

## *Digital Image Processing, 3rd ed.*

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### Chapter 8 Image Compression

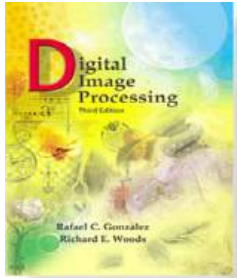
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The size of typical still image (1200x1600)

$$1200 \times 1600 \times 3 \text{byte} = 5760000 \text{byte}$$
$$= 5,760 \text{Kbyte} = 5.76 \text{Mbyte}$$

The size of two hours standard television (720x480) movies

$$30 \frac{\text{frame}}{\text{sec}} \times (760 \times 480) \frac{\text{pixels}}{\text{frame}} \times 3 \frac{\text{bytes}}{\text{pixel}} = 31,104,000 \text{bytes} / \text{sec}$$
$$31,104,000 \times \frac{\text{bytes}}{\text{sec}} \times (60 \times 60) \frac{\text{sec}}{\text{hour}} \times 2 \text{hours} = 2.24 \times 10^{11} \text{bytes}$$
$$= 224 \text{GByte.}$$



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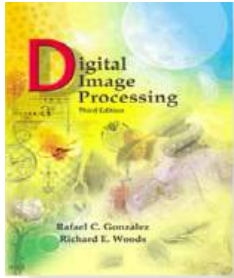
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## Data, Information, and Redundancy

- **Information**
- **Data** is used to represent information
- **Redundancy** in data representation of an information provides no relevant information or repeats a stated information
- Let  $n_1$ , and  $n_2$  are data represents the same information. Then, the relative data redundancy  $R$  of the  $n_1$  is defined as
$$R = 1 - 1/C \quad \text{where } C = n_1/n_2$$



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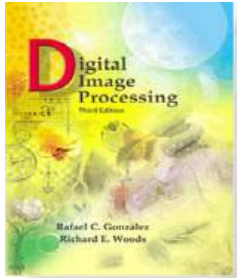
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### Image Compression

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- Redundancy in Digital Images
  - Coding redundancy
    - usually appear as results of the uniform representation of each pixel
  - Spatial/Temopral redundancy
    - because the adjacent pixels tend to have similarity in practical.
  - Irrelevant Information
    - Image contain information which are ignored by the human visual system.

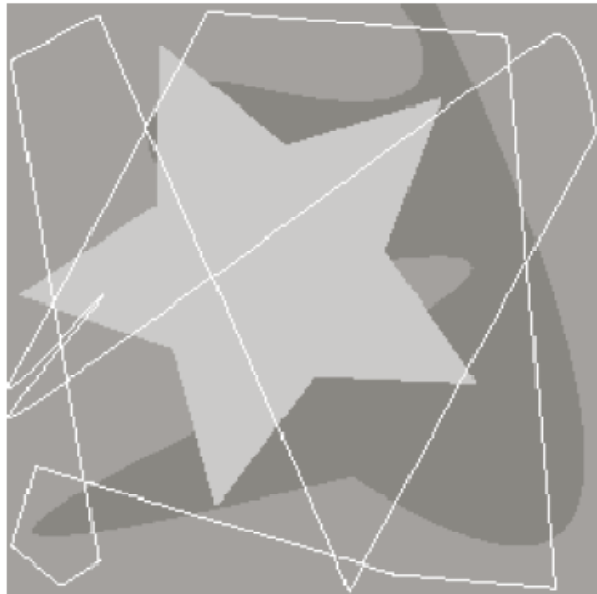


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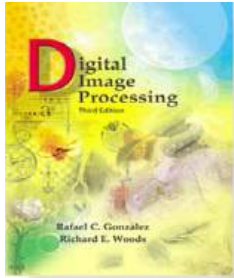
**Coding Redundancy**



**Spatial Redundancy**



**Irrelevant Information**



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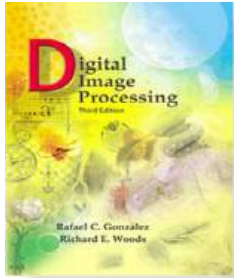
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## Coding Redundancy

- Assume the discrete random variable for  $r_k$  in the interval  $[0,1]$  that represent the gray levels. Each  $r_k$  occurs with probability  $p_k$
- If the number of bits used to represent each value of  $r_k$  by  $l(r_k)$  then

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p(r_k)$$

- The average code bits assigned to the gray level values.
- The length of the code should be inverse proportional to its probability (occurrence).



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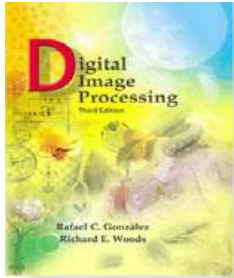
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#### Examples of variable length encoding

$r_k$	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
$r_k$ for $k \neq 87, 128, 186, 255$	0	—	8	—	0



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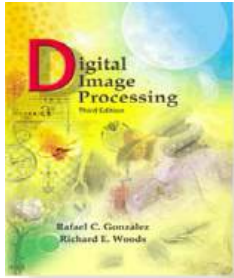
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## **Spatial/Temopral Redundancy**

- Internal Correlation between the pixel result from
  - Respective Autocorrelation
  - Structural Relationship
  - Geometric Relation ship
- The value of a pixel can be reasonably predicted from the values of its neighbors.
- To reduce the inter-pixel redundancies in an image the 2D array is transformed (*mapped*) into more efficient format (Frequency Domain etc.)



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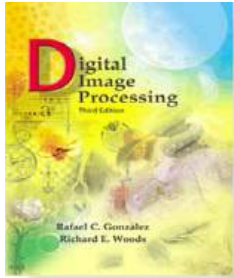
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## **Irrelevant information and Psycho-Visual Redundancy**

- The brightness of a region depend on other factors that the light reflection
- The perceived intensity of the eye is limited an non linear
- Certain information has less relative importance that other information in normal visual processing
- In general, observer searches for distinguishing features such as edges and textural regions.





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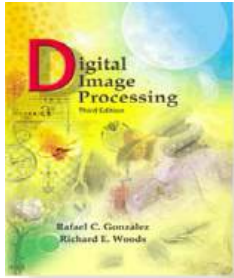
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## Measuring Information

- A random event  $E$  that occurs with probability  $P(E)$  is said to contain  $I(E)$  information where  $I(E)$  is defined as
$$I(E) = \log(1/P(E)) = -\log(P(E))$$
- $P(E) = 1$  contain no information
- $P(E) = 1/2$  requires one bit of information.



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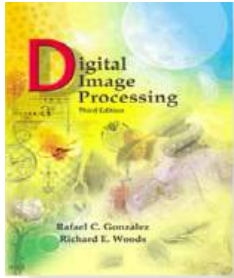
## Measuring Information

- For a source of events  $a_0, a_1, a_2, \dots, a_k$  with associated probability  $P(a_0), P(a_1), P(a_2), \dots, P(a_k)$ .
- The average information per source (entropy) is

$$H = -\sum_{j=0}^k P(a_j) \log(P(a_j))$$

For image, we use the normalized histogram to generate the source probability, which leads to the entropy

$$\tilde{H} = -\sum_{i=0}^{L-1} p_r(r_i) \log(p_r(r_i))$$



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## Fidelity Criteria

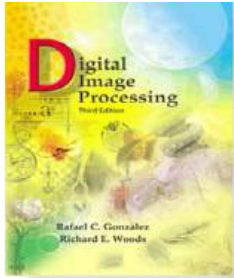
- Objective Fidelity Criteria

- The information loss can be expressed as a function of the encoded and decoded images.
- For image  $I(x,y)$  and its decoded approximation  $I'(x,y)$
- For any value of  $x$  and  $y$ , the error  $e(x,y)$  could be defined as

$$e(x, y) = I'(x, y) - I(x, y)$$

- For the entire Image

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I'(x, y) - I(x, y)$$



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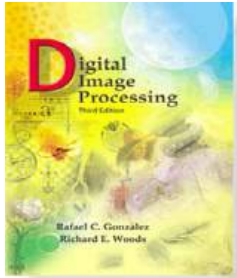
## Fidelity Criteria

- The mean-square-error,  $e_{rms}$  is

$$e_{rms} = \sqrt{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [I'(x, y) - I(x, y)]^2}$$

The mean-square-error signal-to-noise ratio  $SNR_{ms}$  is

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I'(x, y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [I'(x, y) - I(x, y)]^2}$$



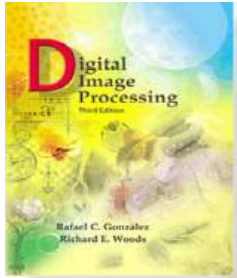
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<b>Value</b>	<b>Rating</b>	<b>Description</b>
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.



