MODULE-4

EC 368-ROBOTICS

Overview

MARIAN ENGINEERING COLLEGE — Page No: 1

Module IV

Forward Kinematics :- Link Coordinates, D-H repn, Appln of DH convention to different serial kinematic arrangements fitted with Spherical wrist.

Inverse Kinematics :- General properties of solutions, Kinematic Decoupling, Inverse kinematic solutions for all basic types of three link robotic arms fitted with a spherical wrist.

No: of hours assigned : 9hrs

No : of hours taken : 7hrs

Forward kinematics : Determines the position and orientation of the end-effector given the values for the joint variables of the robot.

Robot manipulator – set of links connected together by joints.

Revolute, Prismatic | Ball & Socket joint | Spherical Wrist

[single DOF] [2 DOF] [3 DOF]

Revolute → angle of rotation; Prismatic → displacement

Links – 0 to n

Joints – 1 to n

Joint variable denoted by q_i

$q_i = \theta_i$ if joint, i is revolute

$= d_i$ if " prismatic.

$$H = \begin{bmatrix} R_n^0 & O_n^0 \\ 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} R_i^{i-1} & O_i^{i-1} \\ 0 & 1 \end{bmatrix}$$ Homogeneous Transformation.

D-H convention

- Rot(z,θi)* Trans(z,di)*Trans(x,ai)*Rot(x,αi)

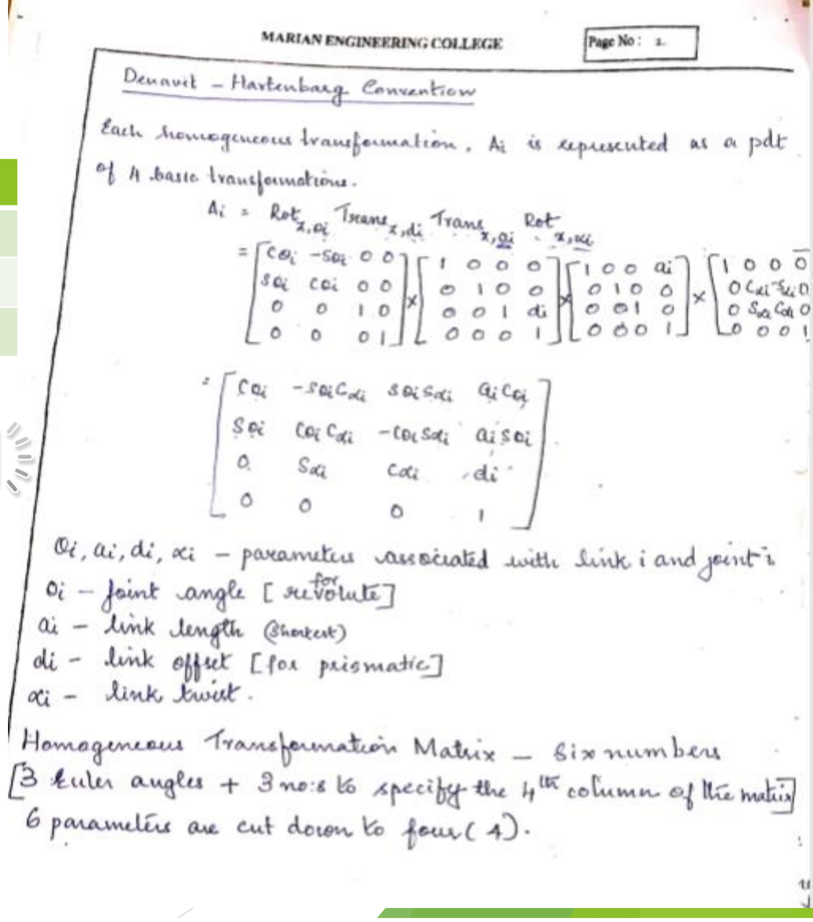
Cθi	-Sθi	0	0
Sθi	Cθi	0	0
0	0	1	di
0	0	0	1

- X

1	0	0	ai
0	Cαi	-Sαi	0
0	Sαi	Cαi	0
0	0	0	1

-

Cθi	-SθiCαi	SαiSθi	aiCθi
Sθi	CαiCθi	-SαiCθi	aiSθi
0	Sαi	Cαi	di
0	0	0	1



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Page No : 2

Denavit – Hartenbarg Convention

Each homogeneous transformation, A_i is represented as a pdt of 4 basic transformations.

$$A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i C\alpha_i & s\theta_i S\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i C\alpha_i & -c\theta_i S\alpha_i & a_i s\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta_i, a_i, d_i, \alpha_i$ – parameters associated with link i and joint i

θ_i – joint angle [for revolute]

a_i – link length (shortest)

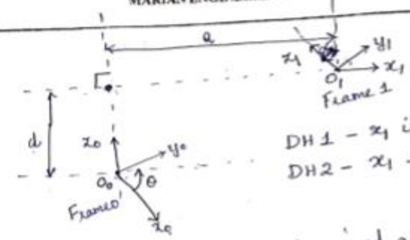
d_i – link offset [for prismatic]

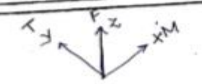
α_i – link twist.

Homogeneous Transformation Matrix – Six numbers [3 Euler angles + 3 no:s to specify the 4^{th} column of the matrix]

6 parameters are cut down to four (4).

Continued....





