Module 5

Electric Field Transport in Nanostructures

- Quantum wells are formed at interfaces between semiconductors of different gaps.

- Conduction band electrons in these quasi-2D wells behave almost as free carriers for their motion along planes parallel to the interfaces of the well.

- Above kind of transport, usually called *parallel transport.*

- Transport through the potential barriers at the interfaces is known as *perpendicular transport* which is based on the quantum tunnelling effect.

PARALLEL TRANSPORT

- Electronic transport is parallel to the potential barriers at the interfaces.
- There are chances for electron scattering during the transport since we are dealing with low-dimensional system.
- Parallel transport in nanostructures was happening in all Nano devices like MOSFET, MODFET etc..
- Electron motion takes place in a region free of charged dopants, and therefore, electrons can reach very high motilities.

Electron scattering mechanisms

- Following are the main scattering mechanisms for parallel transport in semiconductor nanostructures.
- ***(i) Electron–phonon scattering***
- ***(ii) Impurity scattering***
- ***(iii) Surface roughness scattering***
- ***(iv) Intersubband scattering***

Electron–phonon scattering

- The quantum of a lattice vibration is commonly referred as a Phonon.
- It is similar to photon and it carries heat in the material.
- Electron-Phonon interaction is commonly observed in most of the materials.
- In reality at room temperature, the lattice of materials is not static and atoms, they actually vibrate about their mean positions.
- The vibrations of the lattice increase with the increase in temperature.
- Phonon scattering mechanism is the predominant one for temperatures higher than about 50 K.

- The electrons will have a chance to make collisions with the vibrating lattice during their motion and such collision is referred as electron-phonon collision or scattering.

- Phonon scattering becomes very considerable in low- dimensional semiconductors since the width '*a'* of the quantum wells is very small.

- In semiconductors, the electron-phonon scattering results in a decrease in mobility of charge carriers involved.

Impurity scattering

- **Impurity scattering** constitutes the largest contribution to scattering in low-dimensional semiconductors **at low temperatures**.
- In **modulation-doped heterostructures, the charged donors are located in the AlGaAs, while electron motion takes place in a separated region in the GaAs** parallel to the interface which is **separated from impurities.**
- Similarly, in a MOS structure, electrons move within the inversion channel, which is separated from impurities located in the thin gate oxide.

- For the calculation of impurity scattering in MODFET quantum heterostructures some simplifying assumptions are usually made.

  – Impurities are supposed to be located in a 2D plane at a distance *d* of the electron channel.

  – However, there is an optimum value of *d*, because if *d* is too large, the concentration of electrons in the channel diminishes significantly, as a consequence of the decrease in the electric field, and the transconductance of MODFETs is greatly reduced.

  – Electrons in the channel which participate in the scattering events are those with energies very close to the Fermi level.

  – Also **assumed that the concentration of impurities is not too high,**

Surface roughness scattering

- Interface scattering is due to the interaction of electrons with a roughened surface, in contrast to an ideal perfect flat surface.

- Interfaces have a roughness at the atomic level, which produces non-specular reflections of carriers, and therefore, a loss of momentum.

- The role of interface scattering for parallel transport in modulation-doped heterostructures is not very important, due to the high perfection of the interfaces when growth techniques such as molecular beam epitaxy are used.

- In the case of MOS structures, interface scattering becomes more important since the oxide is grown thermally and the interface is not as perfect as in the modulation-doped heterostructure.

- The contribution of interface scattering in MOS structures depends on the quantum well width.

- As the width decreases the electron wave function penetrates deeper into the oxide-semiconductor potential barriers, i.e. the electrons are more exposed to the interface roughness and the corresponding scattering increases.

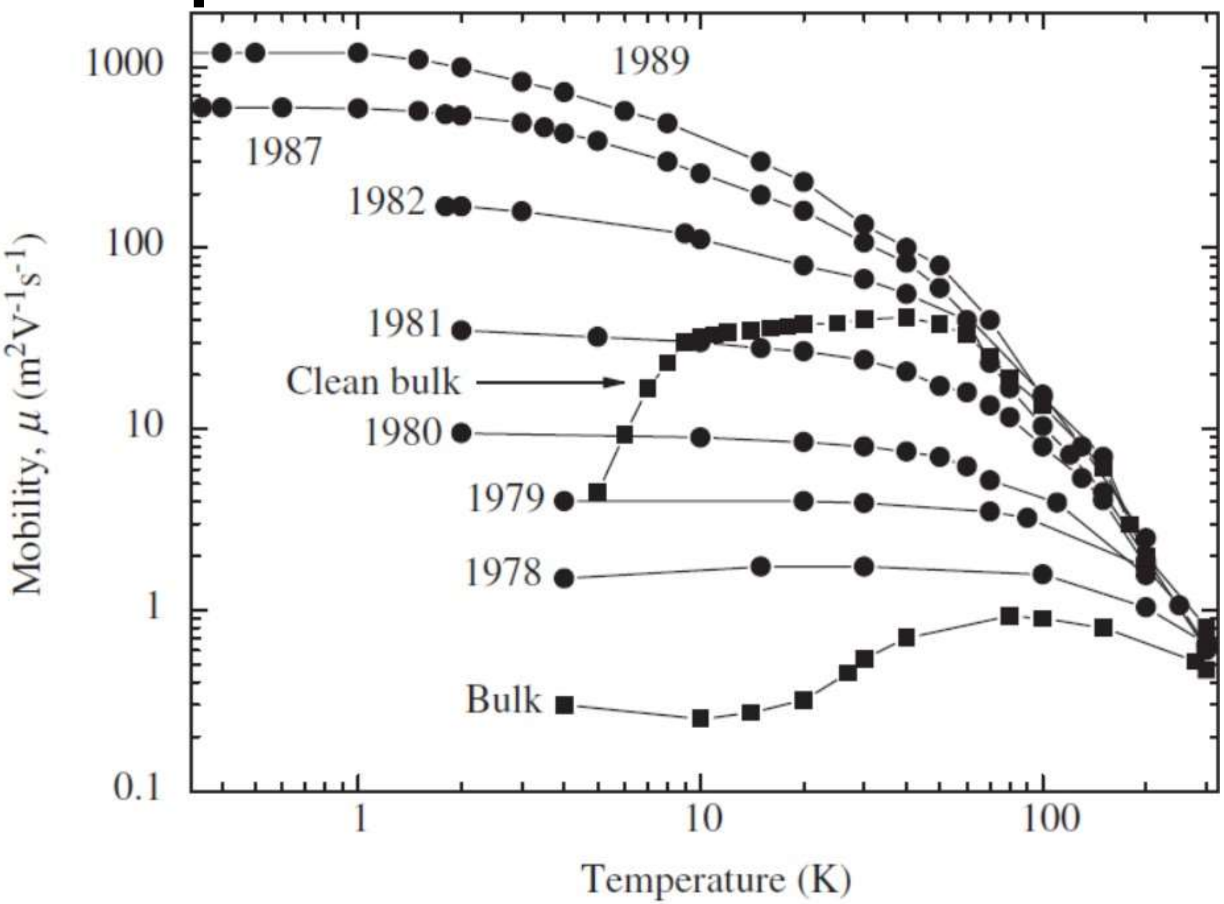
- Roughness scattering, like impurity scattering, only becomes significant at temperatures low enough for phonon scattering to be negligible.

Intersubband scattering

- For large electron concentrations in the well, the levels with energies higher than the first one *E*1 will start to become filled.
- Then, electrons with energies can undergo an intraband scattering transition within the subband n=2 or an interband transition between subbands n = 1 and n = 2.
- As a consequence, the electron mobility should become smaller.

- In summary, **as the electron concentration in a quantum well increases, additional scattering channels start to contribute to the overall scattering rate, and the mobility of the electron decreases.**

Mobility improvements



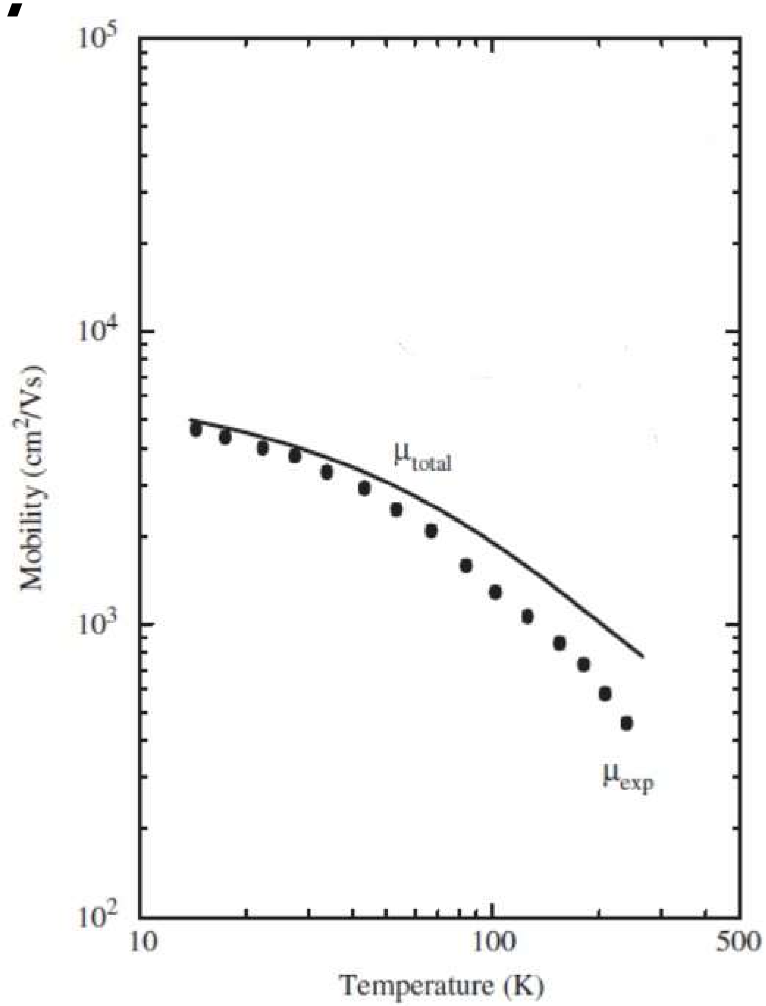
Increase of the mobility for parallel electron motion

- It is observed a large improvement in electron mobility of parallel transport at low temperatures in GaAs-based nanostructures.