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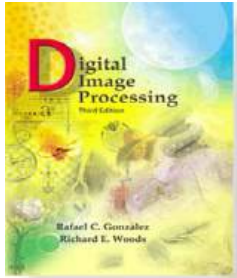
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Three approximations of the same image

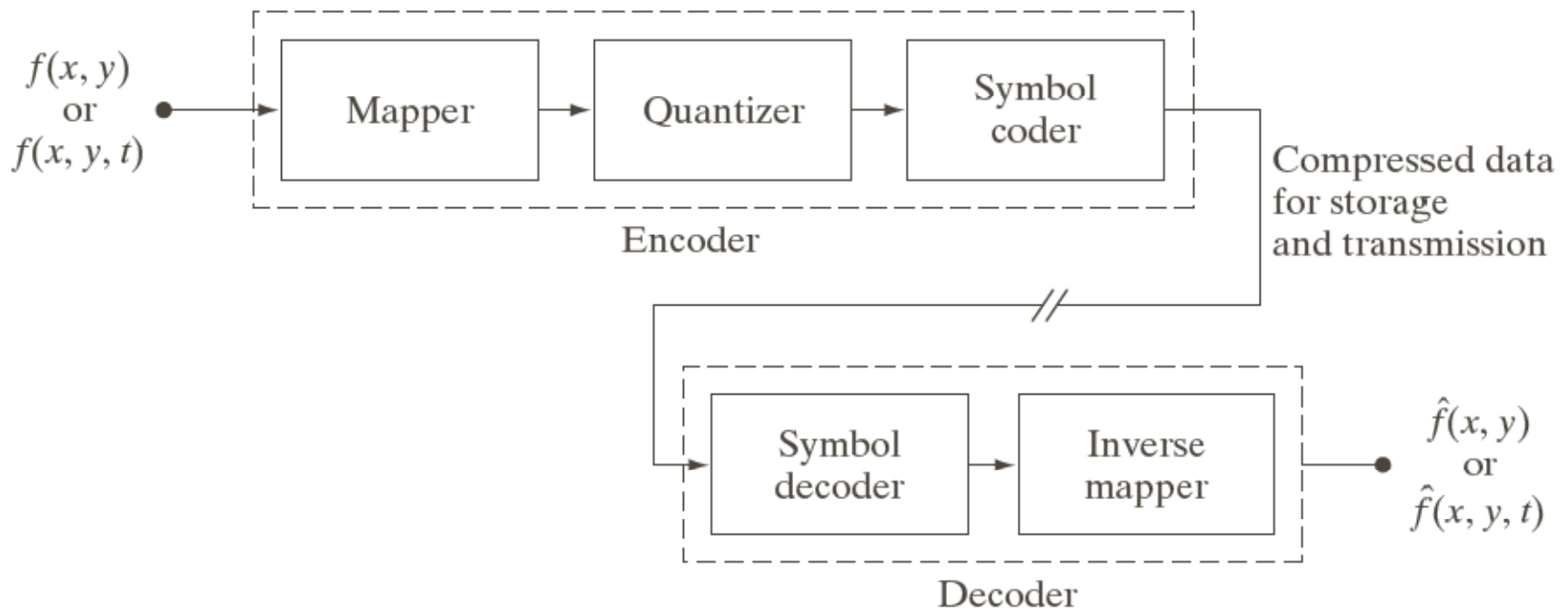


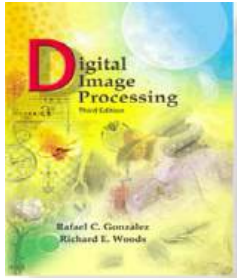
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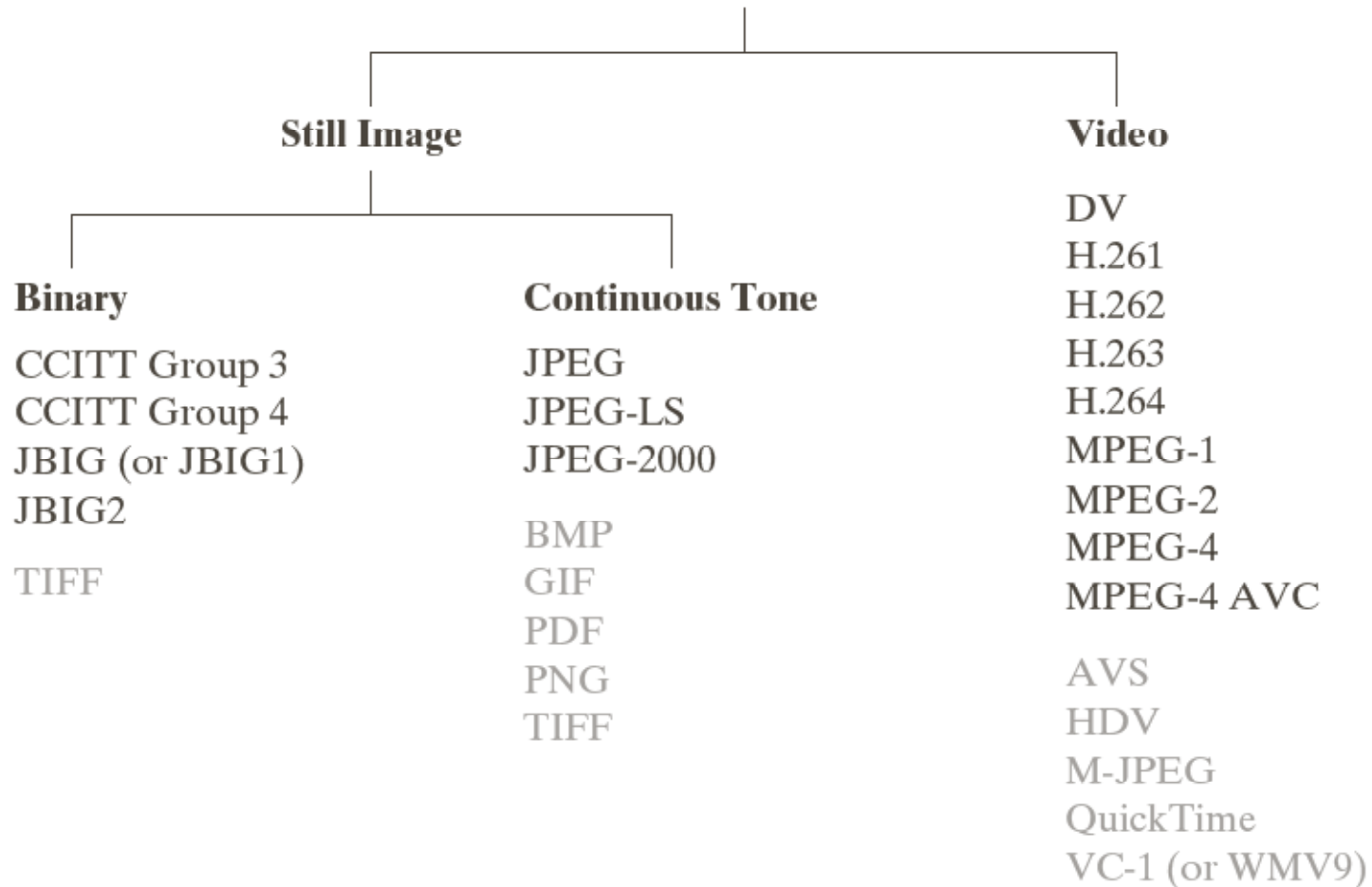
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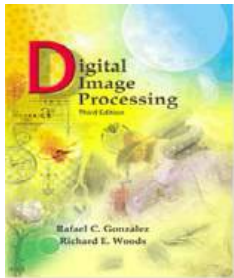
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### Image Compression Standards, Formats, and Containers







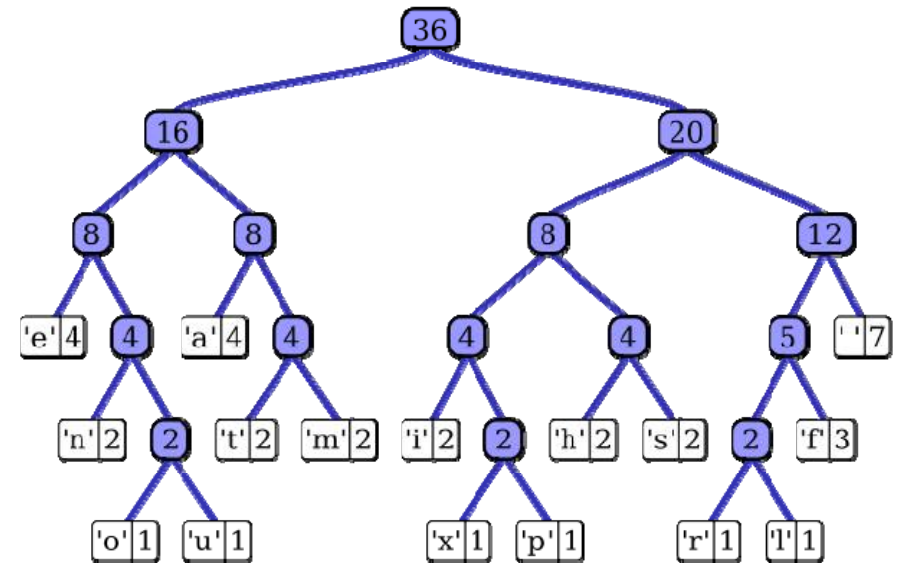
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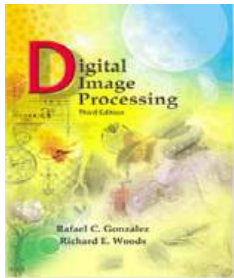
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**Huffman coding** is an entropy encoding algorithm used for lossless data compression. The term refers to the use of a variable-length code table for encoding a source symbol (such as a character in a file) where the variable-length code table has been derived in a particular way based on the estimated probability of occurrence for each possible value of the source symbol.



Original source		Source reduction			
Symbol	Probability	1	2	3	4
$a_2$	0.4	0.4	0.4	0.4	0.6
$a_6$	0.3	0.3	0.3	0.3	
$a_1$	0.1	0.1	0.2	0.3	0.4
$a_4$	0.1	0.1			
$a_3$	0.06	0.1	0.1	0.1	0.1
$a_5$	0.04				



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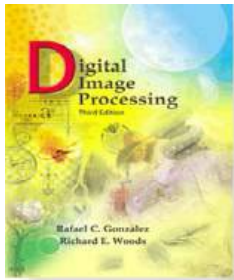
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### Huffman coding Assignment procedure

Original source		Source reduction							
Symbol	Probability	Code	1	2	3	4			
$a_2$	0.4	1	0.4	1	0.4	1	0.4	1	
$a_6$	0.3	00	0.3	00	0.3	00	0.3	00	
$a_1$	0.1	011	0.1	011	0.2	010	0.3	01	
$a_4$	0.1	0100	0.1	0100	0.1	011			
$a_3$	0.06	01010	0.1	0101					
$a_5$	0.04	01011							



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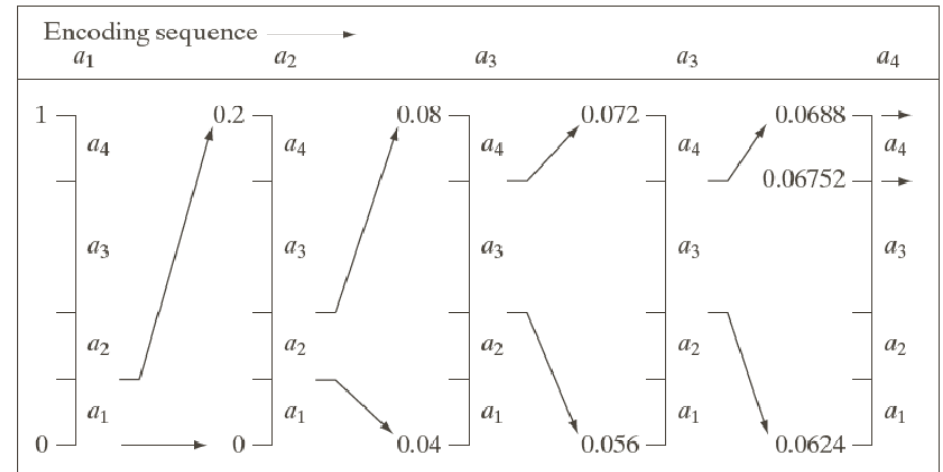
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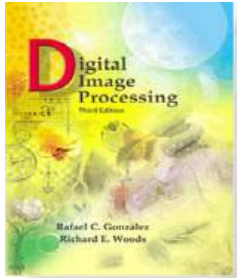
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**Arithmetic coding** is a form of variable-length entropy encoding. A string is converted to arithmetic encoding, usually characters are stored with fewer bits

Arithmetic coding encodes the entire message into a single number, a fraction  $n$  where  $(0.0 \leq n < 1.0)$ .