DH 1 – x_1 is $\perp$ z_0

DH 2 – x_1 intersects the axis z_0.

Every co-ordinate frame is determined and established on the basis of 3 rules :-

① z_{i-1} axis lies along the axis of motion of the i^{th} joint.
[z_0 lies along 1st joint]

② x_i axis is normal to the z_{i-1} axis and pointing away from it.
[x_1 is normal to z_0]

③ The y_i axis completes the right handed coordinate sfm as required.

Location of frame 0 coordinate – in base.

Last frame coordinate (n^{th} frame) – in hand.

as long as x_n normal to z_{n-1}

Four parameters to describe revolute / prismatic.

* θ_i – joint angle from x_{i-1} axis to the x_i axis abt z_{i-1} axis (rt hand rule).
  [$x_0 \longrightarrow x_1$ abt z_0]
* d_i – distance from the origin of $(i-1)^{th}$ coordinate frame to the intersection of the z_{i-1} axis with x_i axis along z_{i-1} axis.

Continued....

* a_i – offset distance from the intersection of the z_{i-1} axis with the x_i axis to the origin of the i^{th} frame along x_i axis (shortest distance b/w the z_{i-1} axis & z_i axis)

* α_i – offset angle from the z_{i-1} axis to the z_i axis abt the x_i axis (rt hand rule) [z_0 to z_1 abt x_1]

Assigning coordinate frames:-

Establish x axes for the links.

* Assign z_i to be the axis of actuation for joint i+1 [z_0 " joint 1]

2 cases:-

(i) if joint i+1 is revolute, z_i axis of revolution of joint i+1

(ii) " prismatic, " translation " ,

when joint i is activated abt axis z_{i-1}, link i and frame $O x_i y_i z_i$ results in a motion.

Establish base frame.

Choose x_0, y_0 in a convenient manner so long as the resulting frame is rt handed. This is frame 0.

After establishing frame 0, then define frame i using i-1

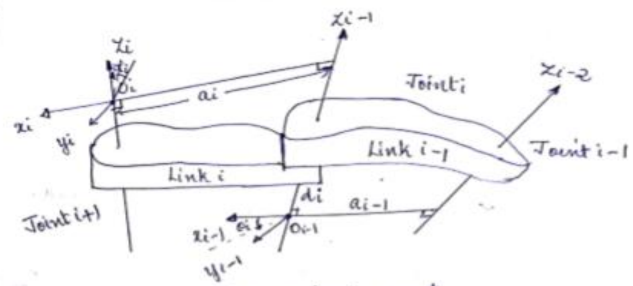
To setup frame i :-

(i) z_{i-1}, z_i are not coplanar

(ii) z_{i-1}, z_i intersect } coplanar

(iii) z_{i-1}, z_i are parallel }

D-H frame Assignment



D-H Frame Assignment

(i) z_{i-1} and z_i are not coplanar $[d_i^*, a_i^*]$

z_{i-1} to z_i → α_i as not coplanar (to make it coplanar)

→ a_i (shortest line segment from z_{i-1} to z_i ⊥ to z_{i-1} and z_i

The line segment is x_i and the point where it intersects z_i is the origin O_i. Satisfies $x_1 \perp z_0$ (DH1)

x_1 intersects z_0 (DH2)

So, $A_i = Rot_{z,\theta_i} \, Trans_{z,d_i} \, Trans_{x,a_i} \, Rot_{x,\alpha_i}$

(ii) z_{i-1} is ∥ to z_i $[\alpha_i = 0, d_i^*]$

So many normals are present. Free to choose origin O_i anywhere along z_i (for simplifying)

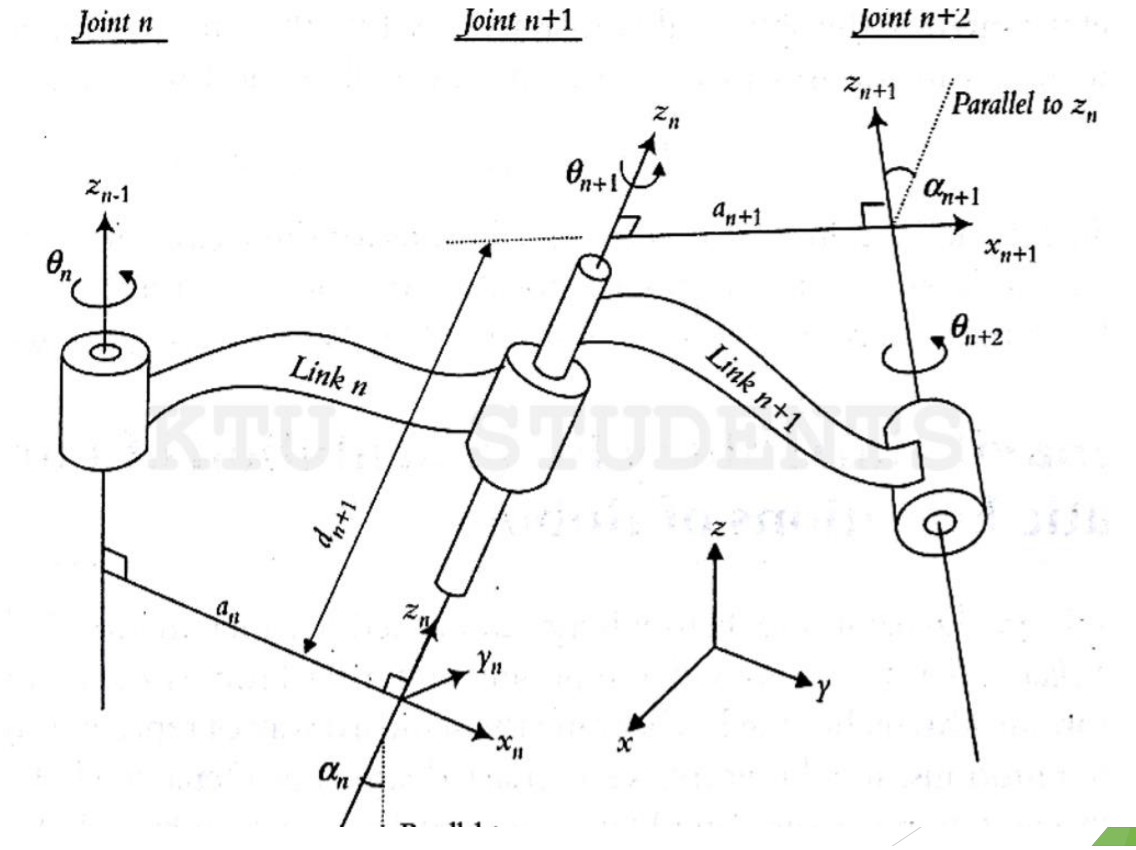
Normal method to choose O_i is to choose the normal that passes thru O_{i-1} as x_i axis. O_i is then the pt. at which this normal intersects z_i. $d_i = 0$.