- There are several reasons for this
  - The main reason, is due to the physical separation between dopants and carriers in modulation-doped heterostructures.
    - A semi-insulating layer, called a spacer, is added between the donor layer and the 2D electrons in the conducting channel.
    - This spacer is especially effective at low temperatures for which the impurity–electron scattering mechanism becomes predominant.

  - Another reason for the large increase in the electron mobility is the high purity of the bulk material, caused by the improvement in the growth techniques such as molecular beam epitaxy.
- As the temperature approaches 100K and gets close to room temperature, the dominant scattering mechanisms are due to phonons.

MOSFET vs MODFET mobility



Temperature dependence of electron mobility in a silicon MOSFET.

- Mobility of electrons in a silicon MOSFET is much lower than in a MODFET.

- There are several reasons for this
  - The effective mass of electrons in Si is much higher than in GaAs.

  - the effect of impurity scattering in a Si MOSFET, caused by charges and impurities in the oxide and the interface, is larger than in the case of AlGaAs/GaAs
  - The silicon–oxide interface, grown thermally, is not as perfect as the AlGaAs/GaAs interface and hence surface roughness scattering is higher at low temp.

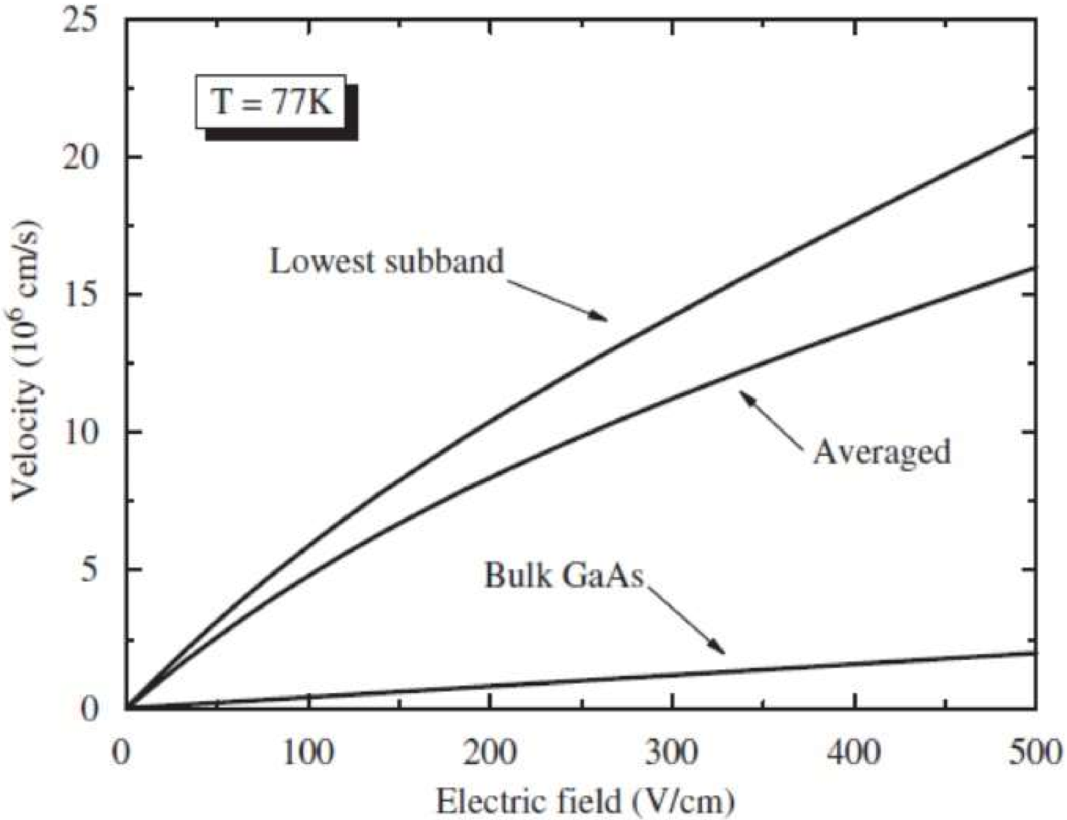
Hot electrons in parallel transport

- Electrons are accelerated by the electric field to kinetic energies much higher than their energies at thermal equilibrium.

- After the acceleration by high electric fields, the electron energy distribution corresponds to an effective temperature higher than that of the crystal lattice, and the electrons receive the name *hot electrons*.
- average energy *E* is defined by the equation $\overline{E} = \frac{3}{2}kT_e$

- The electron velocities reached under the action of an electric field are higher than in bulk GaAs and that the difference becomes larger at low temperatures.

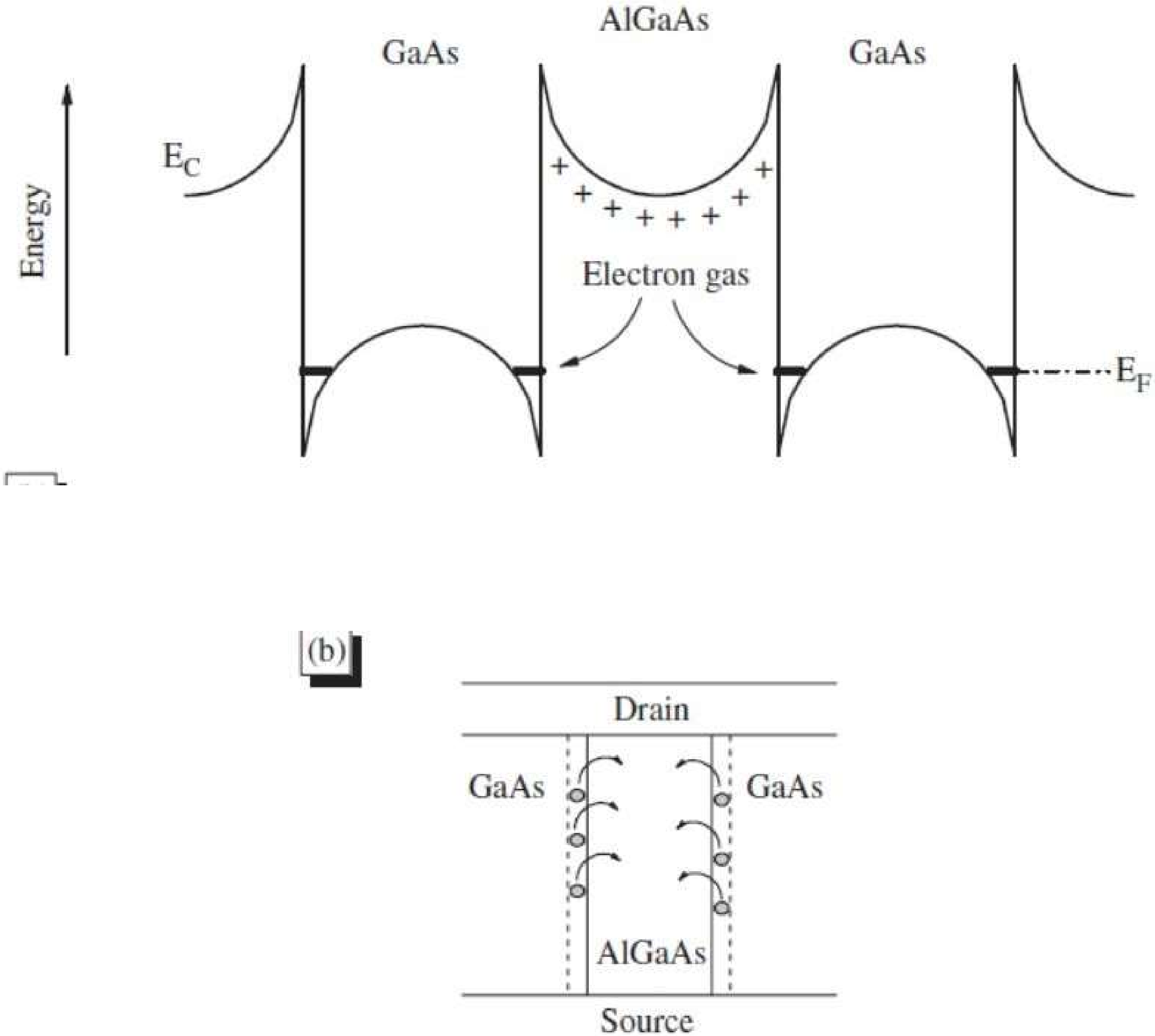
- The value of the velocity is specially high for the lowest subband (E=E_1) in comparison to the second subband (E=E_2) for which the electron wave function extends much more outside the barrier region.



