. These two components are defined as follows:

$$v_s = \sqrt{\frac{2}{T_b}}\int_0^{T_b} s(t)\cos(2\pi f_c t)\,dt \tag{7.24}$$

and

$$v_{cj} = \sqrt{\frac{2}{T_b}}\int_0^{T_b} c(t)j(t)\cos(2\pi f_c t)\,dt \tag{7.25}$$

Consider first the component v_s due to the signal. The despread binary PSK signal $s(t)$ equals

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t), \qquad 0 \le t \le T_b \tag{7.26}$$

where the plus sign corresponds to information bit 1, and the minus sign corresponds to information bit 0. Hence, assuming that the carrier frequency f_c is an integer multiple of $1/T_b$, we have

$$v_s = \pm\sqrt{E_b} \tag{7.27}$$

Consider next the component v_{cj} due to interference. Expressing the PN signal $c(t)$ in the explicit form of a sequence, $\{c_0, c_1, \ldots, c_{N-1}\}$, we may rewrite Equation (7.25) in the corresponding form

$$v_{cj} = \sqrt{\frac{2}{T_b}} \sum_{k=0}^{N-1} c_k \int_{kT_c}^{(k+1)T_c} j(t) \cos(2\pi f_c t)\, dt \tag{7.28}$$

Using Equation (7.14) for $\phi_k(t)$, and then Equation (7.18) for the coefficient j_k, we may redefine v_{cj} as

$$\begin{aligned} v_{cj} &= \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k \int_0^{T_b} j(t)\phi_k(t)\, dt \\ &= \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} c_k j_k \end{aligned} \tag{7.29}$$

We next approximate the PN sequence as an *independent and identically distributed (i.i.d.) binary sequence.* We emphasize the implication of this approximation by recasting Equation (7.29) in the form

$$V_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} C_k j_k \tag{7.30}$$

where V_{cj} and C_k are random variables with sample values v_{cj} and c_k, respectively. In Equation (7.30), the jammer is assumed to be fixed. With the C_k treated as i.i.d. random variables, we find that the probability of the event $C_k = \pm 1$ is

$$P(C_k = 1) = P(C_k = -1) = \tfrac{1}{2} \tag{7.31}$$

Accordingly, the mean of the random variable V_{cj} is zero since, for fixed k, we have

$$\begin{aligned} E[C_k j_k \mid j_k] &= j_k P(C_k = 1) - j_k P(C_k = -1) \\ &= \tfrac{1}{2} j_k - \tfrac{1}{2} j_k \\ &= 0 \end{aligned} \tag{7.32}$$

For a fixed vector $\mathbf{j}$, representing the set of coefficients $j_0, j_1, \ldots, j_{N-1}$, the variance of V_{cj} is given by

$$\text{var}[V_{cj} \mid \mathbf{j}] = \frac{1}{N} \sum_{k=0}^{N-1} j_k^2 \tag{7.33}$$

Since the *spread factor* $N = T_b/T_c$, we may use Equation (7.22) to express this variance in terms of the average interference power J as

$$\text{var}[V_{cj} \mid \mathbf{j}] = \frac{JT_c}{2} \tag{7.34}$$

Thus the random variable V_{cj} has zero mean and variance $JT_c/2$.

From Equation (7.27), we note that the signal component at the coherent detector output (during each bit interval) equals $\pm\sqrt{E_b}$, where E_b is the signal energy per bit. Hence, the peak instantaneous power of the signal component is E_b. Accordingly, we may define the *output signal-to-noise ratio* as the instantaneous peak power E_b divided by the variance of the equivalent noise component in Equation (7.34). We thus write

$$(\text{SNR})_O = \frac{2E_b}{JT_c} \tag{7.35}$$

The average signal power at the receiver input equals E_b/T_b. We thus define an *input signal-to-noise* ratio as