∵ the z_{i-1} and z_i are ∥, $\alpha_i = 0$.

(iii) z_{i-1} intersects z_i $[a_i = 0]$

x_i is normal to the plane formed by z_i and z_{i-1}

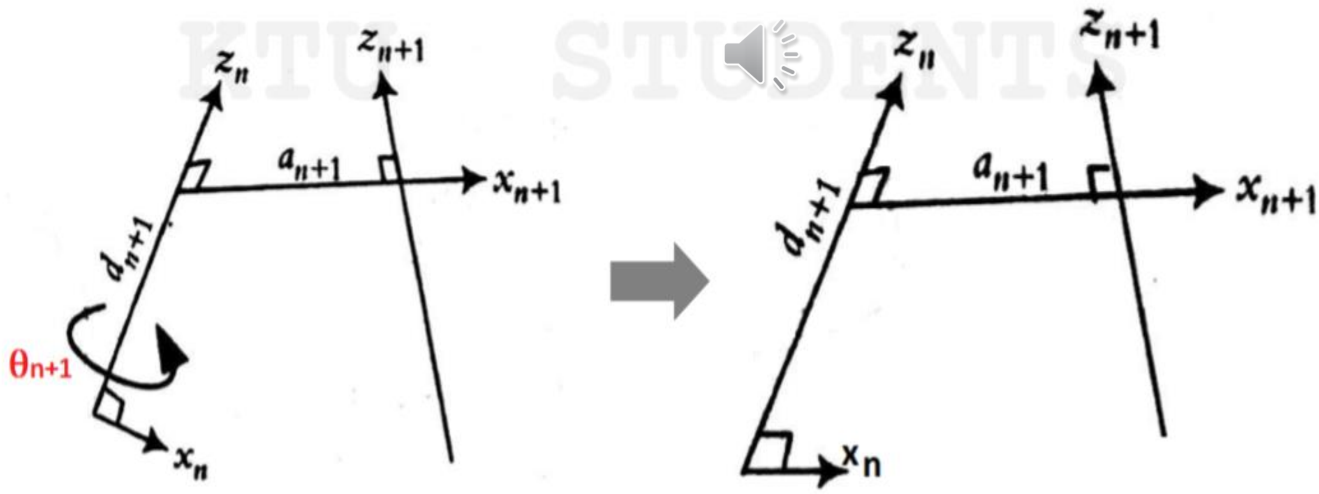
LINK COORDINATES



To find DH parameter (θi-joint angle)

1. Rotate the joint at angle θ_{n+1} about the z_n-axis.

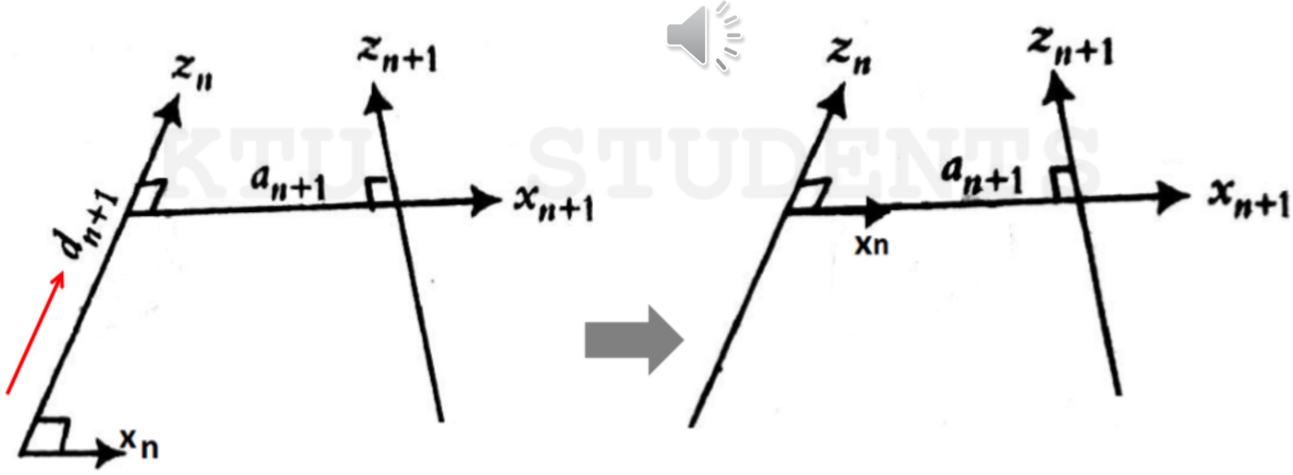
This will make x_n and x_{n+1} parallel to each other