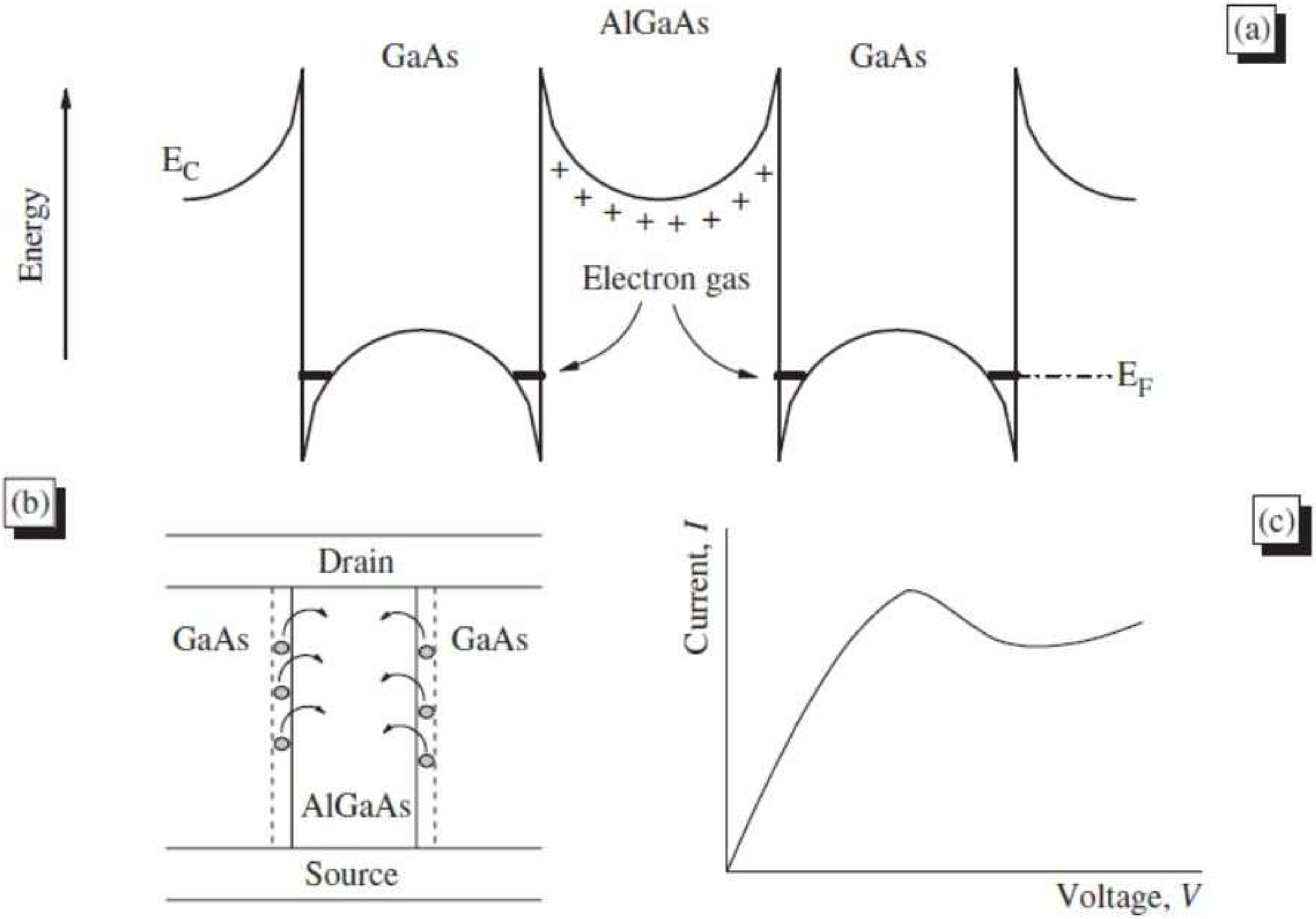
Electron drift velocity for parallel motion in AlGaAs–GaAs modulation-doped heterostructures

Real-Space Transfer (RST)

- An interesting effect, called *real-space transfer (RST)*, arises for hot electron parallel transport in quantum heterostructures.
- If the energy of the hot electrons is high enough, some of them will be able to escape from the well.
- electrons are transferred from the undoped GaAs to the surrounding AlGaAs doped semiconductor.
- It means, electrons can be transferred from a high electron mobility material (GaAs) to one with a lower mobility (AlGaAs) as the voltage between source and drain is increased.
- As a consequence, a *negative differential resistance (NDR)* region in the *I*–*V* characteristics is observed.
- the NDR effect leads to new kinds of devices such as resonant tunnelling transistors.



AlGaAs
GaAs
GaAs
Energy
E_C
Electron gas
E_F
(b)
Drain
GaAs
GaAs
AlGaAs
Source



(a) Schematics of the RST mechanism; (b) structure of a device based on RST; (c) I–V characteristics.

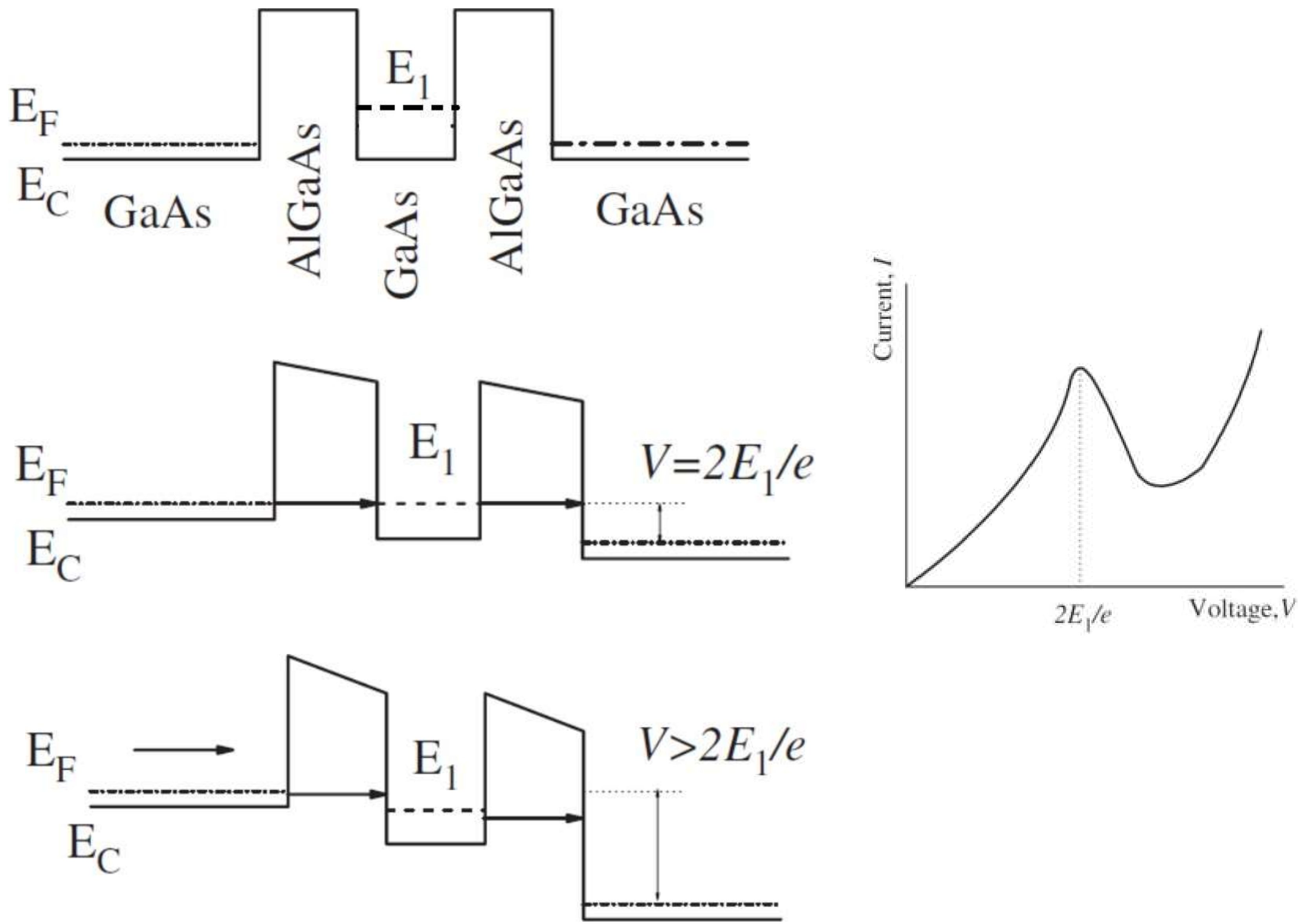
PERPENDICULAR TRANSPORT

- Motion of the carriers perpendicularly to the planes of the potential barriers separating quantum heterostructures.

- This kind of transport is often associated to quantum transmission or tunnelling.

Resonant tunnelling

- It has more applications in high frequency electronic diodes and transistor.
- Considera nanostructure made of undoped GaAs surrounded by AlGaAs in each side.


E1
EF
EC
GaAs
AlGaAs
GaAs
AlGaAs
GaAs
E1
EF
EC
V=2E1/e
E1
EF
EC
V>2E1/e
Current, I
2E1/e
Voltage, V

- RT occurs for a voltage $V1 = 2\ E/e$, where E coincides with the quantized energy level $E1$.