$$(\text{SNR})_I = \frac{E_b/T_b}{J} \tag{7.36}$$

Hence, eliminating E_b/J between Equations (7.35) and (7.36), we may express the output signal-to-noise ratio in terms of the input signal-to-noise ratio as

$$(\text{SNR})_O = \frac{2T_b}{T_c}(\text{SNR})_I \tag{7.37}$$

It is customary practice to express signal-to-noise ratios in decibels. To that end, we introduce a term called the *processing gain* (PG), which is defined as *the gain in SNR obtained by the use of spread spectrum*. Specifically, we write

$$\text{PG} = \frac{T_b}{T_c} \tag{7.38}$$

which represents the gain achieved by processing a spread-spectrum signal over an unspread signal. We may thus write Equation (7.37) in the equivalent form:

$$10 \log_{10}(\text{SNR})_O = 10 \log_{10}(\text{SNR})_I + 3 + 10 \log_{10}(\text{PG}) \text{ dB} \tag{7.39}$$

The 3-dB term on the right-hand side of Equation (7.39) accounts for the gain in SNR that is obtained through the use of coherent detection (which presumes exact knowledge of the signal phase by the receiver). This gain in SNR has nothing to do with the use of spread spectrum. Rather, it is the last term, $10 \log_{10}(\text{PG})$, that accounts for the processing gain. Note that both the processing gain PG and the spread factor N (i.e., PN sequence length) equal the ratio T_b/T_c. Thus, the longer we make the PN sequence (or, correspondingly, the smaller the chip time T_c is), the larger will the processing gain be.

7.6 *Probability of Error*

Let the coherent detector output v in the direct-sequence spread BPSK system of Figure 7.9 denote the sample value of a random variable V. Let the equivalent noise component v_{cj} produced by external interference denote the sample value of a random variable V_{cj}. Then, from Equations (7.23) and (7.27) we deduce that

$$V = \pm\sqrt{E_b} + V_{cj} \tag{7.40}$$

where E_b is the transmitted signal energy per bit. The plus sign refers to sending symbol (information bit) 1, and the minus sign refers to sending symbol 0. The decision rule used by the coherent detector of Figure 7.9 is to declare that the received bit in an interval (0, T_b) is 1 if the detector output exceeds a threshold of zero, and that it is 0 if the detector output is less than the threshold; if the detector output is exactly zero, the receiver makes a random guess in favor of 1 or 0. With both information bits assumed equally likely, we find that (because of the symmetric nature of the problem) the average probability of error P_e is the same as the conditional probability of (say) the receiver making a decision in favor of symbol 1, given that symbol 0 was sent. That is,

$$\begin{aligned} P_e &= P(V > 0 \mid \text{symbol 0 was sent}) \\ &= P(V_{cj} > \sqrt{E_b}) \end{aligned} \tag{7.41}$$

Naturally, the probability of error P_e depends on the random variable V_{cj} defined by Equation (7.30). According to this definition, V_{cj} is the sum of N identically distributed random variables. Hence, from the *central limit theorem*, we deduce that for large N, the random variable V_{cj} assumes a Gaussian distribution. Indeed, the spread factor or PN sequence length N is typically large in the direct-sequence spread-spectrum systems encountered in practice, under which condition the application of the central limit theorem is justified.

Earlier we evaluated the mean and variance of V_{cj}; see Equations (7.32) and (7.34). We may therefore state that the equivalent noise component V_{cj} contained in the coherent detector output may be approximated as a Gaussian random variable with zero mean and variance $JT_c/2$, where J is the average interference power and T_c is the chip duration. With this approximation at hand, we may then proceed to calculate the probability of the event $V_{cj} > \sqrt{E_b}$, and thus express the average probability of error in accordance with Equation (7.41) as

$$P_e \simeq \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right) \tag{7.42}$$

This simple formula, which invokes the Gaussian assumption, is appropriate for DS/BPSK binary systems with large spread factor N.