To find DH parameter (di-link offset)

2. Translate along z_n - axis a distance d_{n+1}.

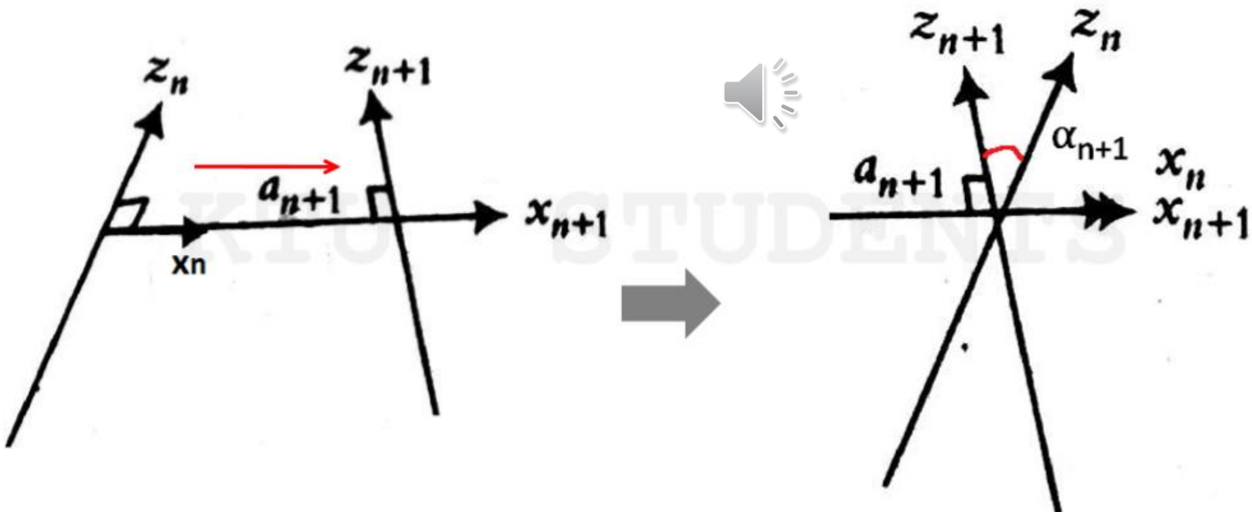
This will make x_n and x_{n+1} colinear.



To find DH parameter (ai-link length)

3. **Translate along x_n - axis a distance a_{n+1}.**

This will bring the origins of x_n and x_{n+1}. That is the two origins will be the same.

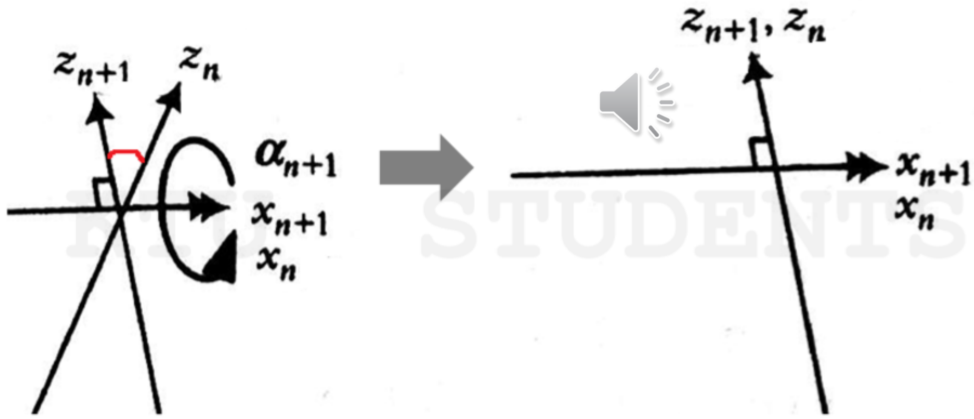


To find DH parameter (αi-link twist)

4. Rotate the *z_n-axis* at angle α_{n+1} about the *x_{n+1}-axis.*

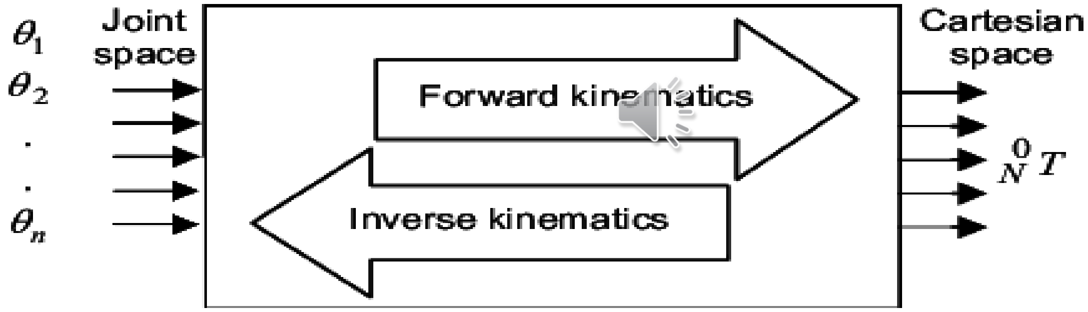
This will make z_n to align with z_{n+1} axis.