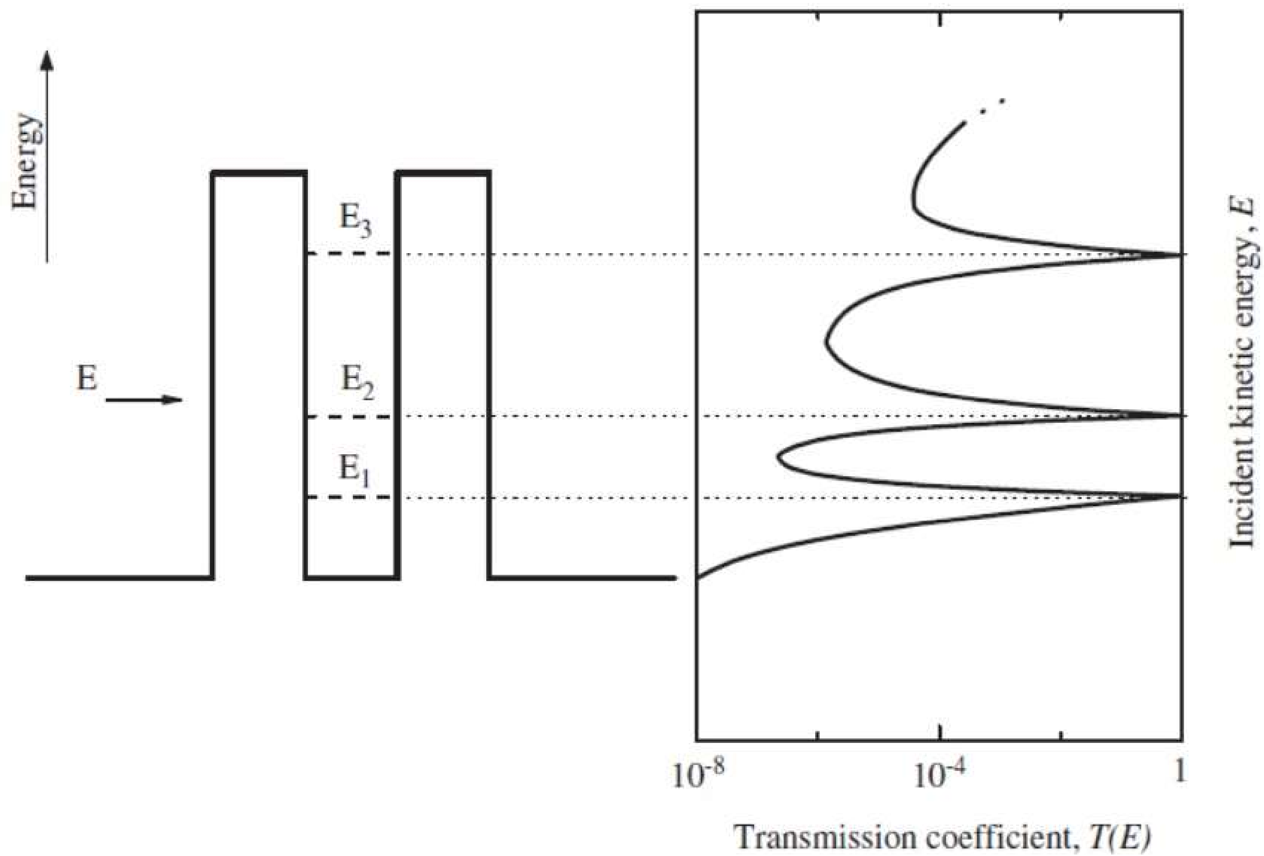
- In this situation, the Fermi level EF of the metallic contact on the left coincides with the n=1 level in the well.

- Then, the tunnelling transmission coefficient approaches unity and a large current flows through the structure.

- As the voltage increases over $2E1/e$, EF surpasses $E1$ and the current through the structure decreases.

- In the I–V characteristics of Figure, after the maximum, the slope of the curve becomes negative, i.e. there exists a region with differential negative resistance.

Dependence of $T(E)$ as a function of E



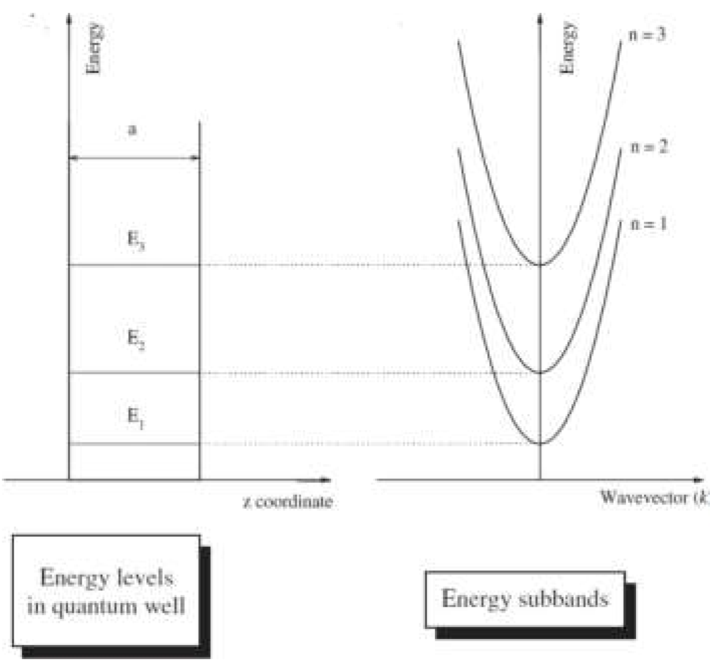
- Consider an RT structure with three energy levels in the quantum well.
- Observe that the transmission coefficient is one, at energies corresponding to the three levels. Or when the energy of the incident electron is aligned with these levels.

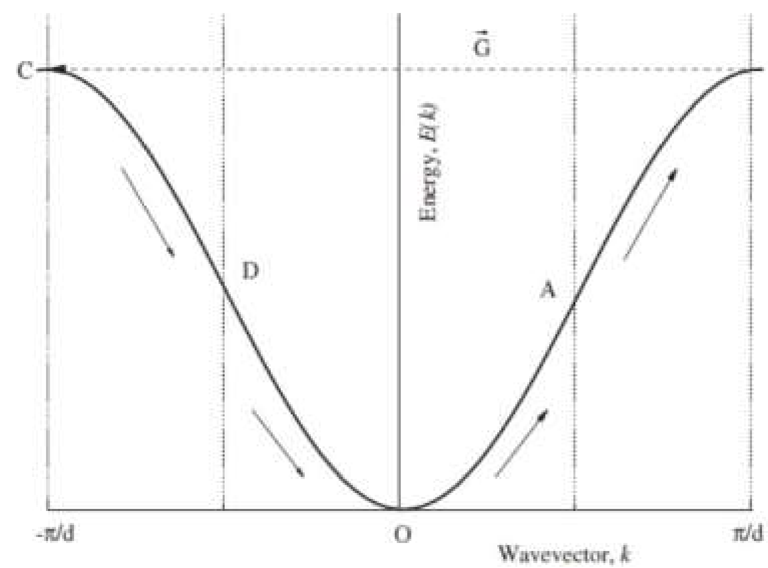
Electric field effects in superlattices

- Electron states in superlattices are grouped in electronic bands or minibands, which are very narrow in comparison with bands in crystals.

- Electrons in narrow bands, under the action of an electric field will reveal some observable effects, such as *Bloch oscillations*.

- Suppose an electronic band in *k*-space such as the one shown in Figure which is similar to the first miniband of a superlattice.





- The equation of motion for an electron in this band, under the action of an electric field is $\hbar\frac{dk}{dt} = -eF$

- The solution of above equation for the wave number is

$$k(t) = k(0) - \frac{eF}{\hbar}t$$

- Electron is initially at rest at the origin O.

- Electron starts to move from O towards A until it reaches the point B.

- At B, the electron is transferred to point C and it moves in *k*- space towards D by the action of the field, closing one cycle in *k*-space when the electron reaches O again.

- The motion of the electron is periodic and the velocity is given by equation: $v = \frac{1}{\hbar}\frac{dE}{dk}$

- The period *T*B of the oscillatory motion.

$$T_B = \frac{2\pi}{\omega_B} = \frac{2\pi\hbar}{eFd}$$

- In order to experimentally observe Bloch oscillations, *T*B should be shorter that the relaxation time due to scattering.

- Bloch oscillations cannot be observed in bulk solids because their typical values of *T*B are much longer than the corresponding ones in a superlattices.

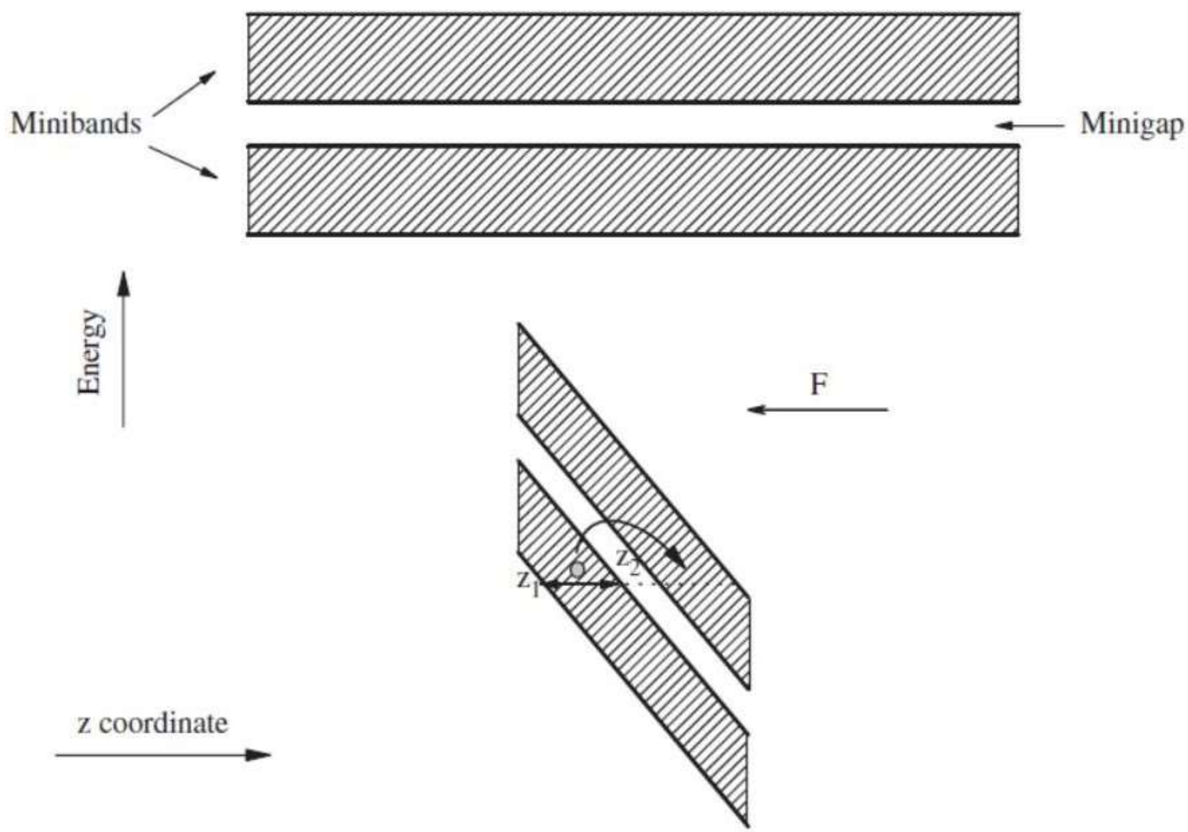
- In practice, the value of *T*B cannot be made very low by making the values of *F* very high since this would produce Zenner tunnelling and Bloch oscillations would not be produced.