Source Symbol	Probability	Initial Subinterval
$a_1$	0.2	$[0.0, 0.2)$
$a_2$	0.2	$[0.2, 0.4)$
$a_3$	0.4	$[0.4, 0.8)$
$a_4$	0.2	$[0.8, 1.0)$



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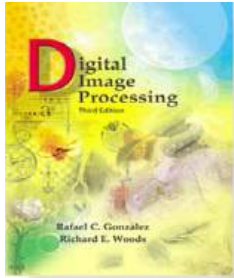
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# Compression Algorithms



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### Symbol compression

This approach determines a set of symbols that constitute the image, and takes advantage of their multiple appearance. It converts each symbol into a token, generates a token table, and represents the compressed image as a list of tokens.

This approach is good for document images.

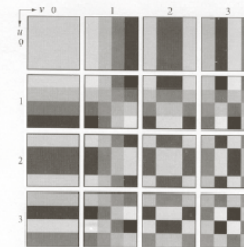


FIGURE 8.30 Discrete-cosine basis functions for  $N = 4$ . The origin of each block is at its top left.

where

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } u = 0 \\ \sqrt{\frac{2}{N}} & \text{for } u = 1, 2, \dots, N - 1 \end{cases} \quad (8.5-33)$$

and similarly for  $\alpha(v)$ . Figure 8.30 shows  $g(x, y, u, v)$  for the case  $N = 4$ . The computation follows the same format as explained for Fig. 8.29, with the difference that the values of  $g$  are not integers. In Fig. 8.30, the lighter gray levels correspond to larger values of  $g$ .

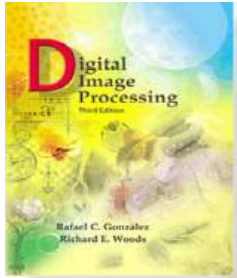
Figures 8.31(a), (c), and (e) show three approximations of the  $512 \times 512$  monochrome image in Fig. 8.23. These pictures were obtained by dividing the original image into subimages of size  $8 \times 8$ , representing each subimage using one of the transforms just described (i.e., the DFT, WHT, or DCT transform), truncating 50% of the resulting coefficients, and taking the inverse transform of the truncated coefficient arrays.

In each case, the 32 retained coefficients were selected on the basis of maximum magnitude. When we disregard any quantization or coding issues, this process amounts to compressing the original image by a factor of 2. Note that in all cases, the 32 discarded coefficients had little visual impact on reconstructed image quality. Their elimination, however, was accompanied by some mean-square error, which can be seen in the scaled error images of Figs. 8.31(b), (d), and (f). The actual rms errors were 1.28, 0.86, and 0.68 gray levels, respectively.



$$r N = 4. T$$

EXAMPLE 8.19: Transform coding with the DFT, WHT, and DCT.

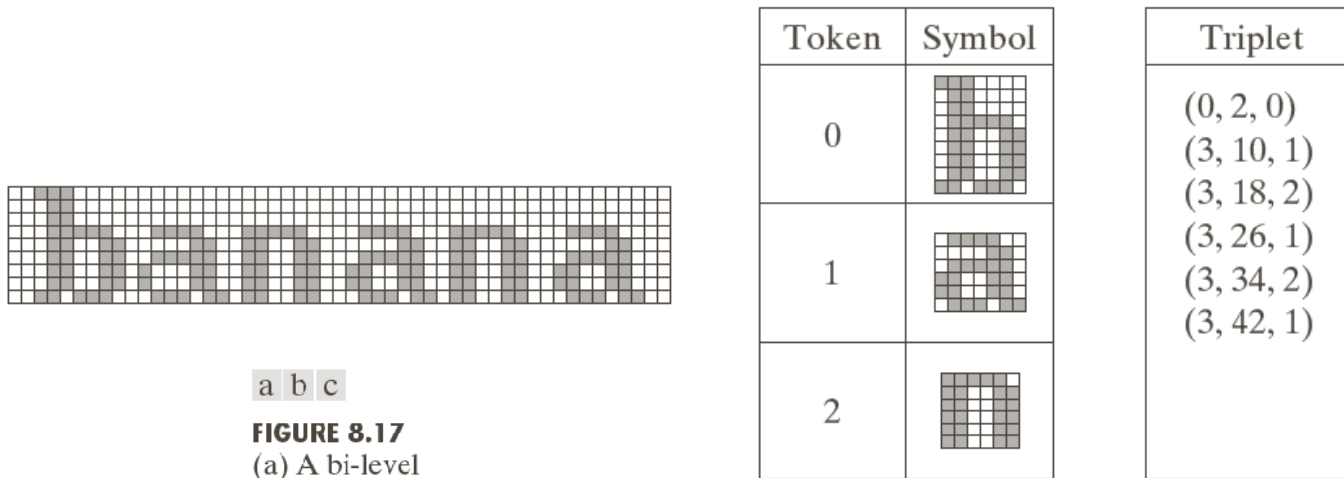


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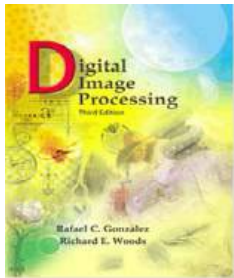
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a b c

**FIGURE 8.17**

(a) A bi-level document, (b) symbol dictionary, and (c) the triplets used to locate the symbols in the document.



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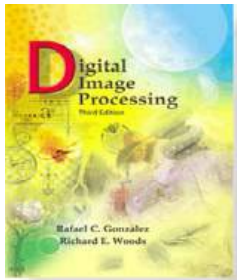
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Images of size just described resulting coefficient arrays. retained coefficients when we disregard

Images of size just described resulting coefficient arrays. retained coefficients when we disregard

a b c

**FIGURE 8.18**  
JBIG2 compression comparison: (a) lossless compression and reconstruction; (b) perceptually lossless; and (c) the scaled difference between the two.



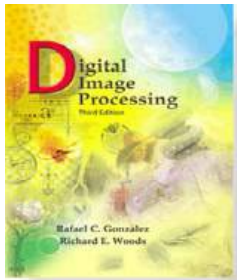
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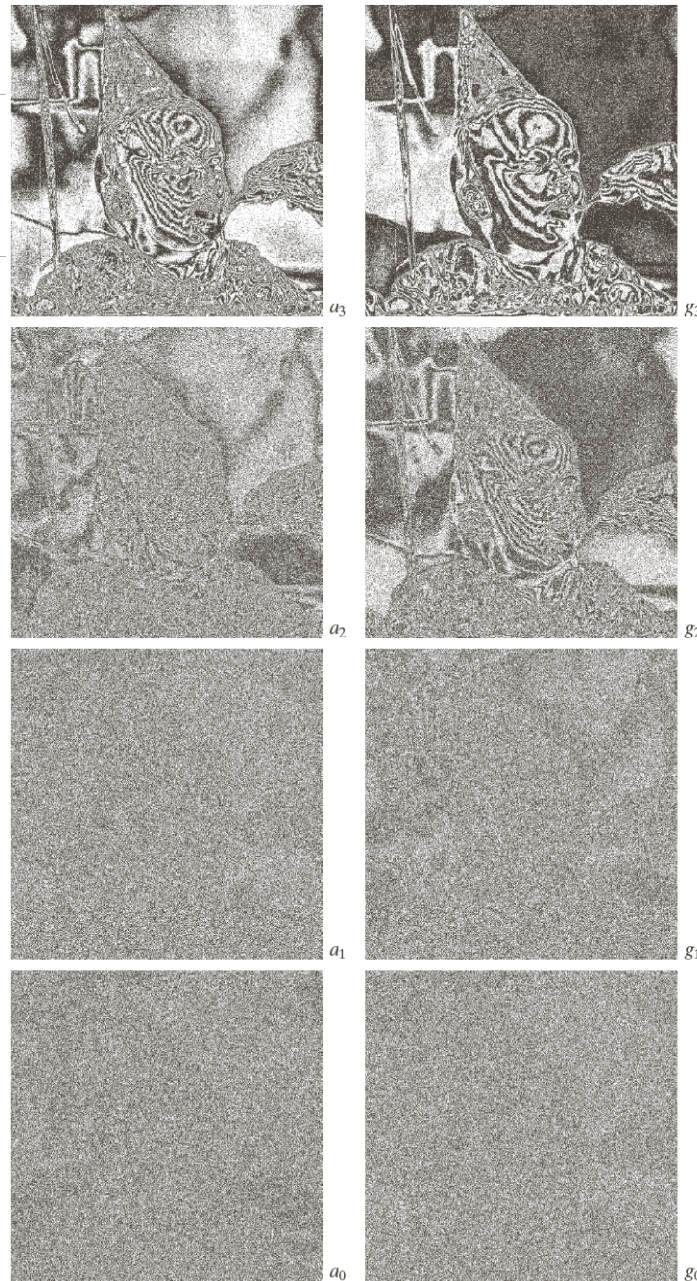
a	b
c	d
e	f
g	h

**FIGURE 8.19**  
 (a) A 256-bit monochrome image. (b)–(h) The four most significant binary and Gray-coded bit planes of the image in (a).