So the frame n and n+1 will be exactly same and we have transformed from one frame to the next.

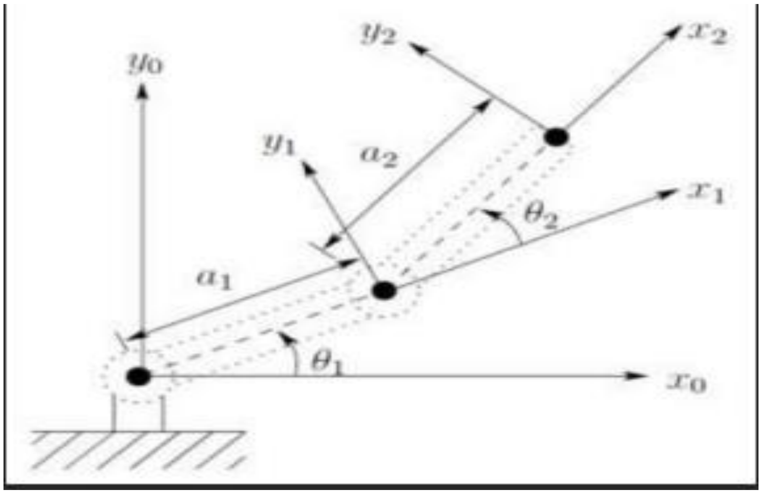


Angle is taken positive when rotation is made counter clockwise.

What is Forward Kinematics and Inverse Kinematics?



2 link Planar Manipulator(RR)



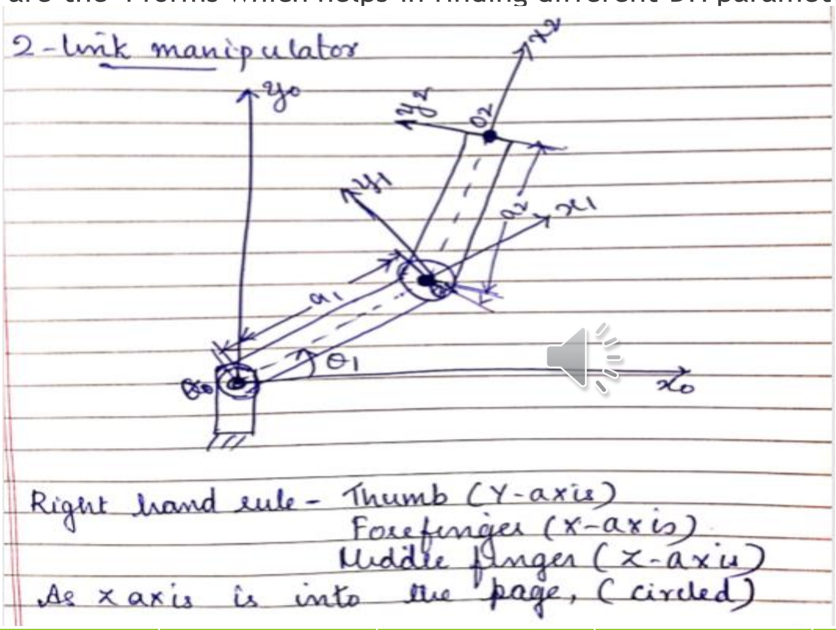
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

* variable

- Explained earlier

1.How to find DH parameter table for a 2 link planar manipulator

- These are the 4 forms which helps in finding different DH parameters



Link	θi	di	ai	αi
1	Θ1*	0	a1	0
2	Θ2*	0	a2	0

To find Homogenous Transformation Matrix

- Transformation matrix is represented as $A_i = Rot(z,\theta_i) * Trans(z,d_i) * Trans(x,a_i) * Rot(x,\alpha_i)$

$$Rot(z,\theta_i) = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans(z,d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans(x,a_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(x,\alpha_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By matrix multiplication

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For 2 link manipulator (from the general matrix)

Link 1 → $\theta_1, a_1, \alpha_1 = 0, d_1 = 0$.

$$A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d_1)

Link 2 → $\theta_2, a_2, \alpha_2 = 0, d_2 = 0$.

$$A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_1 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