■ ANTIJAM CHARACTERISTICS

It is informative to compare Equation (7.42) with the formula for the average probability of error for a coherent binary PSK system reproduced here for convenience of presentation [see Equation (6.20)]

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{7.43}$$

Based on this comparison, we see that insofar as the calculation of bit error rate in a direct-sequence spread binary PSK system is concerned, the interference may be treated as wideband noise of power spectral density $N_0/2$, defined by

$$\frac{N_0}{2} = \frac{JT_c}{2} \tag{7.44}$$

This relation is simply a restatement of an earlier result given in Equation (7.34).

Since the signal energy per bit $E_b = PT_b$, where P is the average signal power and T_b is the bit duration, we may express the signal energy per bit-to-noise spectral density ratio as

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right) \tag{7.45}$$

Using the definition of Equation (7.38) for the processing gain PG we may reformulate this result as

$$\frac{J}{P} = \frac{\text{PG}}{E_b/N_0} \tag{7.46}$$

The ratio J/P is termed the *jamming margin*. Accordingly, the jamming margin and the processing gain, both expressed in decibels, are related by

$$(\text{Jamming margin})_{\text{dB}} = (\text{Procesing gain})_{\text{dB}} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{\text{min}} \tag{7.47}$$

where $(E_b/N_0)_{\text{min}}$ is the minimum value needed to support a prescribed average probability of error.

EXAMPLE 7.3

A spread-spectrum communication system has the following parameters:

$$\text{Information bit duration, } T_b = 4.095 \text{ ms}$$
$$\text{PN chip duration, } T_c = 1\ \mu\text{s}$$

Hence, using Equation (7.38) we find that the processing gain is

$$\text{PG} = 4095$$

Correspondingly, the required period of the PN sequence is $N = 4095$, and the shift-register length is $m = 12$.

For a satisfactory reception, we may assume that the average probability of error is not to exceed 10^{-5}. From the formula for a coherent binary PSK receiver, we find that $E_b/N_0 = 10$ yields an average probability of error equal to 0.387×10^{-5}. Hence, using this value for E_b/N_0, and the value calculated for the processing gain, we find from Equation (7.47) that the jamming margin is

$$\begin{aligned}(\text{Jamming margin})_{\text{dB}} &= 10\log_{10} 4095 - 10\log_{10}(10)\\ &= 36.1 - 10\\ &= 26.1 \text{ dB}\end{aligned}$$

That is, information bits at the receiver output can be detected reliably even when the noise or interference at the receiver input is up to 409.5 times the received signal power. Clearly, this is a powerful advantage against interference (jamming), which is realized through the clever use of spread-spectrum modulation.

7.7 *Frequency-Hop Spread Spectrum*

In the type of spread-spectrum systems discussed in Section 7.4, the use of a PN sequence to modulate a phase-shift-keyed signal achieves *instantaneous* spreading of the transmission bandwidth. The ability of such a system to combat the effects of jammers is determined by the processing gain of the system, which is a function of the PN sequence period. The processing gain can be made larger by employing a PN sequence with narrow chip duration, which, in turn, permits a greater transmission bandwidth and more chips per bit. However, the capabilities of physical devices used to generate the PN spread-spectrum signals impose a practical limit on the attainable processing gain. Indeed, it may turn out that the processing gain so attained is still not large enough to overcome the effects of some jammers of concern, in which case we have to resort to other methods. One such alternative method is to force the jammer to cover a wider spectrum by *randomly hopping* the data-modulated carrier from one frequency to the next. In effect, the spectrum of the transmitted signal is spread *sequentially* rather than instantaneously; the term "sequentially" refers to the pseudo-random-ordered sequence of frequency hops.

The type of spread spectrum in which the carrier hops randomly from one frequency to another is called *frequency-hop (FH) spread spectrum*. A common modulation format for FH systems is that of *M-ary frequency-shift keying* (MFSK). The combination of these two techniques is referred to simply as FH/MFSK. (A description of M-ary FSK is presented in Chapter 6.)

Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to consider the rate at which the hops occur. In this context, we may identify two basic (technology-independent) characterizations of frequency hopping:

1. *Slow-frequency hopping*, in which the *symbol rate* R_s of the MFSK signal is an integer multiple of the *hop rate* R_h. That is, several symbols are transmitted on each frequency hop.
2. *Fast-frequency hopping*, in which the hop rate R_h is an integer multiple of the MFSK symbol rate R_s. That is, the carrier frequency will change or hop several times during the transmission of one symbol.

Obviously, slow-frequency hopping and fast-frequency hopping are the converse of one another. In the following, these two characterizations of frequency hopping are considered in turn.

▪ SLOW-FREQUENCY HOPPING

Figure 7.10*a* shows the block diagram of an FH/MFSK transmitter, which involves *frequency modulation* followed by *mixing*. First, the incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and the output from a digital *frequency synthesizer* are then applied to a mixer that consists of a multiplier followed by a band-pass filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. In particular, successive k-bit segments of a PN sequence drive the frequency synthesizer, which enables the carrier frequency to hop over 2^k distinct values. On a single hop, the bandwidth of the transmitted signal is the same as that resulting from the use of a conventional MFSK with an alphabet of $M = 2^K$ orthogonal signals. However, for a complete range of 2^k frequency hops, the transmitted FH/MFSK signal occupies a much larger bandwidth. Indeed, with present-day technology, FH bandwidths on the order of several GHz are attainable, which is an order of magnitude larger than that achievable with direct-sequence spread spectra. An implication of these large FH bandwidths is that coherent detection is possible only within each hop, because frequency synthesizers are unable to maintain phase coherence over successive hops. Accordingly, most frequency-hop spread-spectrum communication systems use noncoherent M-ary modulation schemes.

In the receiver depicted in Figure 7.10*b*, the frequency hopping is first removed by *mixing* (down-converting) the received signal with the output of a local frequency synthesizer that is synchronously controlled in the same manner as that in the transmitter. The resulting output is then band-pass filtered, and subsequently processed by a *noncoherent* M-ary FSK detector. To implement this M-ary detector, we may use a bank of M noncoherent matched filters, each of which is matched to one of the MFSK tones. (Noncoherent matched filters are described in Chapter 6.) An estimate of the original symbol transmitted is obtained by selecting the largest filter output.

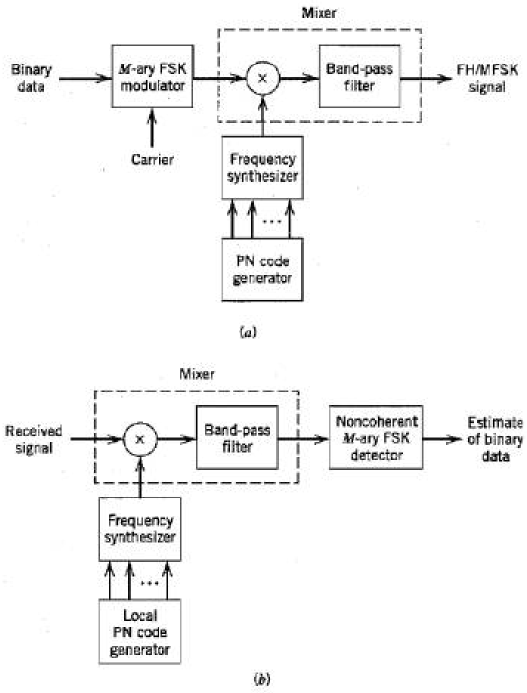


FIGURE 7.10 Frequency-hop spread M-ary frequency-shift keying. (*a*) Transmitter. (*b*) Receiver.

An individual FH/MFSK tone of shortest duration is referred to as a *chip*; this terminology should not be confused with that used in Section 7.4 describing DS/BPSK. The *chip rate*, R_c, for an FH/MFSK system is defined by

$$R_c = \max(R_h, R_s) \tag{7.48}$$

where R_h is the *hop rate*, and R_s is the *symbol rate*.