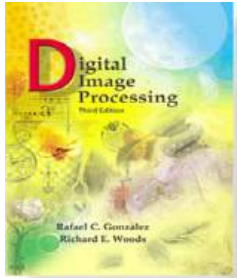


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a	b
c	d
e	f
g	h

**FIGURE 8.20**  
(a)–(h) The four least significant binary (left column) and Gray-coded (right column) bit planes of the image in Fig. 8.19(a).

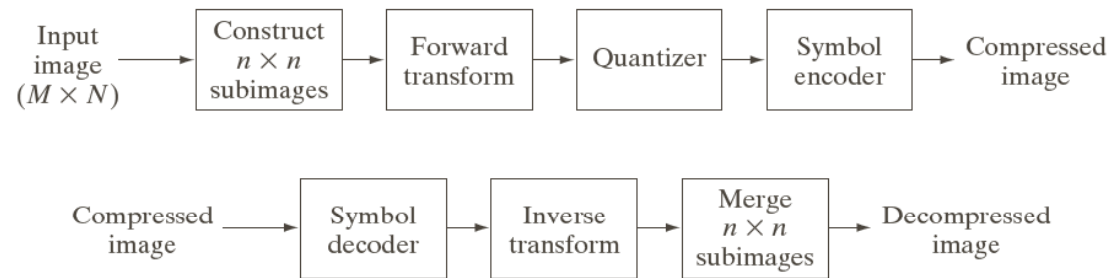


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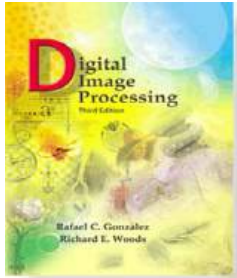
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a  
b  
**FIGURE 8.21**  
A block transform coding system:  
(a) encoder;  
(b) decoder.

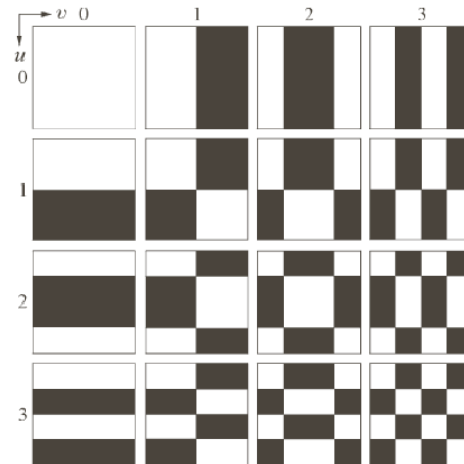


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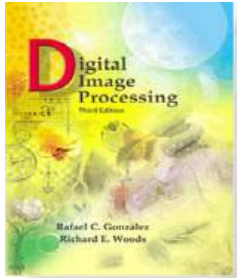
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**FIGURE 8.22**  
Walsh-Hadamard  
basis functions for  
 $n = 4$ . The origin  
of each block is at  
its top left.

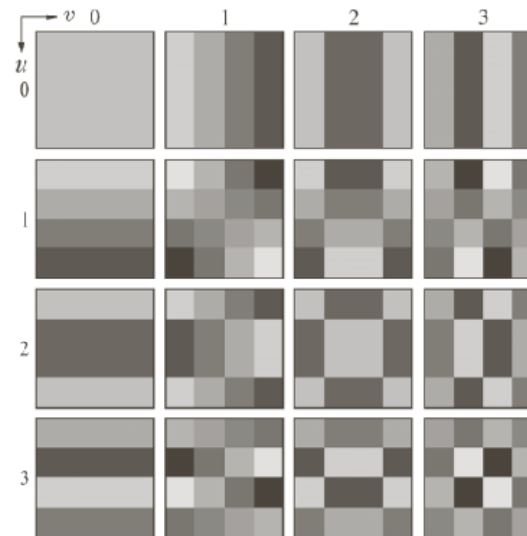


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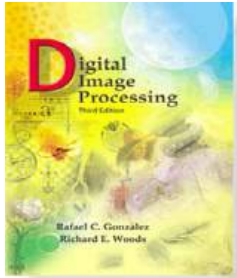
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**FIGURE 8.23**  
Discrete-cosine  
basis functions for  
 $n = 4$ . The origin  
of each block is at  
its top left.



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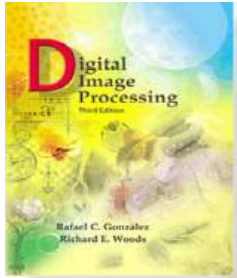
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a	b	c
d	e	f

**FIGURE 8.24** Approximations of Fig. 8.9(a) using the (a) Fourier, (b) Walsh-Hadamard, and (c) cosine transforms, together with the corresponding scaled error images in (d)–(f).



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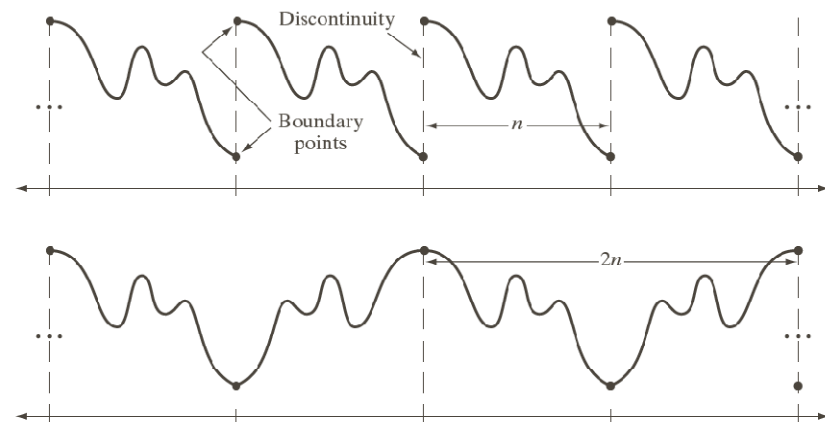
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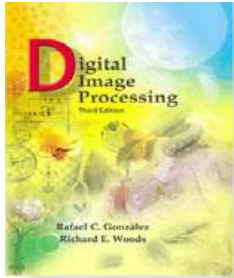
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### DFT and DCT

The periodicity implicit in the 1-D DFT and DCT. The DCT provide better continuity that the general DFT.





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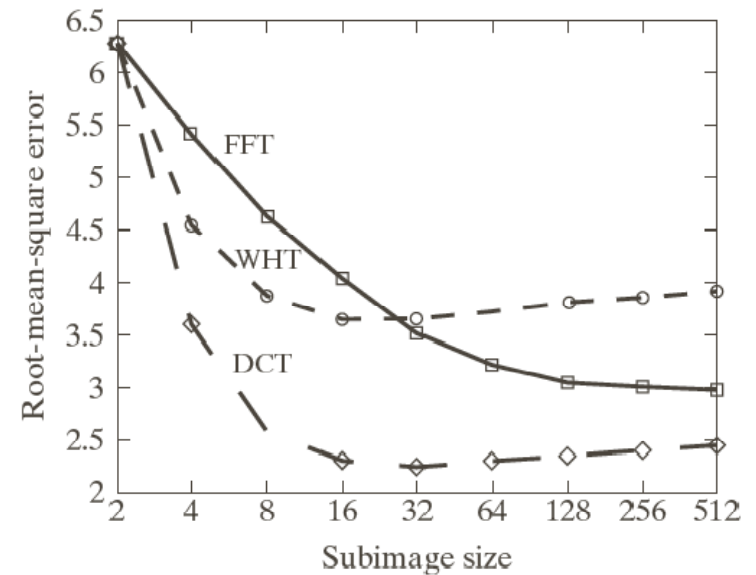
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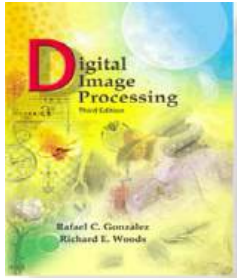
## Chapter 8 Image Compression

### Block Size vs. Reconstruction Error

The DCT provide the least error at almost any sub-image size.

The error takes its minimum at sub-images of sizes between 16 and 32.



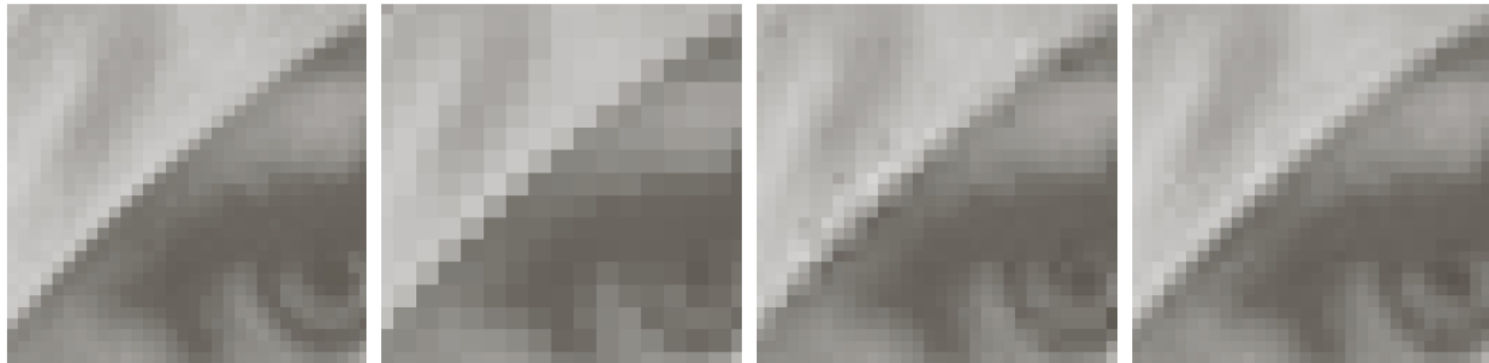


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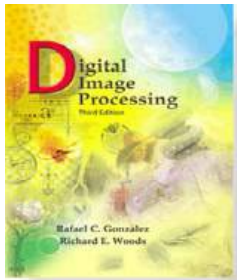
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a b c d

**FIGURE 8.27** Approximations of Fig. 8.27(a) using 25% of the DCT coefficients and (b)  $2 \times 2$  subimages, (c)  $4 \times 4$  subimages, and (d)  $8 \times 8$  subimages. The original image in (a) is a zoomed section of Fig. 8.9(a).