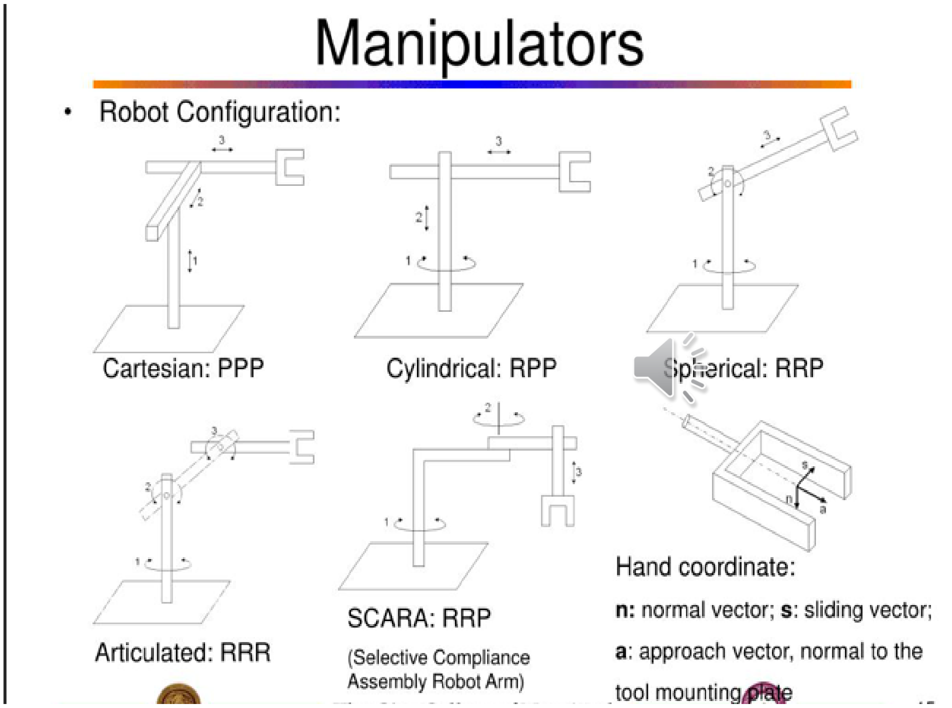
$$A^0_2 = A_1 \cdot A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d_2)

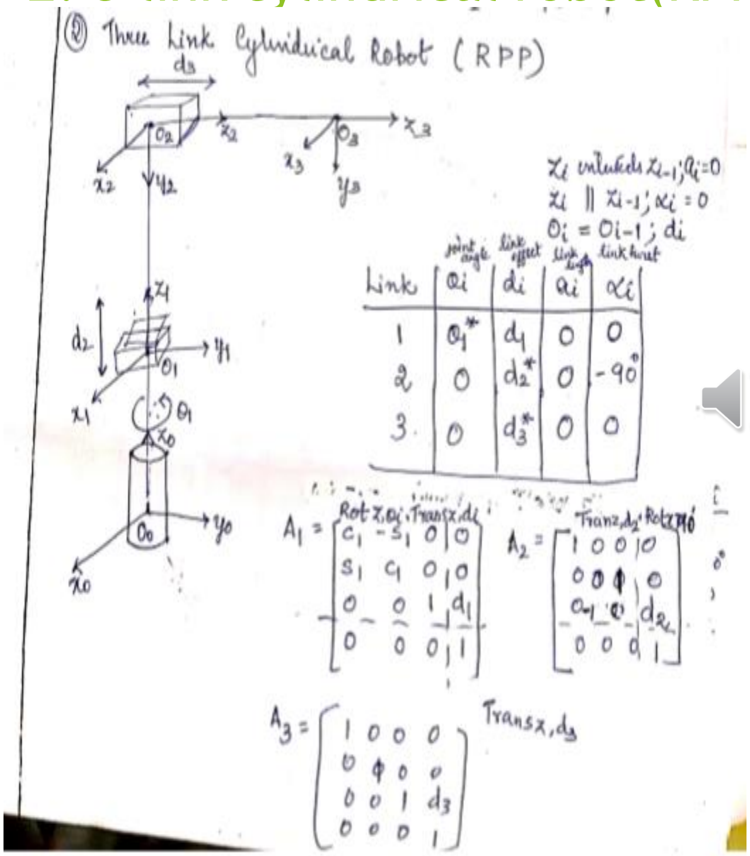
(Homogenous Transformation Matrix)

$c_{12} = \cos(\theta_1 + \theta_2)$

$s_{12} = \sin(\theta_1 + \theta_2)$


Manipulators
Robot Configuration:
Cartesian: PPP
Cylindrical: RPP
Spherical: RRP
Articulated: RRR
SCARA: RRP
(Selective Compliance Assembly Robot Arm)
Hand coordinate:
n: normal vector; s: sliding vector;
a: approach vector, normal to the tool mounting plate

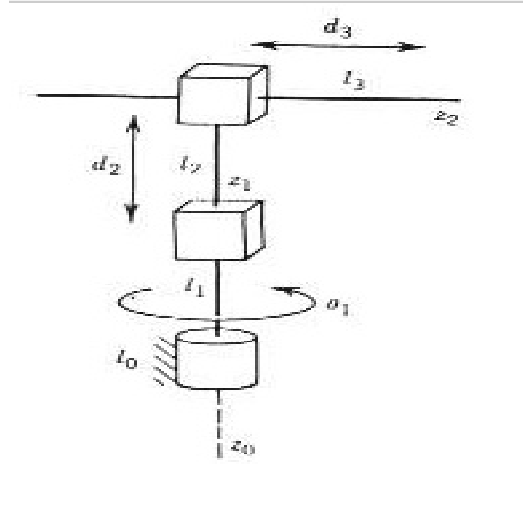
2. 3 link cylindrical robot(RPP)



Link	θ_i (joint angle)	d_i (link offset)	a_i (link length)	α_i (link twist)
1	θ_1^*	d_1	0	0
2	0	d_2^*	0	$-90°$
3	0	d_3^*	0	0

$$A_1 = \text{Rot}_{z,\theta_i} \cdot \text{Trans}_{z,d_i} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \text{Trans}_{z,d_2} \cdot \text{Rot}_{x,-90°} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