- If a constant electric field *F* is applied in the *z*-direction, the bands become tilted with a slope equal to –*eF*.

- *Now t*he expression of the potential energy becomes

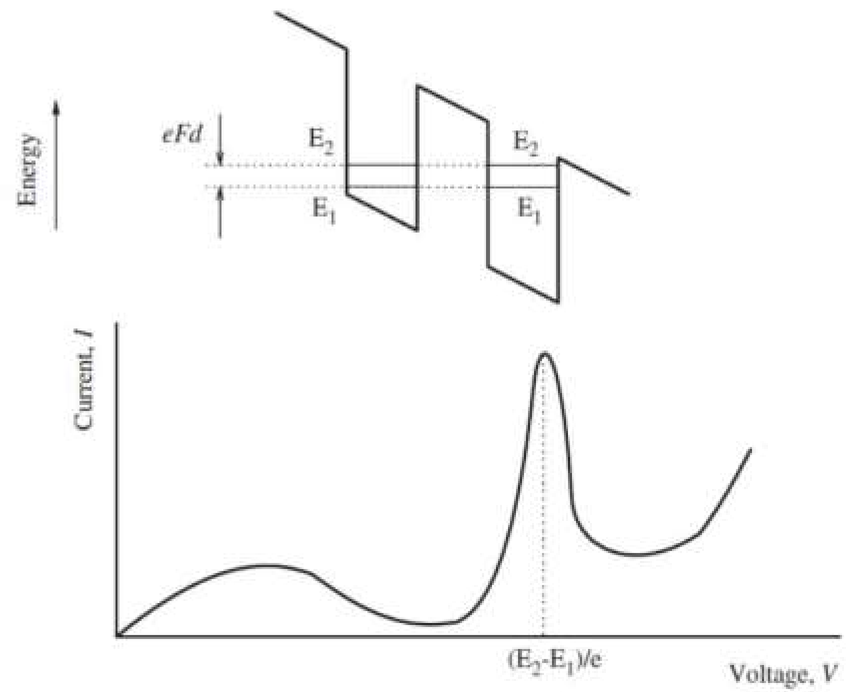
$$E(z) = E0 - eF\,z$$

where E0 is the energy at origin.



- An electron with total energy E_T will oscillate in space between locations $z1$ and $z2$.
- Energy levels in each quantum well of width a of the superlattice will form a ladder of step height eFa, where F is the applied electric field.
- Resonant tunnelling occurs when high electric fields are applied through the structure and the successive quantum wells differ in energy by about eFd.

$$E2 - E1 = eF\,d$$

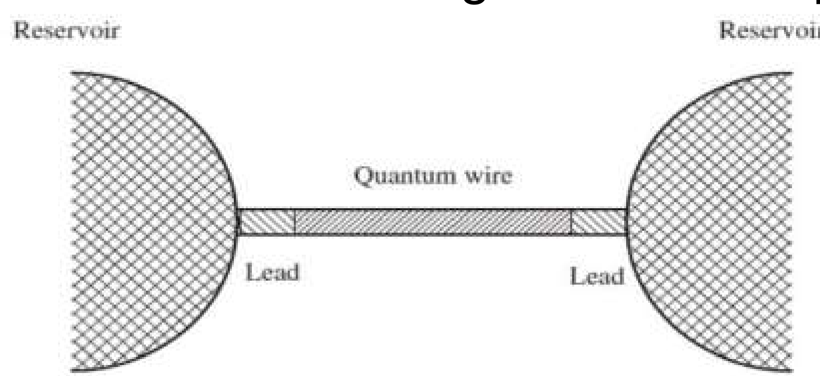


QUANTUM TRANSPORT IN NANOSTRUCTURES

- It happens when nanostructures are connected to an external current by means of contacts or leads.
- This transport is also called mesoscopic transport.
- In order to observe quantum transport effects in semiconductor nanostructures, some conditions must be met.
- Quantum transport will be more easily revealed in nanostructures in which the electron effective mass is small, since this implies high electron mobilities.
- Transport in mesoscopic devices is usually ballistic, since the dimensions of the devices are smaller that the mean free path of electrons.

Quantized conductance

- Let us consider a 1D mesoscopic semiconductor structures like quantum wires.
- The 1D quantum wire is connected through ideal contacts, which do not produce scattering events, to reservoirs characterized by Fermi levels E_{F1} and E_{F2}.
- In order for the current to flow through the quantum wire, a small voltage V is applied between the reservoirs.
- As a consequence, there is a potential energy eV between the two reservoirs equal to $E_{F1} - E_{F2}$.
- If the wire is short enough, i.e. shorter than the electron mean free path in the material, there would be no scattering and the transport is ballistic.



Schematics of a 1D mesoscopic system

- Current through the wire is defined as

$$I = \frac{2e^2}{h} V$$

- The value of the conductance *G* ≡ *(I/V)* is therefore:

$$G = \frac{2e^2}{h}$$

- It is interesting to observe that the conductance of the quantum wire is length independent.

- Since the quantity $2e^2/h$ appears very often, it is usually called *fundamental conductance*.

- $G_0 = \frac{e^2}{h}$ is called the *quantum unit of conductance* and corresponds to a *quantum resistance* of value $R_0 = \frac{h}{e^2} = 25.812807\,\mathrm{k\Omega}$

- Assuming the existence of several channels(Subbands) between the reservoirs.

- let us suppose that the leads can inject electrons in any channel or mode m and after interacting the electrons emerge through any channel n.

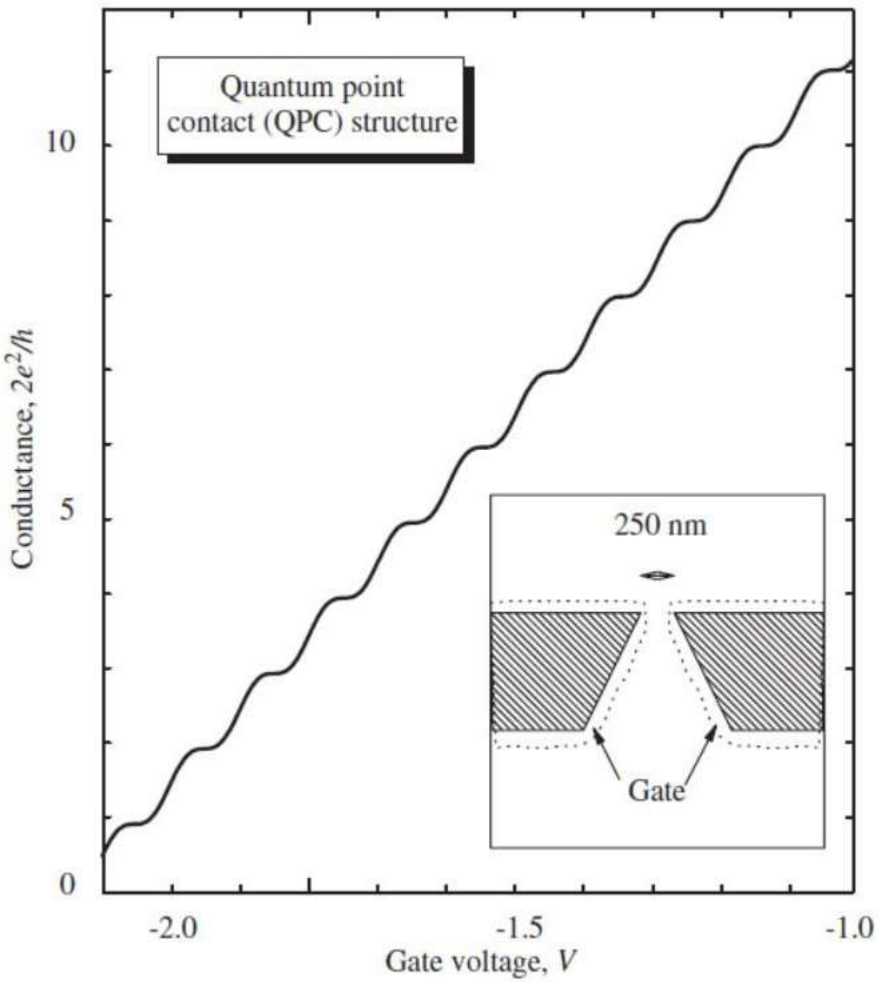
- The total conductance will be obtained by adding over all channels:

$$G = \frac{2e^2}{h} \sum_{\mathrm{n,m}}^{N} |t_{\mathrm{nm}}|^2$$

  - where *N* is the number of quantum channels
  - $|t\mathrm{nm}|^2$ is the quantum mechanical transmission probability.

- The above expression is known as *Landauer formula*.