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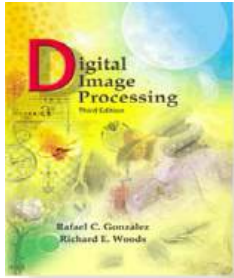
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a b  
c d

**FIGURE 8.28**

Approximations of Fig. 8.9(a) using 12.5% of the  $8 \times 8$  DCT coefficients: (a)–(b) threshold coding results; (c)–(d) zonal coding results. The difference images are scaled by 4.



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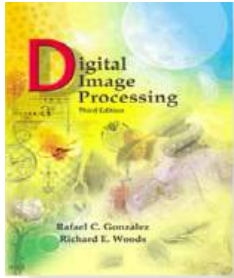
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1	1	1	1	1	0	0	0	8	7	6	4	3	2	1	0
1	1	1	1	0	0	0	0	7	6	5	4	3	2	1	0
1	1	1	0	0	0	0	0	6	5	4	3	3	1	1	0
1	1	0	0	0	0	0	0	4	4	3	3	2	1	0	0
1	0	0	0	0	0	0	0	3	3	3	2	1	1	0	0
0	0	0	0	0	0	0	0	2	2	1	1	1	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	0	0	0	0	1	5	6	14	15	27	28
1	1	1	1	0	0	0	0	2	4	7	13	16	26	29	42
1	1	0	0	0	0	0	0	3	8	12	17	25	30	41	43
1	0	0	0	0	0	0	0	9	11	18	24	31	40	44	53
0	0	0	0	0	0	0	0	10	19	23	32	39	45	52	54
0	1	0	0	0	0	0	0	20	22	33	38	46	51	55	60
0	0	0	0	0	0	0	0	21	34	37	47	50	56	59	61
0	0	0	0	0	0	0	0	35	36	48	49	57	58	62	63

a b  
c d

**FIGURE 8.29**

A typical (a) zonal mask, (b) zonal bit allocation, (c) threshold mask, and (d) thresholded coefficient ordering sequence. Shading highlights the coefficients that are retained.

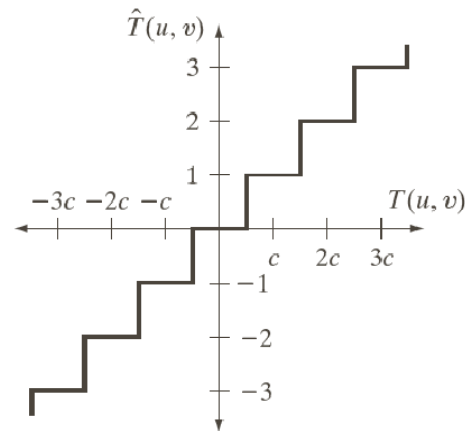


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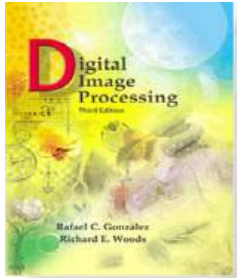
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16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

**FIGURE 8.30**  
(a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.



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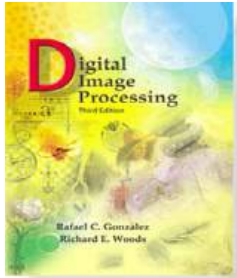
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**FIGURE 8.31** Approximations of Fig. 8.9(a) using the DCT and normalization array of Fig. 8.30(b): (a)  $Z$ , (b)  $2Z$ , (c)  $4Z$ , (d)  $8Z$ , (e)  $16Z$ , and (f)  $32Z$ .



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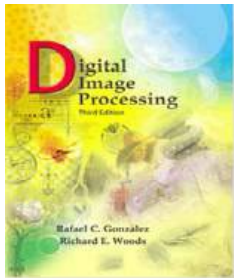
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a	b	c
d	e	f

**FIGURE 8.32** Two JPEG approximations of Fig. 8.9(a). Each row contains a result after compression and reconstruction, the scaled difference between the result and the original image, and a zoomed portion of the reconstructed image.



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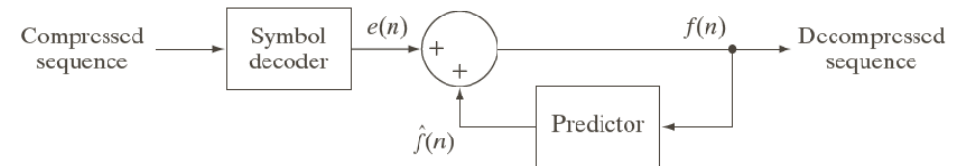
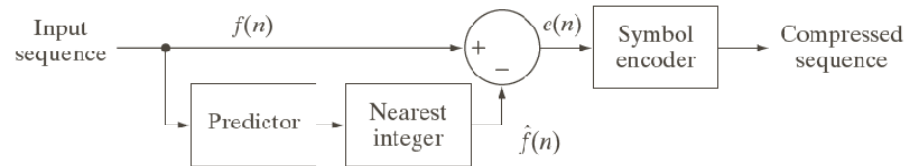
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## Chapter 8 Image Compression

### Lossless Predictive coding

The encoder expects a discrete sample of a signal  $f(n)$ .

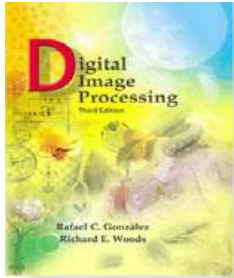
1. A predictor is applied and its output is rounded to the nearest integer.  $\hat{f}(n)$
2. The error is estimated as  $e(n) = f(n) - \hat{f}(n)$
3. The compressed stream consist of first sample and the errors, encoded using variable length coding



The decoder uses the predictor and the error stream to reconstructs the original signal  $f(n)$ .

1. The predictor is initialized using the first sample.
2. The received error is added to predictor result.

$$f(n) = \hat{f}(n) + e(n)$$



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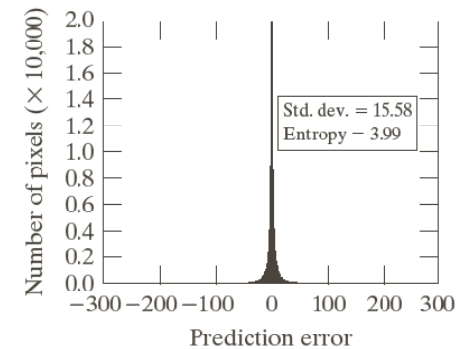
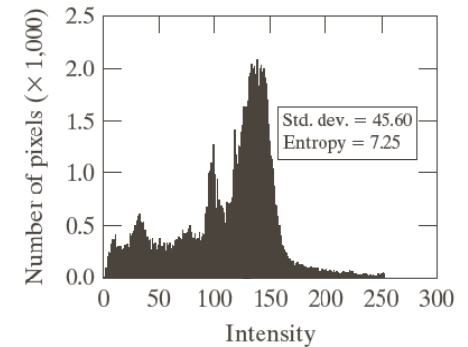
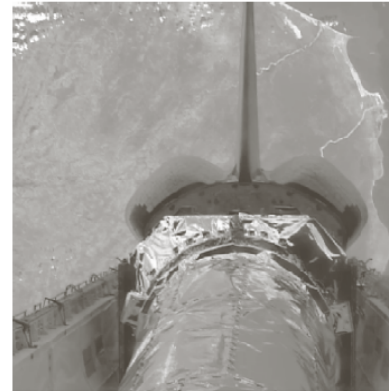
### Lossless Predictive coding

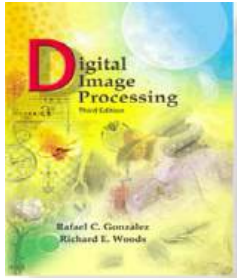
Linear predictors usually have the form:

$$\hat{f}(n) = \text{round} \left[ \sum_{i=0}^m a_i f(n-i) \right]$$

Original Image (view of the earth).  
The prediction error and its histogram.