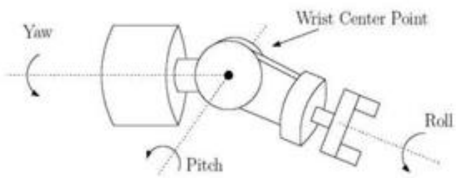
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Trans}_{z,d_3}$$

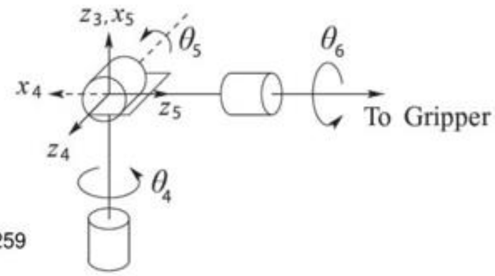


$$T_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3: spherical wrist

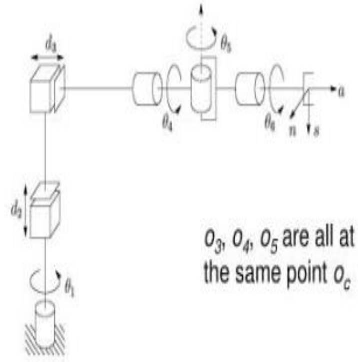
- 3DOF: need to assign four coordinate frames
  - yaw, pitch, roll (θ_4, θ_5, θ_6) all intersecting at one point o (wrist center)
  1. Choose z_3 axis (axis of rotation for joint 4)
  2. Choose z_4 axis (axis of rotation for joint 5)
  3. Choose z_5 axis (axis of rotation for joint 6)
  4. Choose tool frame:
     - z_6 (a) is collinear with z_5
     - y_6 (s) is in the direction the gripper closes
     - x_6 (n) is chosen with a right-handed convention





Example 4: cylindrical robot with spherical wrist

- 6DOF: need to assign seven coordinate frames
  - But we already did this for the previous two examples, so we can fill in the table of DH parameters:



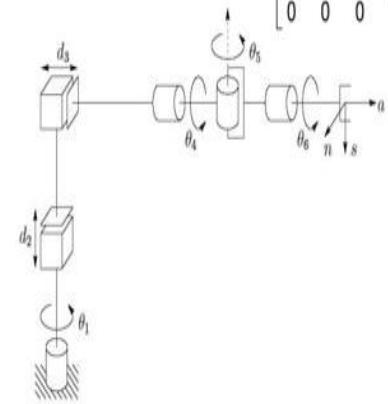
o_3, o_4, o_5 are all at the same point o_c

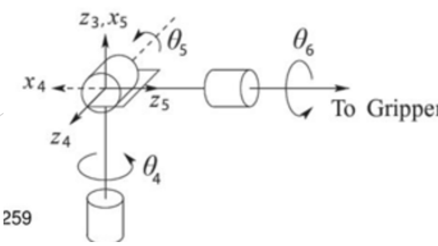
link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	-90	d_2	0
3	0	0	d_3	0
4	0	-90	0	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

- Note that z_3 (axis for joint 4) is collinear with z_2 (axis for joint 3), thus we can make the following combination:

$$T_6^0 = T_3^0 T_6^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} r_{11} = c_1c_4c_5c_6 - c_1s_4s_6 + s_1s_5c_6 \\ r_{21} = s_1c_4c_5c_6 - s_1s_4s_6 - c_1s_5c_6 \\ r_{31} = -s_4c_5c_6 - c_4s_6 \\ r_{12} = -c_1c_4c_5s_6 - c_1s_4c_6 - s_1s_5c_6 \\ r_{22} = -s_1c_4c_5s_6 - s_1s_4s_6 + c_1s_5c_6 \\ r_{32} = s_4c_5c_6 - c_4c_6 \\ r_{13} = c_1c_4s_5 - s_1c_5 \\ r_{23} = s_1c_4s_5 + c_1c_5 \\ r_{33} = -s_4s_5 \\ d_x = c_1c_4s_5d_6 - s_1c_5d_6 - s_1d_3 \\ d_y = s_1c_4s_5d_6 + c_1c_5d_6 + c_1d_3 \\ d_z = -s_4s_5d_6 + d_1 + d_2 \end{cases}$$

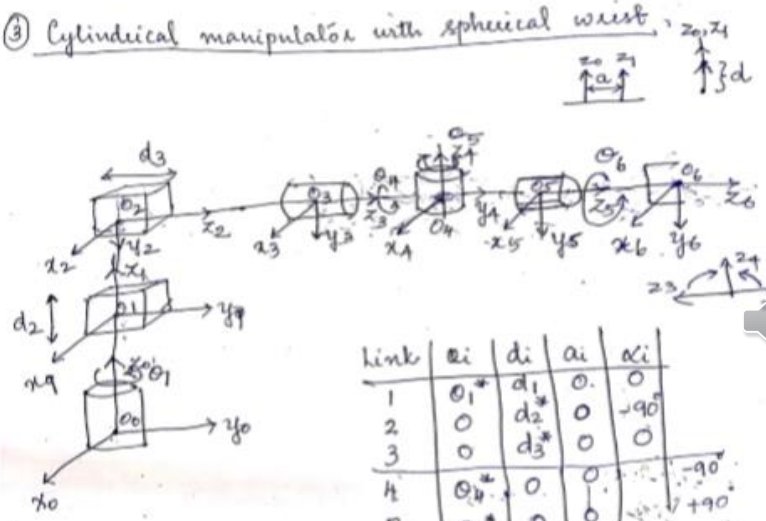




259

3.Cylindrical manipulator with spherical wrist (RPP+spherical wrist(3 DOF)=6 DOF)