Quantum conductance as a function of voltage

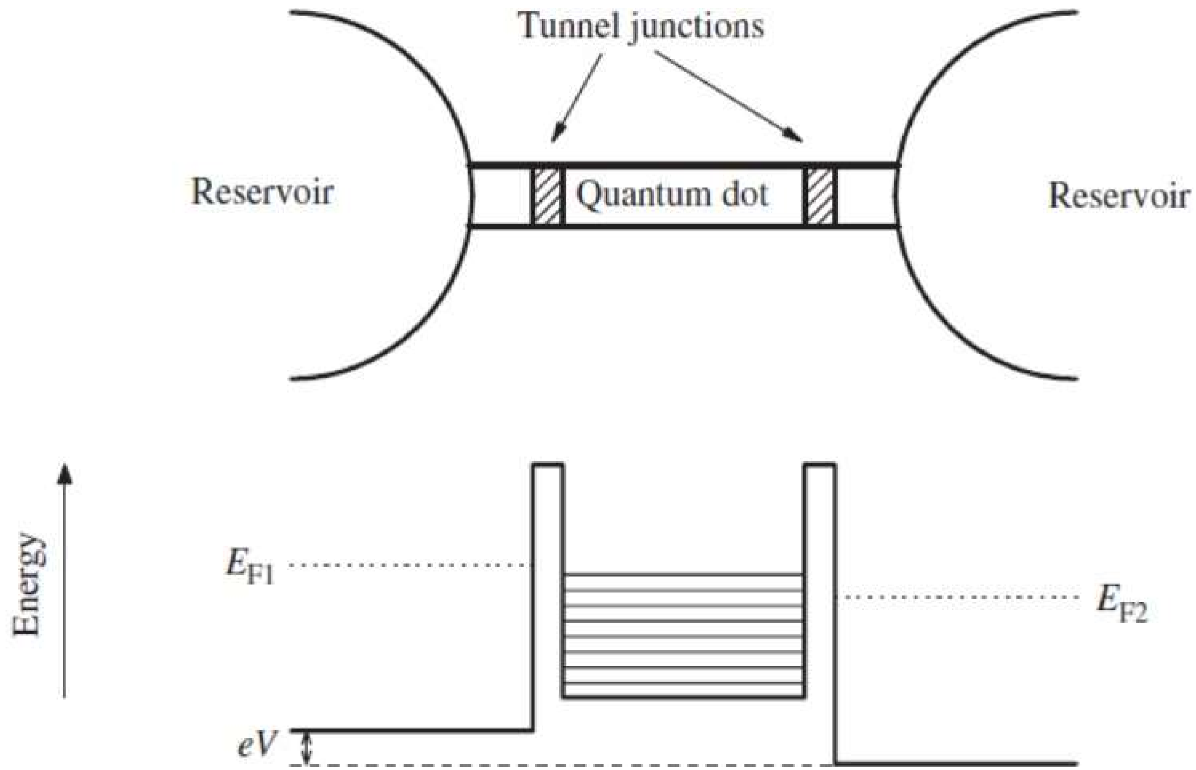


- Consider the structure illustrated in the inset of Figure with an external voltage is applied to the gate .
- The electrons in the 2D plane are constrained to travel through a very small or 1D region, as a consequence of the distribution in electrical voltage.
- This structure is called *quantum point contact (QPC).*
- It is observed that values of the conductance are quantized in multiples of the fundamental Conductance $2e^2/h$ when the voltage is varied.

Coulomb blockade

- In semiconductor devices, the magnitude of the currents are reduced as the feature size of the device shrinks.
- A typical semiconductor device utilizes many electron; for example there can be $10^{11} – 10^{12}$ electrons in 1 cm^2 area of a typical MOSFET device.
- If the device size is very small, then a single electron may be involved in the device application.
- What happens when the current is transported by just one single electron.
- Even the change of one elementary charge in such small systems has a measurable effect in the electrical and transport properties of the dot. This phenomenon is known as *Coulomb blockade.*

- Let us imagine a semiconductor quantum dot structure, connected to electron reservoirs at each side by potential barriers or tunnel junctions.



- In order to allow the transport of electrons to or from the reservoirs, the barriers will have to be sufficiently thin, so that electrons can cross them by tunnel effect.

- Suppose the number of electrons in the dot is *N*.

- we wish to change the number *N* of electrons in the dot by adding just one electron.

- The above electron will have to tunnel for instance from the left reservoir into the dot.

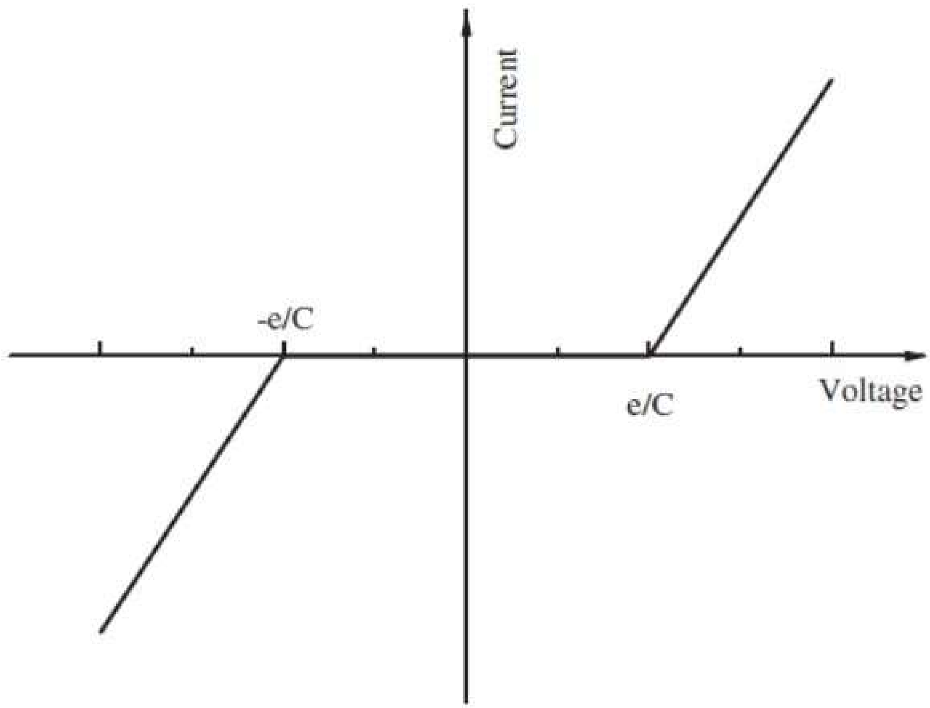
- For tunneling, we will have to provide the potential energy eV to the electron by means of a voltage source.

- Potential energy in a capacitor is
  $(1/2)CV^2$ Q=CV

- Potential energy $=Q^2/2C$
- Therefore an energy of at least $e^2/2C$ will have to be provided to the electron.
  E=QV

- It means that for the electron to enter the dot, the voltage will have to be raised to at least $e/2C$.
- we see that electrons cannot tunnel if $V< e/2C$

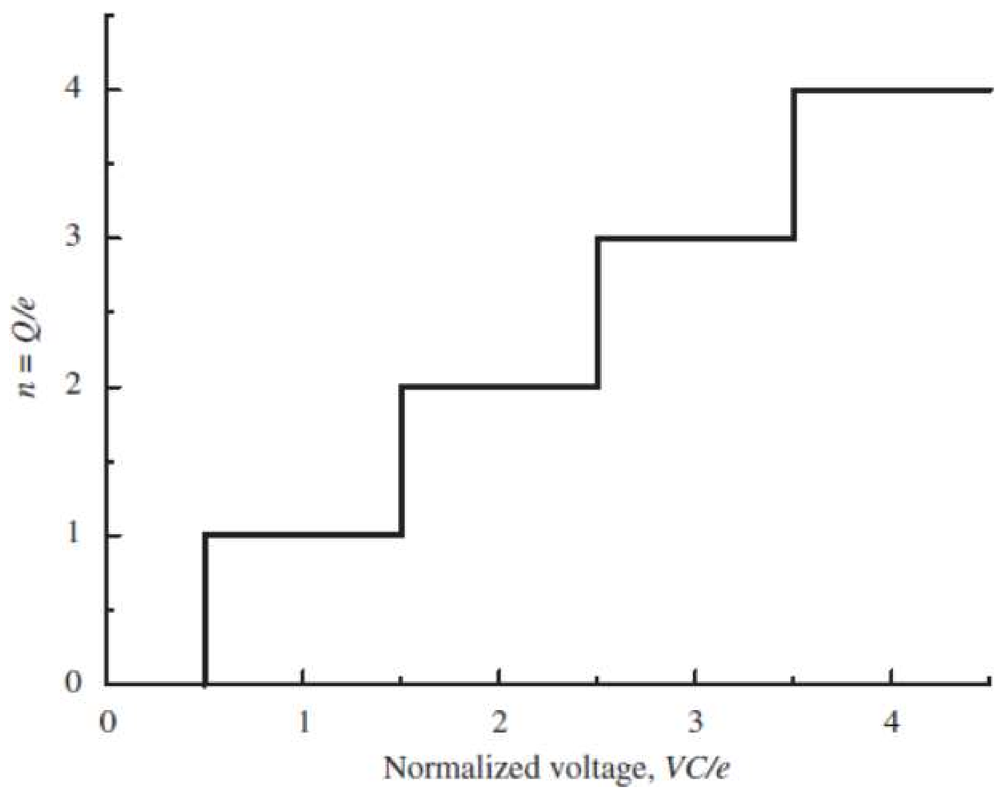
- Therefore, there is a voltage range, between $-e/2C$ and $e/2C$, in which current cannot go through the dot, hence known as Coloumb Blockade.



I–V characteristics in a quantum dot showing the Coulomb blockade effect.

- Evidently if the process is continued and we keep adding more electrons then it can observe discontinuities in the current through the quantum dot whenever the voltage acquires the values expressed by:

$$V = \left(\frac{1}{2C}\right)(2n+1)e, \qquad n = 0,\ 1,\ 2,\ \ldots$$



- As the size of the quantum dot is reduced, and therefore C gets smaller, the value of the energy necessary to change the number of electrons in the dot increases.

- In this case, it will be easier to observe the Coulomb blockade, since the changes in voltage and electric energy for electrons to enter the dot also increase.

- This change in electric energy has to be much larger than the thermal energy kT at the working temperature, in order to observe measurable Coulomb blockade effects.
- SO it is better to have $C \ll \frac{e^2}{kT}$

- For this condition to be fulfilled, either the capacitance of the dot should be very or we should work at very low temperatures, usually smaller than 1 K.

Effect of Magnetic field on a crystal

- Application of high magnetic fields to a crystal will collapse energy states of conduction electrons into different levels.

- Above levels are called Landau levels.