A slow FH/MFSK signal is characterized by having multiple symbols transmitted per hop. Hence, each symbol of a slow FH/MFSK signal is a chip. Correspondingly, in a slow FH/MFSK system, the bit rate R_b of the incoming binary data, the symbol rate R_s of the MFSK signal, the chip rate R_c, and the hop rate R_h are related by

$$R_c = R_s = \frac{R_b}{K} \geq R_h \tag{7.49}$$

where $K = \log_2 M$.

At each hop, the MFSK tones are separated in frequency by an integer multiple of the chip rate $R_c = R_s$, ensuring their orthogonality. The implication of this condition is that any transmitted symbol will not produce any crosstalk in the other $M - 1$ noncoherent matched filters constituting the MFSK detector of the receiver in Figure 7.10*b*. By "crosstalk" we mean the spillover from one filter output into an adjacent one. The resulting performance of the slow FH/MFSK system is the same as that for the noncoherent detection of conventional (unhopped) MFSK signals in additive white Gaussian noise. Thus the interfering (jamming) signal has an effect on the FH/MFSK receiver, in terms of average probability of symbol error, equivalent to that of additive white Gaussian noise on a conventional noncoherent M-ary FSK receiver experiencing no interference. On the basis of this equivalence, we may use Equation (6.140) for approximate evaluation of the probability of symbol error in the FH/MFSK system.

Assuming that the jammer decides to spread its average power J over the entire frequency-hopped spectrum, the jammer's effect is equivalent to an AWGN with power spectral density $N_0/2$, where $N_0 = J/W_c$ and W_c is the FH bandwidth. The spread-spectrum system is thus characterized by the *symbol energy-to-noise spectral density ratio*:

$$\frac{E}{N_0} = \frac{P/J}{W_c/R_s} \tag{7.50}$$

where the ratio P/J is the reciprocal of the jamming margin. The other ratio in the denominator of Equation (7.50) is the processing gain of the slow FH/MFSK system, which is defined by

$$\begin{aligned} \text{PG} &= \frac{W_c}{R_s} \\ &= 2^k \end{aligned} \tag{7.51}$$

That is, the processing gain (expressed in decibels) is equal to $10 \log_{10} 2^k \simeq 3k$, where k is the length of the PN segment employed to select a frequency hop.

This result assumes that the jammer spreads its power over the entire FH spectrum. However, if the jammer decides to concentrate on just a few of the hopped frequencies, then the processing gain realized by the receiver would be less than $3k$ decibels.

▶ EXAMPLE 7.4

Figure 7.11*a* illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is $2^4 - 1 = 15$. The FH/MFSK signal has the following parameters:

Number of bits per MFSK symbol	$K = 2$
Number of MFSK tones	$M = 2^K = 4$
Length of PN segment per hop	$k = 3$
Total number of frequency hops	$2^k = 8$

In this example, the carrier is hopped to a new frequency after transmitting two symbols or equivalently, four information bits. Figure 7.11*a* also includes the input binary data, and the PN sequence controlling the selection of FH carrier frequency. It is noteworthy that although there are eight distinct frequencies available for hopping, only three of them are utilized by the PN sequence.

Figure 7.11*b* shows the variation of the dehopped frequency with time. This variation is recognized to be the same as that of a conventional MFSK signal produced by the given input data. ◀

■ FAST-FREQUENCY HOPPING

A fast FH/MFSK system differs from a slow FH/MFSK system in that there are multiple hops per M-ary symbol. Hence, in a fast FH/MFSK system, each hop is a chip. In general,