1. The error is small in uniform regions
2. Large close to edges and sharp changes in pixel intensities



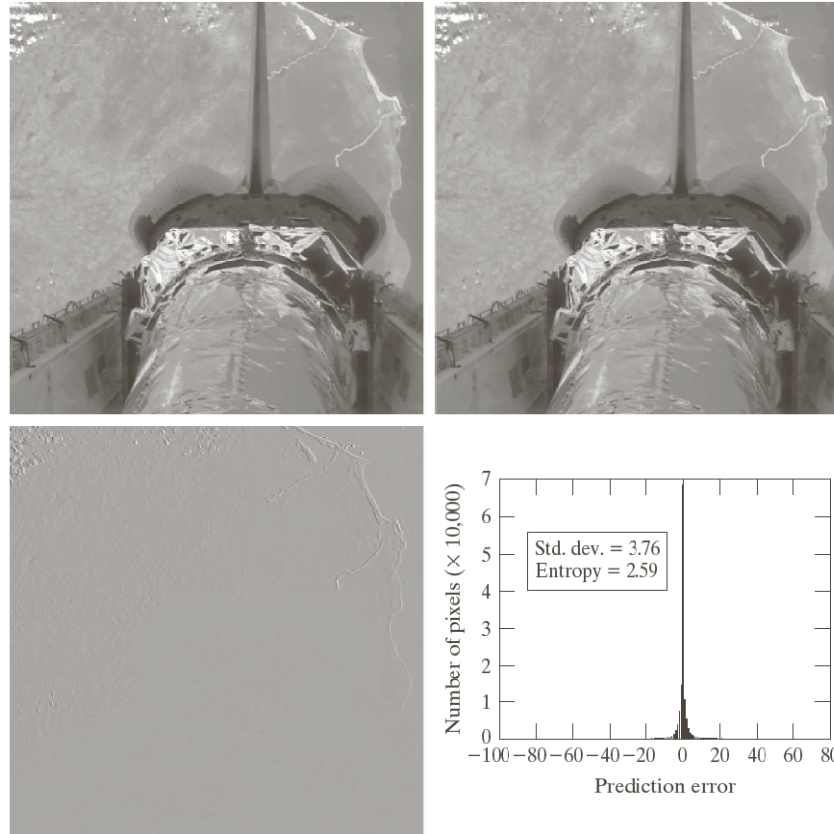


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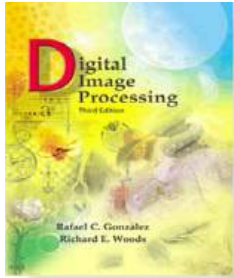
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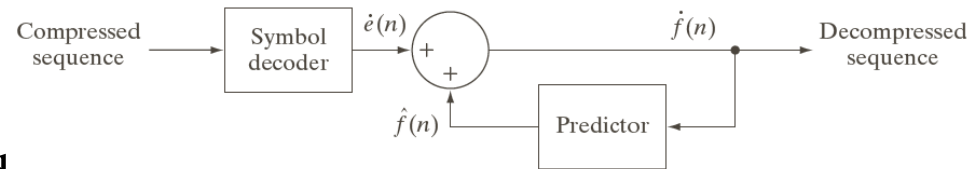
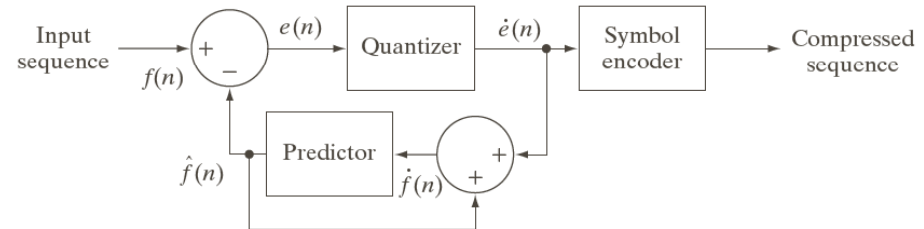
## Chapter 8

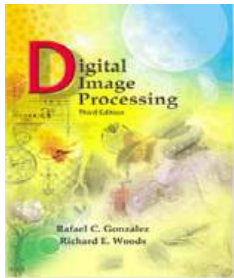
### Image Compression

#### Lossy Predictive coding

*The encoder expects a discrete samples of a signal  $f(n)$ .*

1. A predictor is applied and its output is rounded to the nearest integer,  $\hat{f}(n)$
2. The error is mapped into limited range of values (quantized)  $\dot{e}(n)$
3. The compressed stream consist of first sample and the mapped errors, encoded using variable length coding





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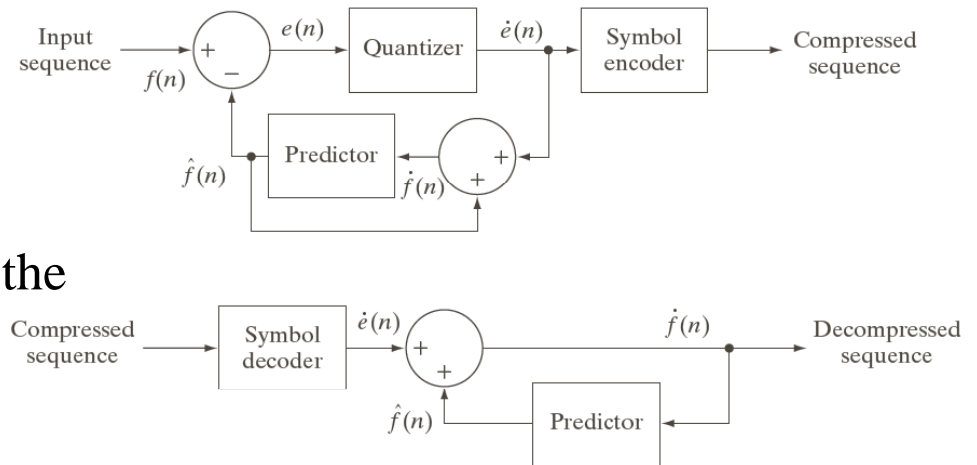
### Lossy Predictive coding

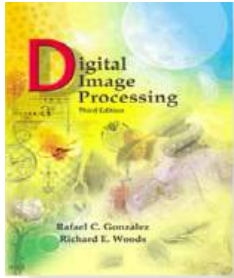
The decoder uses error stream to reconstructs an approximation of the original signal,  $\hat{f}(n)$

1. The predictor is initialized using the first sample.
2. The received error is added to predictor result.

$$\hat{f}(n) = \hat{e}(n) + \hat{f}(n)$$

$$\hat{e}(n) = \begin{cases} +\xi & e(n) > 0 \\ -\xi & \text{otherwise} \end{cases}$$



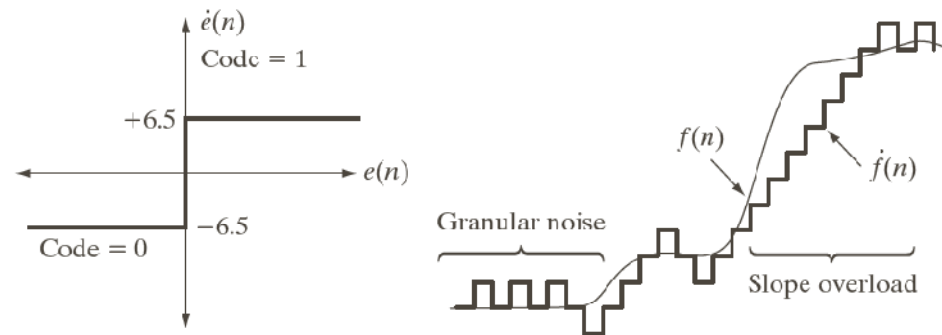


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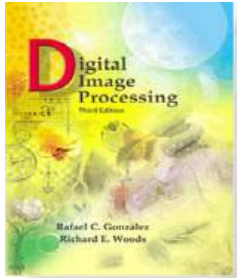
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Input		Encoder			Decoder		Error	
$n$	$f(n)$	$\hat{f}(n)$	$e(n)$	$\dot{e}(n)$	$\dot{f}(n)$	$\hat{f}(n)$	$\dot{\hat{f}}(n)$	$f(n) - \hat{f}(n)$
0	14	—	—	—	14.0	—	14.0	0.0
1	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
2	14	20.5	-6.5	-6.5	14.0	20.5	14.0	0.0
3	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
14	29	20.5	8.5	6.5	27.0	20.5	27.0	2.0
15	37	27.0	10.0	6.5	33.5	27.0	33.5	3.5
16	47	33.5	13.5	6.5	40.0	33.5	40.0	7.0
17	62	40.0	22.0	6.5	46.5	40.0	46.5	15.5
18	75	46.5	28.5	6.5	53.0	46.5	53.0	22.0
19	77	53.0	24.0	6.5	59.6	53.0	59.6	17.5
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.



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### Prediction Error

The following images show the prediction error of the predictor

$$\hat{f}(x, y) = 0.97 f(x, y - 1)$$

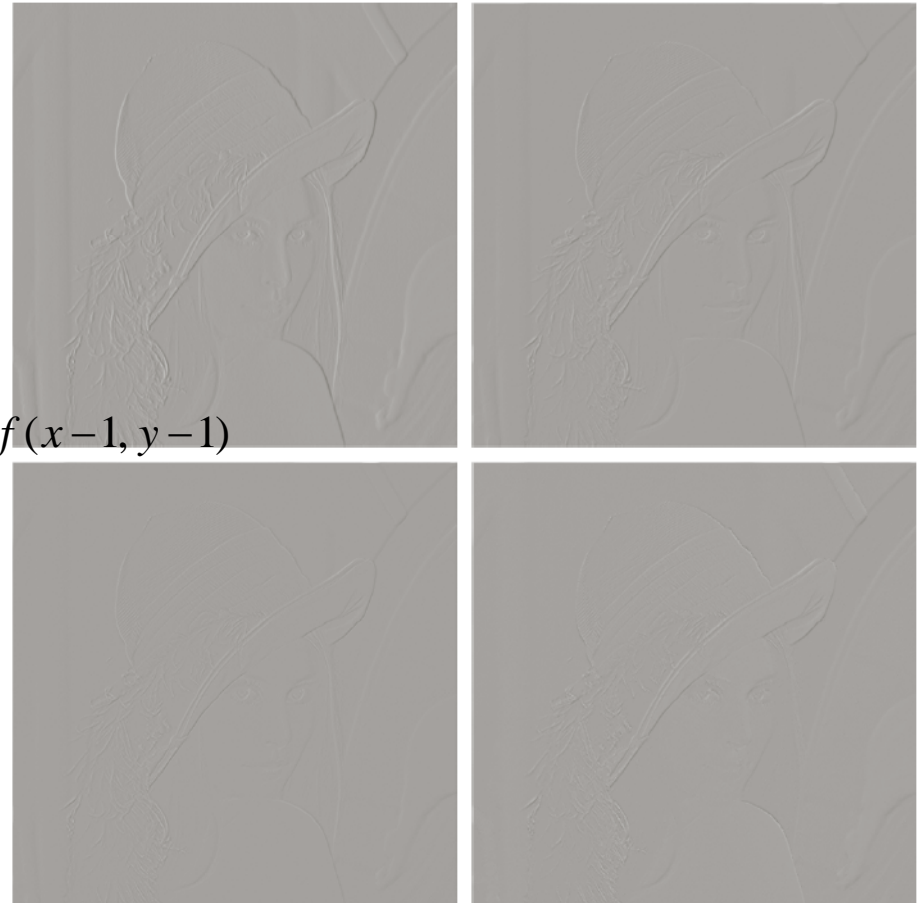
$$\hat{f}(x, y) = 0.5 f(x, y - 1) + 0.5 f(x - 1, y)$$