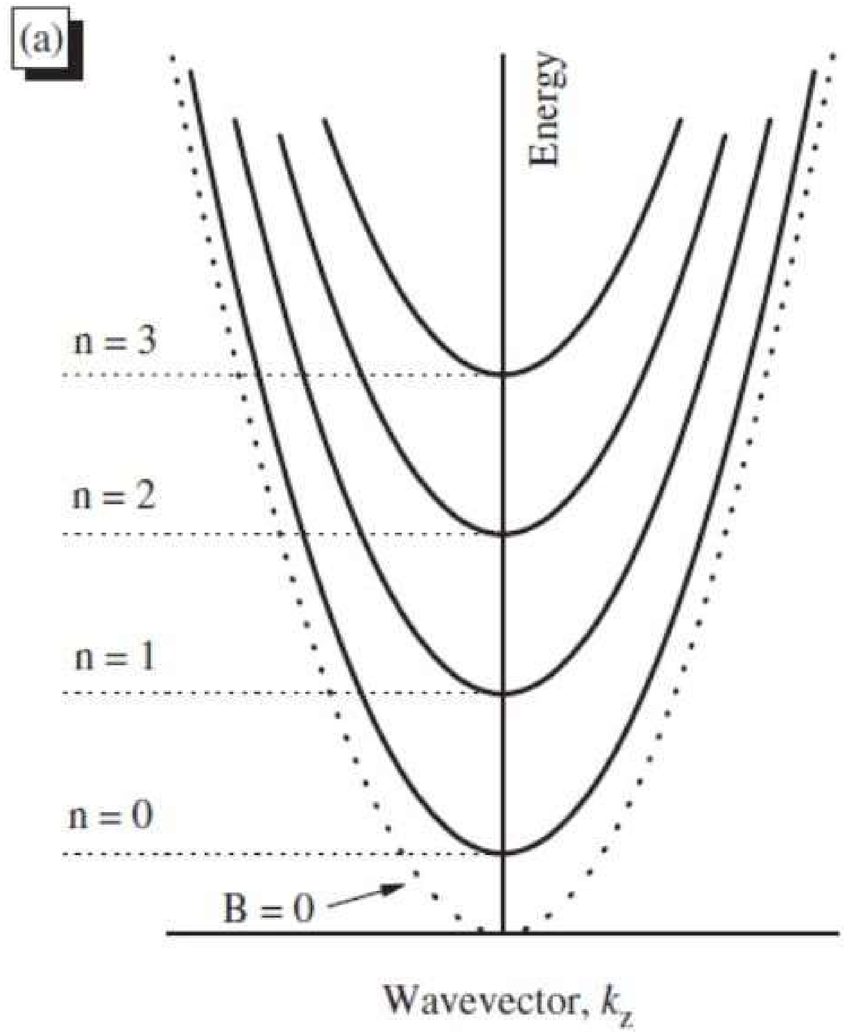
- Consider a magnetic field *Bz*, applied in the *z*-direction.

- This will quantize energies in the conduction band into landau levels.

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m_e^*}, \quad n = 0, 1, 2, \ldots$$

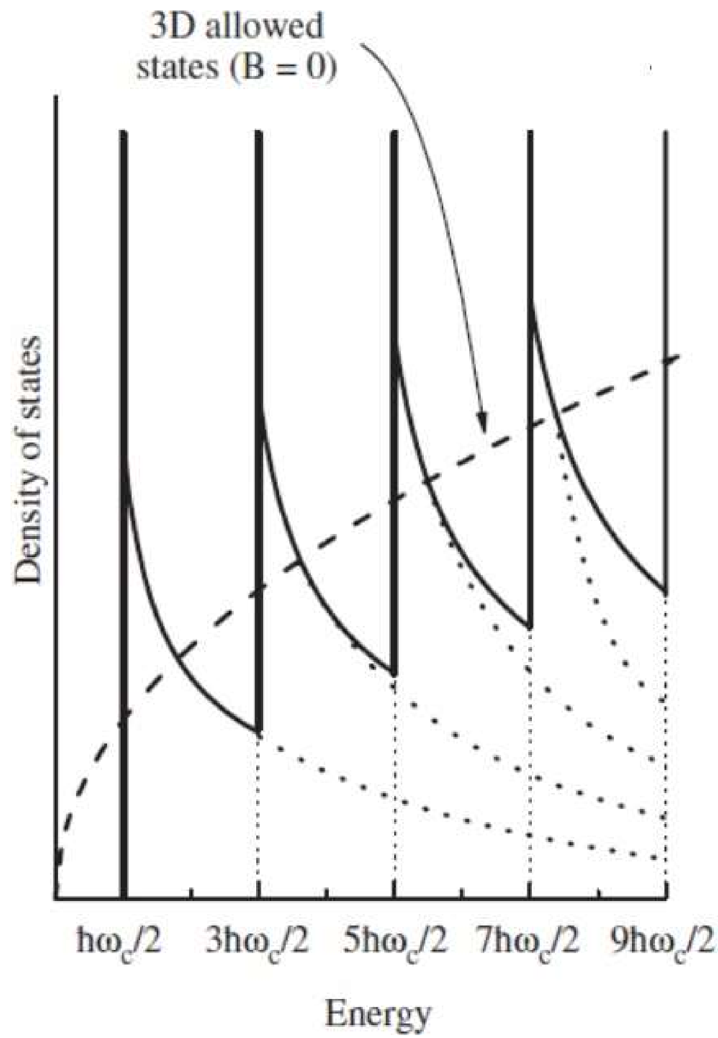
- where *ωc* is the frequency given by $\omega_c = \dfrac{eB_z}{m_e^*}$

- In previous example, the field does not alter the motion of the electrons along the *z*- direction.
- The electrons behave with respect to the *z*-direction as if they were free.
- On the other hand, the electron motion in the *x* and *y* directions is quantized.



Electron energy bands for a 3D solid vs the *z*-direction wave vector for different Landau levels (*n* = 0, 1, 2 . . .)

Effect of magnetic field on 3D density of states



- For a 3D material DOS is proportional to the root of E.

- When a magnetic field Bz is applied, the 3D allowed states in k-space collapse with a degeneracy factor $g_n = eB/\pi h$.

LOW-DIMENSIONAL SYSTEMS IN MAGNETIC FIELDS

- In a 2D electron system, under the action of B, the energy spectrum becomes completely quantized.

- Schrödinger's equation for an electron in a 2D system under the action of a magnetic field applied in a direction (z) perpendicular to the low- dimensional system is given as

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{2m}\left(i\hbar\frac{\partial}{\partial y}+eBx\right)^2\right]\psi(x,y)=E\psi(x,y)$$

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-\frac{i\hbar eBx}{m}+\frac{(eBx)^2}{2m}\right]\psi(x,y)=E\psi(x,y)$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega_c^2(x-x_0)^2\right]\varphi(x)=E_n\varphi(x)$$

$$x_0=\frac{hk}{eB}$$

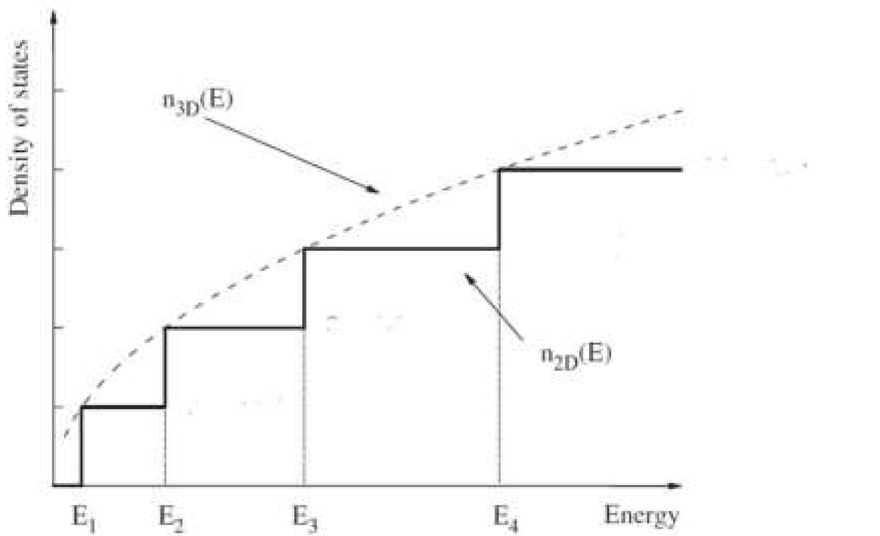
- Energy of landau levels is defined as

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c, \quad n = 0, 1, 2, 3, \ldots \qquad \omega_c = \frac{eB_z}{m_e^*}$$

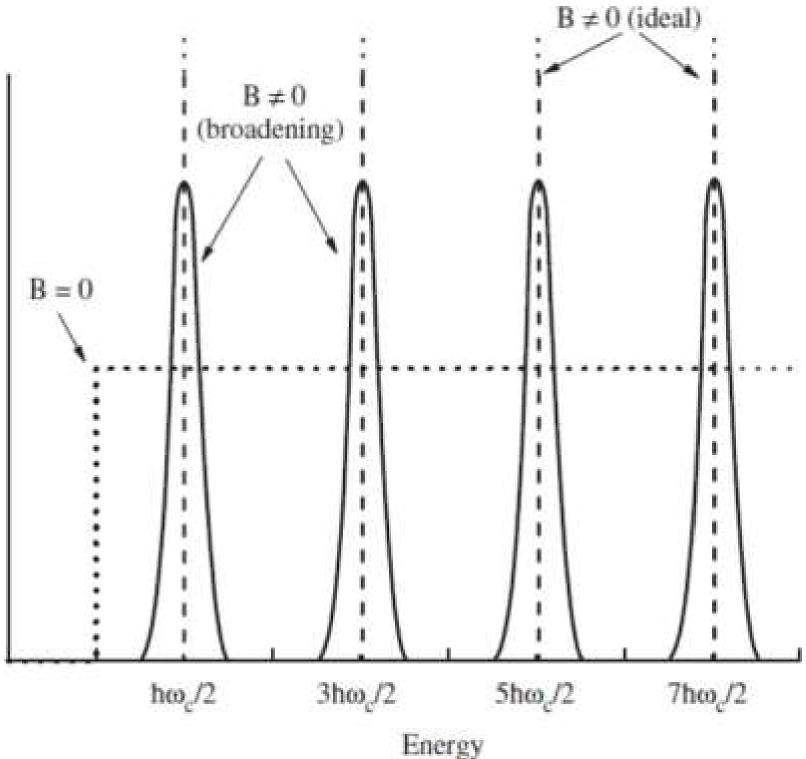
- Note that the energy attributed to the magnetic field depends only on the quantum number *n* and the magnetic field *B* through *ω*c.