(3) Cylindrical manipulator with spherical wrist



Link	θ_i	d_i	a_i	α_i
1	θ_1^*	d_1	0	0
2	0	d_2^*	0	-90°
3	0	d_3^*	0	0
4	θ_4^*	0	0	-90°
5	θ_5^*	0	0	+90°
6	θ_6^*	d_6	0	0

$$A_4 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} \cos\theta_5 & 0 & +\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_1 \cdot A_2 \cdot A_3 = T_3^0$

* 3 link wrist mechanism for which joint axes z_3, z_4, z_5 intersect at O – wrist center

$T_6^3 = A_4 \cdot A_5 \cdot A_6$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^3 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & C_4S_5d_6 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & S_4S_5d_6 \\ -S_5C_6 & S_5S_6 & C_5 & C_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_6^0 = T_3^0 \cdot T_6^3$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_{11} = C_1C_4C_5C_6 - C_1S_4S_6 + S_1S_5C_6$

$r_{21} = S_1C_4C_5C_6 - S_1S_4S_6 - C_1S_5C_6$

$r_{31} = -S_4C_5C_6 - C_4S_6$

$r_{12} = -C_1C_4C_5S_6 - C_1S_4C_6 - S_1S_5C_6$

$r_{22} = -S_1C_4C_5S_6 - S_1S_4S_6 + C_1S_5C_6$

$r_{32} = S_4C_5C_6 - C_4C_6$

$r_{33} = -S_4S_5$

$d_x = C_1C_4S_5d_6 - S_1C_5d_6 - S_1d_3$

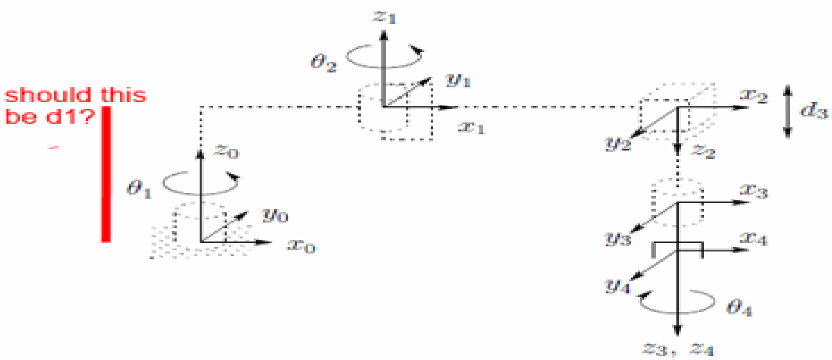


Fig. 3.11 DH coordinate frame assignment for the SCARA manipulator

Table 3.5 Joint parameters for SCARA

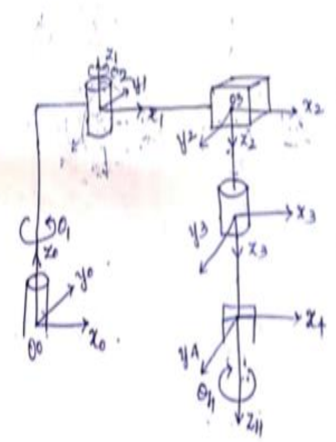
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ^*
2	a_2	180	0	θ^*
3	0	0	d^*	0
4	0	0	d_4	θ^*

Why not d1?

* joint variable

4.SCARA Manipulator(RRP+1 DOF=4 DOF)

(4) SCARA Manipulator (RRP + 1 DOF for wrist) John Craig



Link	θ_i	d_i	a_i	α_i
1	θ_1^*	0	a_1	0
2	θ_2^*	0	a_2	180°
3	0	d_3^*	0	0
4	θ_4^*	d_4	0	0

link	θ_i	d_i	a_i	α_i
1	θ_1^*	0	a_1	0
2	θ_2^*	0	a_2	180
3	0	d_3^*	0	0
4	θ_4^*	d_4	0	0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

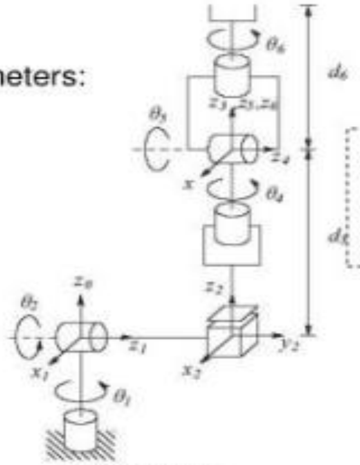
$$T_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

$$= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 5: the Stanford manipulator

- 6DOF: need to assign seven coordinate frames:
  1. Choose z_0 axis (axis of rotation for joint 1, base frame)
  2. Choose z_1-z_5 axes (axes of rotation/translation for joints 2-6)
  3. Choose x_i axes
  4. Choose tool frame
  5. Fill in table of DH parameters:

link	a_i	α_i	d_i	θ_i
1	0	-90	0	θ_1
2	0	90	d_2	θ_2
3	0	0	d_3	0
4	0	-90	0	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6



Suggested insertion: photo of the Stanford manipulator

ES159/259