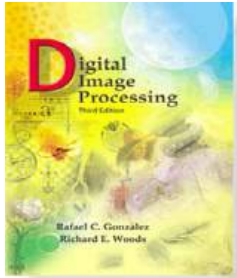
$$\hat{f}(x, y) = 0.75 f(x, y - 1) + 0.75 f(x - 1, y) - 0.5 f(x - 1, y - 1)$$

$$\hat{f}(x, y) = \begin{cases} 0.97 f(x, y - 1) & \Delta h \leq \Delta v \\ 0.97 f(x - 1, y) & \text{otherwise} \end{cases}$$

$$\Delta h = |f(x - 1, y) - f(x - 1, y - 1)|$$

$$\Delta v = |f(x, y - 1) - f(x - 1, y - 1)|$$





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### Chapter 8

### Image Compression

## Optimal Predictors

What are the parameters of a linear predictor that minimize error

$$E\{e^2(n)\} = E\left\{\left[f(n) - \hat{f}(n)\right]^2\right\}$$

While taking into account

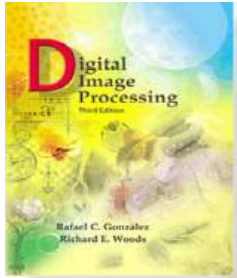
$$\hat{f}(n) = \hat{e}(n) + \hat{f}(n) \cong e(n) + \hat{f}(n) = f(n)$$

Using the definition of linear predictor

$$E\{e^2(n)\} = E\left\{\left[f(n) - \sum_{i=1}^m \alpha_i f(n-i)\right]^2\right\}$$

We assume that  $f(n)$  has a mean zero and variance  $\sigma^2$

$$\alpha = R^{-1}r$$



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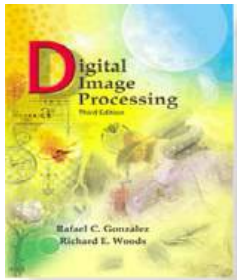
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And  $R^{-1}$  is the  $m \times n$  autocorrelation matrix

$$R = \begin{bmatrix} E\{f(n-1)f(n-1)\} & E\{f(n-1)f(n-2)\} & \dots & E\{f(n-1)f(n-m)\} \\ E\{f(n-2)f(n-1)\} & E\{f(n-2)f(n-2)\} & \dots & E\{f(n-2)f(n-m)\} \\ \vdots & \vdots & \vdots & \vdots \\ E\{f(n-m)f(n-1)\} & E\{f(n-m)f(n-1)\} & \dots & E\{f(n-m)f(n-1)\} \end{bmatrix}$$

$$r = \begin{bmatrix} E\{f(n)f(n-1)\} \\ \vdots \\ E\{f(n-1)f(n-m)\} \end{bmatrix}$$

$$a = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix}$$

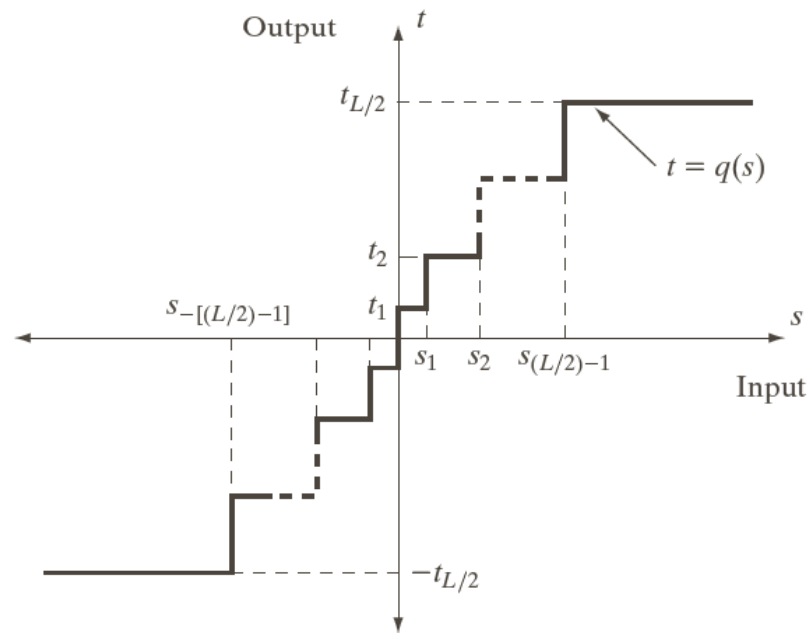


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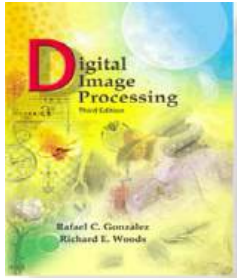
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**FIGURE 8.44**  
A typical  
quantization  
function.



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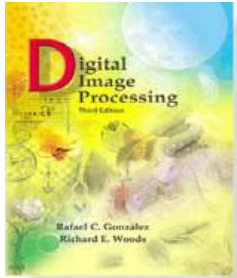
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Levels	2		4		8	
	$s_i$	$t_i$	$s_i$	$t_i$	$s_i$	$t_i$
1	$\infty$	0.707	1.102	0.395	0.504	0.222
2			$\infty$	1.810	1.181	0.785
3					2.285	1.576
4					$\infty$	2.994
$\theta$	1.414		1.087		0.731	

**TABLE 8.12**  
Lloyd-Max  
quantizers for a  
Laplacian  
probability  
density function  
of unit variance.



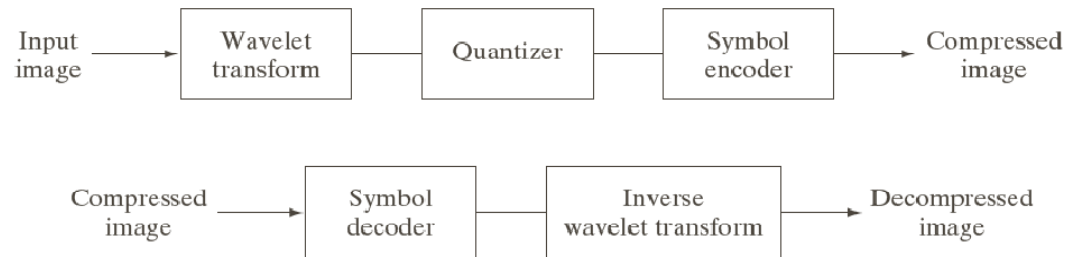


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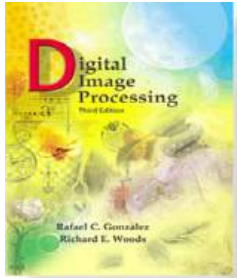
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a  
b

**FIGURE 8.45**  
A wavelet coding system:  
(a) encoder;  
(b) decoder.

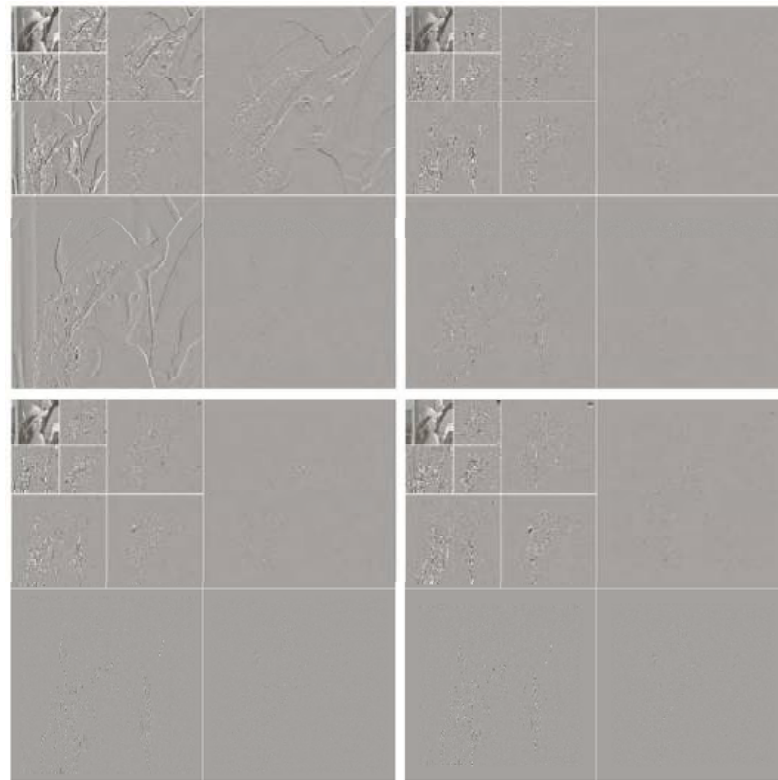


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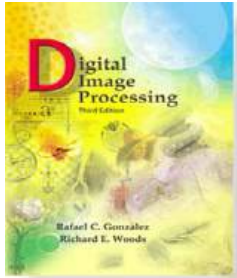
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a b  
c d

**FIGURE 8.46**  
Three-scale wavelet transforms of Fig. 8.9(a) with respect to (a) Haar wavelets, (b) Daubechies wavelets, (c) symlets, and (d) Cohen-Daubechies Feauveau biorthogonal wavelets.



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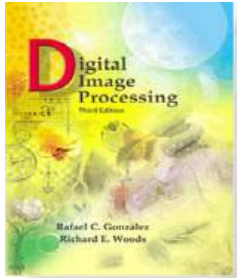
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Wavelet	Filter Taps (Scaling + Wavelet)	Zeroed Coefficients
Haar (see Ex. 7.10)	2 + 2	33.8%
Daubechies (see Fig. 7.8)	8 + 8	40.9%
Symlet (see Fig. 7.26)	8 + 8	41.2%
Biorthogonal (see Fig. 7.39)	17 + 11	42.1%

**TABLE 8.13**

Wavelet transform filter taps and zeroed coefficients when truncating the transforms in Fig. 8.46 below 1.5.