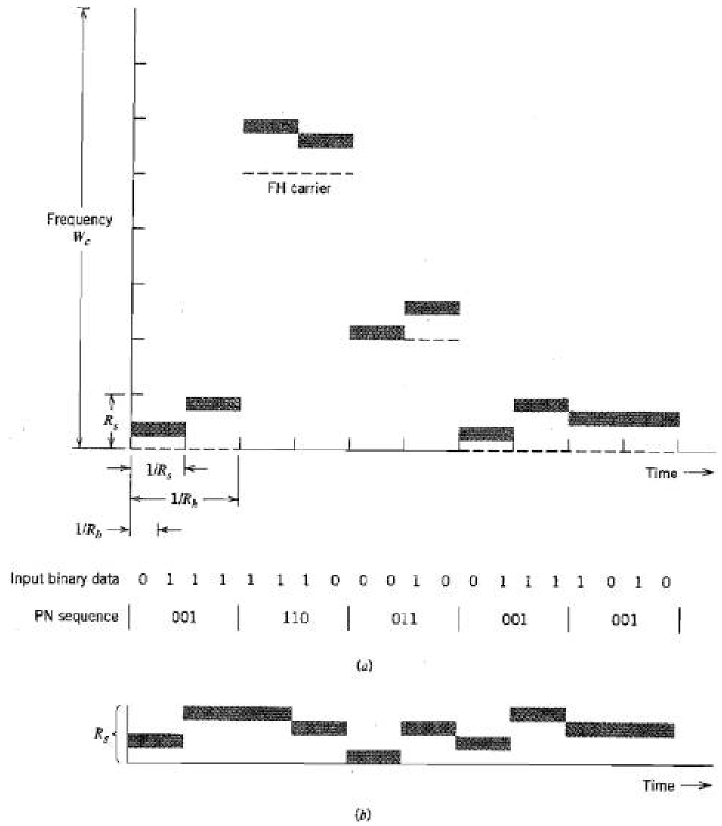


FIGURE 7.11 Illustrating slow-frequency hopping. (*a*) Frequency variation for one complete period of the PN sequence. (*b*) Variation of the dehopped frequency with time.

fast-frequency hopping is used to defeat a smart jammer's tactic that involves two functions: measurements of the spectral content of the transmitted signal, and retuning of the interfering signal to that portion of the frequency band. Clearly, to overcome the jammer, the transmitted signal must be hopped to a new carrier frequency *before* the jammer is able to complete the processing of these two functions.

For data recovery at the receiver, noncoherent detection is used. However, the detection procedure is quite different from that used in a slow FH/MFSK receiver. In particular, two procedures may be considered:

1. For each FH/MFSK symbol, separate decisions are made on the K frequency-hop chips received, and a simple rule based on *majority vote* is used to make an estimate of the dehopped MFSK symbol.
2. For each FH/MFSK symbol, likelihood functions are computed as functions of the total signal received over K chips, and the largest one is selected.

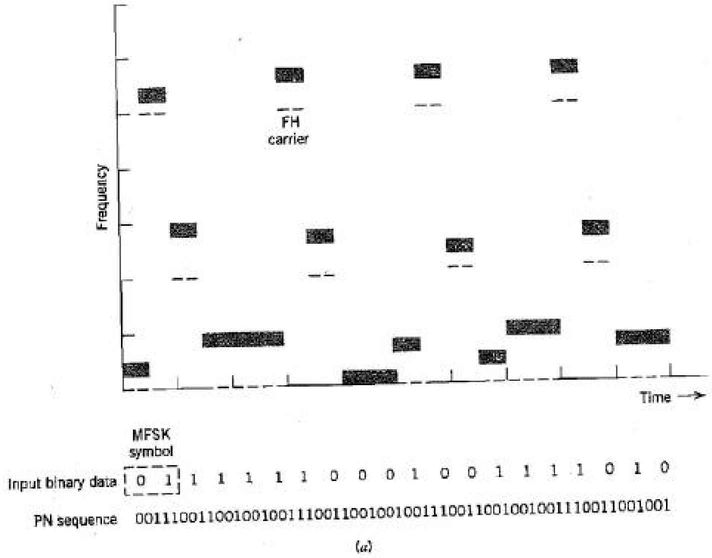
A receiver based on the second procedure is optimum in the sense that it minimizes the average probability of symbol error for a given E_b/N_0.

▶ EXAMPLE 7.5

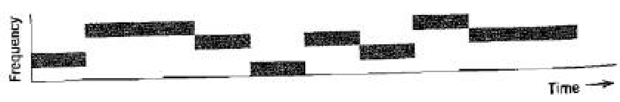
Figure 7.12*a* illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time. The signal has the following parameters:

Number of bits per MFSK symbol	$K = 2$
Number of MFSK tones	$M = 2^K = 4$
Length of PN segment per hop	$k = 3$
Total number of frequency hops	$2^k = 8$

In this example, each MFSK symbol has the same number of bits and chips; that is, the chip rate R_c is the same as the bit rate R_b. After each chip, the carrier frequency of the transmitted MFSK signal is hopped to a different value, except for few occasions when the k-chip segment of the PN sequence repeats itself.



(*a*)



(*b*)

FIGURE 7.12 Illustrating fast-frequency hopping. (*a*) Variation of the transmitter frequency with time. (*b*) Variation of the dehopped frequency with time.

Figure 7.12*b* depicts the time variation of the frequency of the dehopped MFSK signal, which is the same as that in Example 7.4.

7.8 Computer Experiments: Maximal-Length and Gold Codes

Code-division multiplexing (CDM) provides an alternative to the traditional methods of frequency-division multiplexing (FDM) and time-division multiplexing (TDM). It does not require the bandwidth allocation of FDM (discussed in Chapter 2) nor the time synchronization needed in TDM (discussed in Chapter 3). Rather, users of a common channel are permitted access to the channel through the assignment of a "spreading code" to each individual user under the umbrella of spread-spectrum modulation. The purpose of this computer experiment is to study a certain class of spreading codes for CDM systems that provide a satisfactory performance.

In an ideal CDM system, the cross-correlation between any two users of the system is zero. For this ideal condition to be realized, we require that the cross-correlation function between the spreading codes assigned to any two users of the system be zero for all cyclic shifts. Unfortunately, ordinary PN sequences do not satisfy this requirement because of their relatively poor cross-correlation properties.

As a remedy for this shortcoming of ordinary PN sequences, we may use a special class of PN sequences called *Gold sequences (codes)*,[5] the generation of which is embodied in the following theorem:

> Let $g_1(X)$ and $g_2(X)$ be a preferred pair of primitive polynomials of degree n whose corresponding shift registers generate maximal-length sequences of period $2^n - 1$ and whose cross-correlation function has a magnitude less than or equal to
>
> $$2^{(n+1)/2} + 1 \qquad \text{for } n \text{ odd} \tag{7.52}$$
>
> or
>
> $$2^{(n+2)/2} + 1 \qquad \text{for } n \text{ even and } n \neq 0 \bmod 4 \tag{7.53}$$
>
> Then the shift register corresponding to the product polynomial $g_1(X) \cdot g_2(X)$ will generate $2^n + 1$ different sequences, with each sequence having a period of $2^n - 1$, and the cross-correlation between any pair of such sequences satisfying the preceding condition.

Hereafter, this theorem is referred to as *Gold's theorem*.

To understand Gold's theorem, we need to define what we mean by a primitive polynomial. Consider a polynomial $g(X)$ defined over a *binary field* (i.e., a finite set of two elements, 0 and 1, which is governed by the rules of binary arithmetic). The polynomial $g(X)$ is said to be an *irreducible polynomial* if it cannot be factored using any polynomials from the binary field. An irreducible polynomial $g(X)$ of degree m is said to be a *primitive polynomial* if the smallest integer m for which the polynomial $g(X)$ divides the factor $X^n + 1$ is $n = 2^m - 1$. Further discussion of this topic is deferred to Chapter 8; in particular, see Example 8.3.