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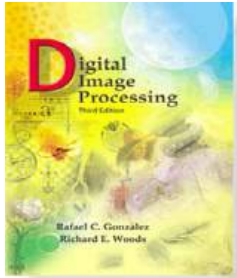
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Decomposition Level (Scales or Filter Bank Iterations)	Approximation Coefficient Image	Truncated Coefficients (%)	Reconstruction Error (rms)
1	256 × 256	74.7%	3.27
2	128 × 128	91.7%	4.23
3	64 × 64	95.1%	4.54
4	32 × 32	95.6%	4.61
5	16 × 16	95.5%	4.63

**TABLE 8.14**  
Decomposition level impact on wavelet coding the 512 × 512 image of Fig. 8.9(a).

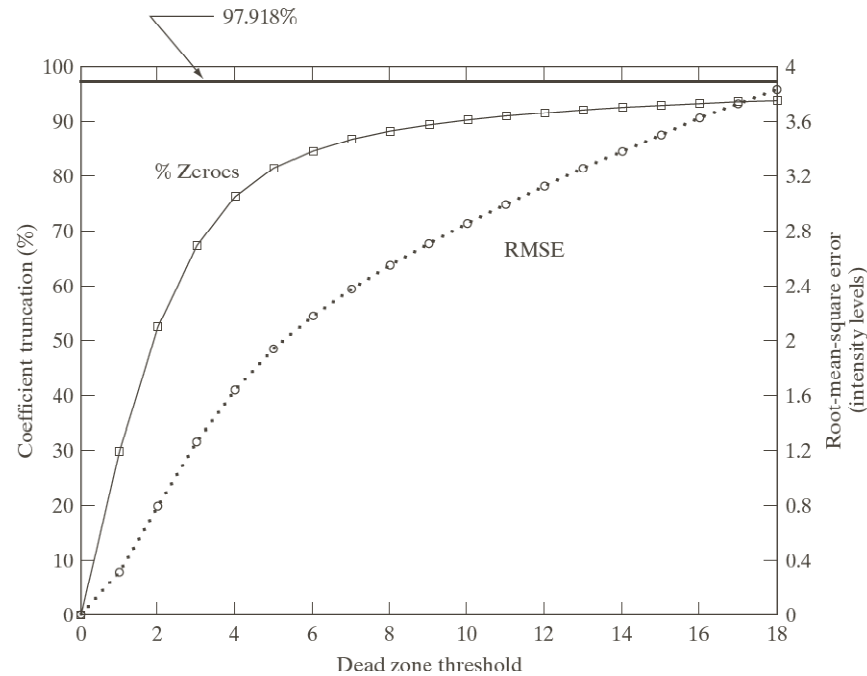


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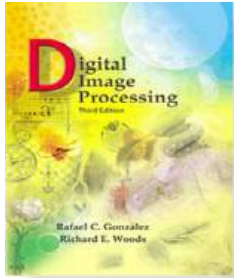
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**FIGURE 8.47** The impact of dead zone interval selection on wavelet coding.



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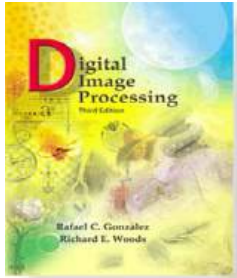
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Filter Tap	Highpass Wavelet Coefficient	Lowpass Scaling Coefficient
0	-1.115087052456994	0.6029490182363579
$\pm 1$	0.5912717631142470	0.2668641184428723
$\pm 2$	0.05754352622849957	-0.07822326652898785
$\pm 3$	-0.09127176311424948	-0.01686411844287495
$\pm 4$	0	0.02674875741080976

**TABLE 8.15**  
Impulse responses of the low- and highpass analysis filters for an irreversible 9-7 wavelet transform.



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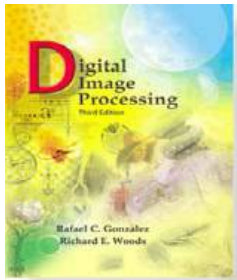
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## Chapter 8 Image Compression

$a_{2LL}(u, v)$ 0	$a_{2HL}(u, v)$ 1	$a_{1HL}(u, v)$ 1
$a_{2LH}(u, v)$ 1	$a_{2HH}(u, v)$ 2	
$a_{1LH}(u, v)$ 1		$a_{1HH}(u, v)$ 2

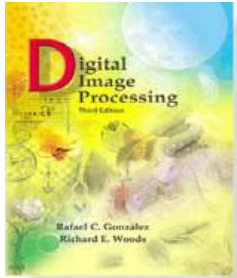
**FIGURE 8.48**  
JPEG 2000  
two-scale wavelet  
transform  
tile-component  
coefficient  
notation and  
analysis gain.

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**FIGURE 8.49** Four JPEG-2000 approximations of Fig. 8.9(a). Each row contains a result after compression and reconstruction, the scaled difference between the result and the original image, and a zoomed portion of the reconstructed image. (Compare the results in rows 1 and 2 with the JPEG results in Fig. 8.32.)





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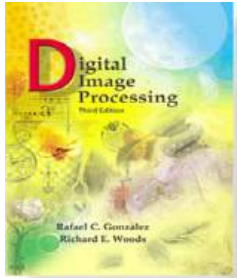
### Digital Image Processing



a  
b c

**FIGURE 8.50**

A simple visible watermark: (a) watermark; (b) the watermarked image; and (c) the difference between the watermarked image and the original (non-watermarked) image.

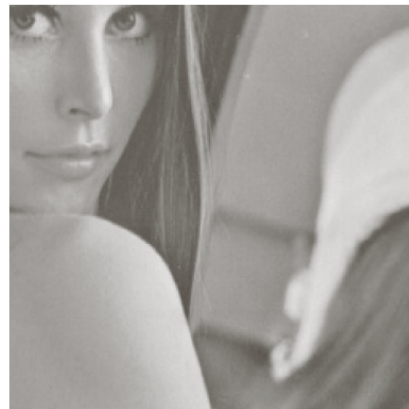
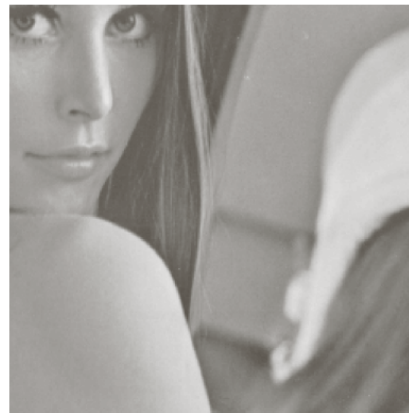


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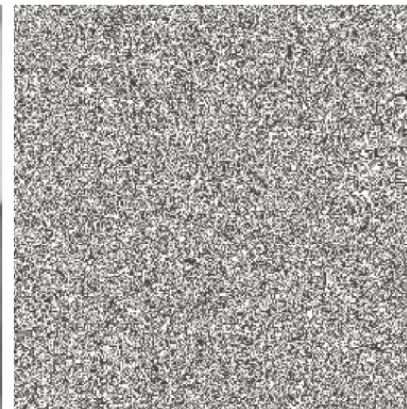
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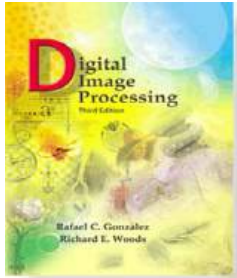


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a b  
c d

**FIGURE 8.51** A simple invisible watermark: (a) watermarked image; (b) the extracted watermark; (c) the watermarked image after high quality JPEG compression and decompression; and (d) the extracted watermark from (c).

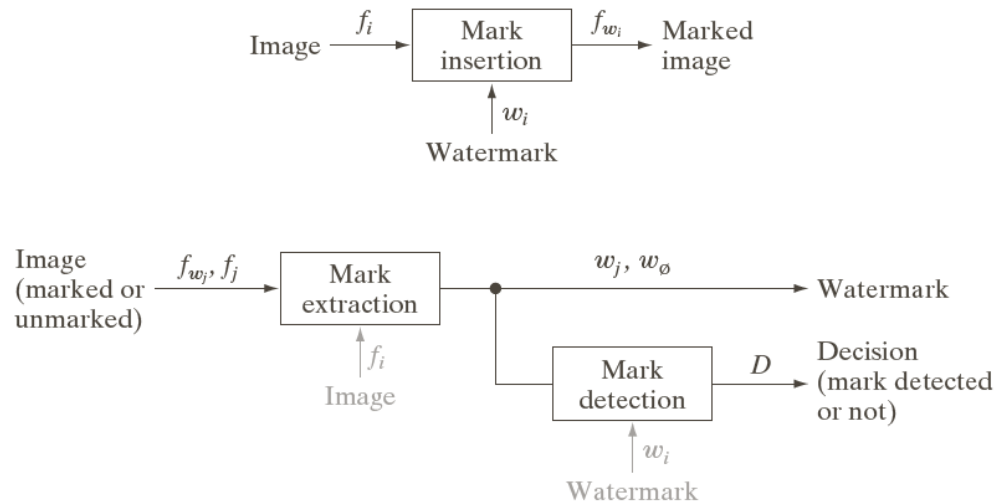


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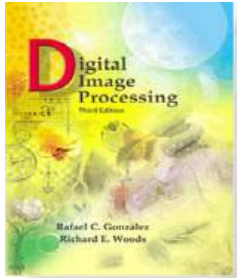
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a  
b

**FIGURE 8.52**  
A typical image watermarking system:  
(a) encoder;  
(b) decoder.



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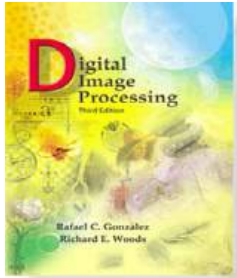
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## Chapter 8 Image Compression



a b  
c d

**FIGURE 8.53** (a) and (c) Two watermarked versions of Fig. 8.9(a); (b) and (d) the differences (scaled in intensity) between the watermarked versions and the unmarked image. These two images show the intensity contribution (although scaled dramatically) of the pseudo-random watermarks on the original image.



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a b c  
d e f

**FIGURE 8.54** Attacks on the watermarked image in Fig. 8.53(a): (a) lossy JPEG compression and decompression with an rms error of 7 intensity levels; (b) lossy JPEG compression and decompression with an rms error of 10 intensity levels (note the blocking artifact); (c) smoothing by spatial filtering; (d) the addition of Gaussian noise; (e) histogram equalization; and (f) rotation. Each image is a modified version of the watermarked image in Fig. 8.53(a). After modification, they retain their watermarks to varying degrees, as indicated by the correlation coefficients below each image.