DENSITY OF STATES OF A 2D SYSTE IN A MAGNETIC FIELD

- The DOS function of the 2D gas (with B = 0), collapses into a δ-function at each Landau level as a consequence of the application of B.



Density of states function for a 2D electron system, as a function of energy.



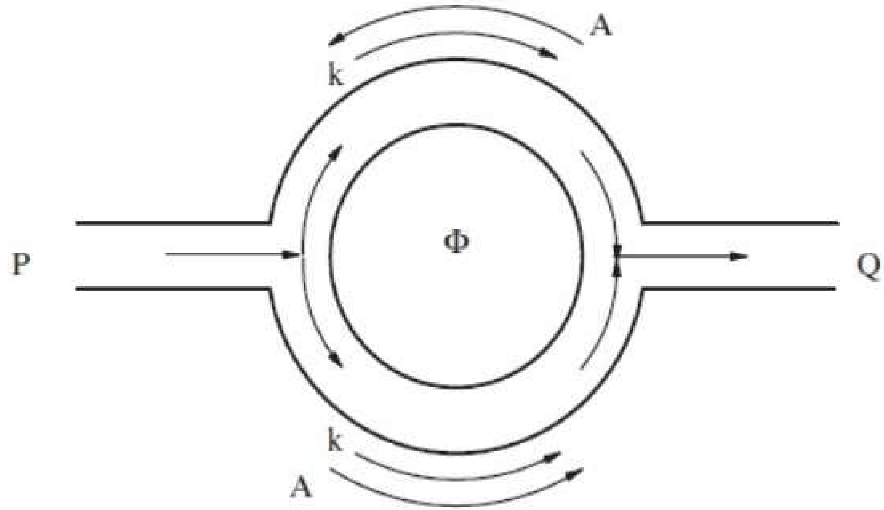
- Since all the levels in an interval *ω*c collapse into the same Landau level when the field *B* is applied, the *degeneracy D* of each Landau level should be given by

$$D = \frac{m_e^*}{2\pi\hbar^2}\hbar\omega_c = \frac{eB}{2\pi\hbar}$$

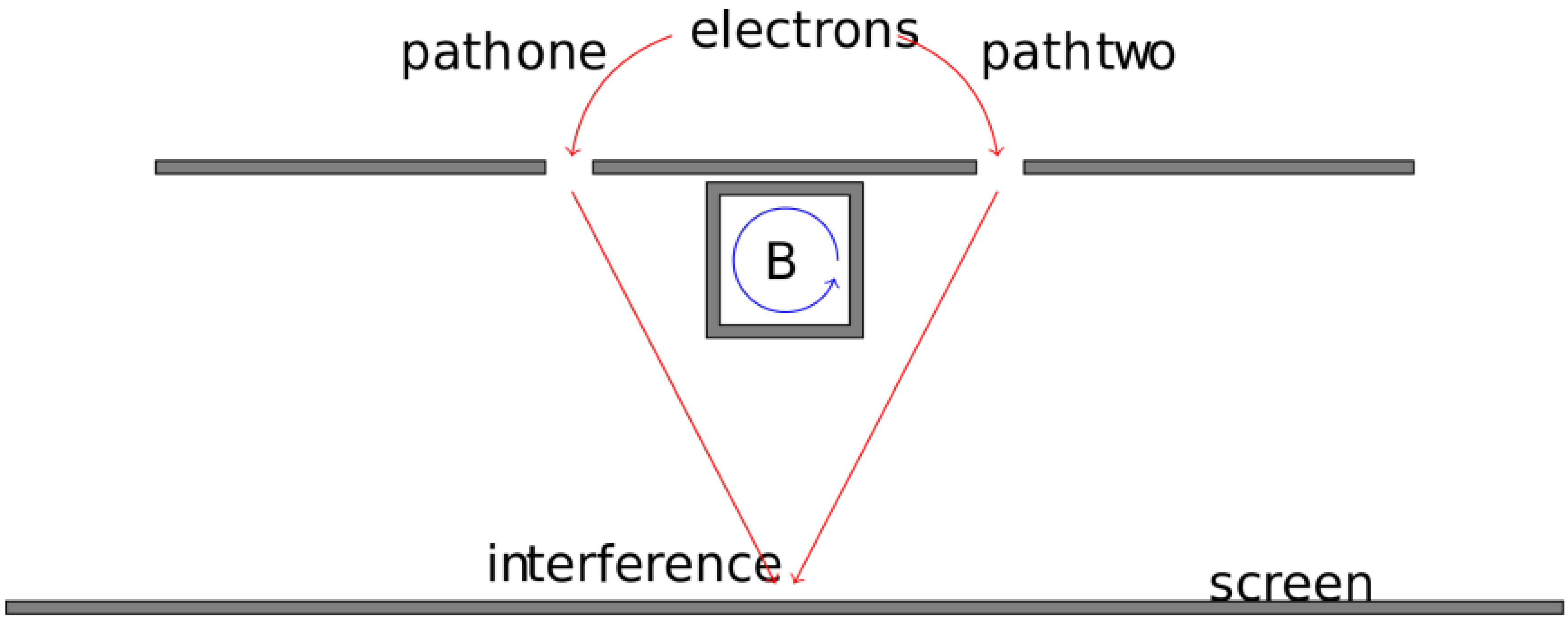
- It is observed that the degeneracy of the Landau levels increases linearly with the magnetic field, a fact which will have important consequences in the explanation of the quantum Hall effects

AHARONOV–BOHM EFFECT

- Magnetic fields can produce and control interference effects between the electrons in solids.
- In 1959, Aharonov and Bohm proposed that an electron wave in a solid has a phase factor which could be controlled by a magnetic field.
- This phenomenon was proved by Webb in 1985 at IBM with a structure similar to below.



- It consisting of a metallic ring of diameter 800 nm made of a wire about 50 nm thick.
- The electrons entering the ring at *P* from the left have their wave function amplitude divided in two equal parts, each one travelling through a different arm of the ring.
- When the waves reach the exit at *Q*, they can interfere.
- Supposethat a magnetic flux produced by a solenoid passes through a region inside the ring and concentric to it.



Schematic of double-slit experiment in which the Aharonov–Bohm effect can be observed: electrons pass through two slits, interfering at an observation screen, with the interference pattern shifted when a magnetic field **B** is turned on in the cylindrical solenoid

- For an electron in a magnetic field *B,* its momentum(p) should be substituted by *p + e A* where *A* is the vector potential *(B = curl A)*.

- As the electron moves from *P* to *Q* the change in phase is given by

$$\vartheta(\vec{r}) = \frac{e}{\hbar}\int_P^Q \vec{A}\cdot d\vec{s}$$

- The difference in phase between a wave travelling around the upper path and the lower one in previous picture is

$$\Delta\vartheta = \vartheta_1 - \vartheta_2 = \frac{e}{\hbar}\left[\int_{\text{lower arm}} \vec{A}\cdot d\vec{s} - \int_{\text{upper arm}} \vec{A}\cdot d\vec{s}\right] = \frac{e}{\hbar}\int_{\text{circle}} \vec{A}\cdot d\vec{s}$$

- since in the top and bottom branches the electron waves advance in opposite directions to *A*

- Applying Stoke's theorem to previous equation $\int_C \vec{F} \cdot d\vec{r} = \iint_S \mathrm{curl}\vec{F} \cdot d\vec{S}$

$$\Delta\vartheta = \frac{e}{\hbar}\Phi$$

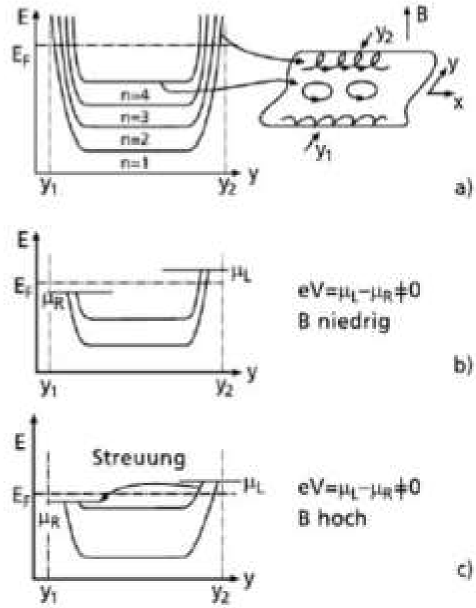
- Where

$$\Phi = \int_{\text{area}} (\mathrm{curl}\vec{A}) \cdot d\vec{S} = \int_{\text{circle}} \vec{B} \cdot d\vec{S}$$

- The quantity $\Phi_0 = \frac{h}{e}$ is defined as the *quantum of flux*.
- One very interesting aspect of the Aharonov–Bohm effect is the observation that variations in phase can be induced by changing *B*, even if the electron waves are not directly subjected to the action of *B*.
- a vector potential *A* indeed exists in the region around the ring , and the changes in phase are produced by *A* accordingly.
- Aharonov–Bohm quantum interference effects are frequently observed even in samples of size in the micrometre range

SHUBNIKOV–DE HAAS EFFECT

- This defines the effect of magnetic fields on the electronic and transport properties of the 2D systems.
- An oscillation in the conductivity of a material that occurs at low temperatures in the presence of very intense magnetic fields is the **Shubnikov–de Haas effect** (**SdH**).
- At sufficiently low temperatures and high magnetic fields, the free electrons in the conduction band of a metal, semimetal, or narrow band gap semiconductor will behave like simple harmonic oscillators.
- When the magnetic field strength is changed, the oscillation period of the simple harmonic oscillators changes proportionally.
- The resulting energy spectrum is made up of Landau levels.
- In each Landau level the energies and the number of electron states all increase linearly with increasing magnetic field.
- Thus, as the magnetic field increases, the Landau levels move to higher energy.
- As each energy level passes through the Fermi energy , it depopulates as the electrons become free to flow as current. This causes periodic oscillation producing a measurable oscillation in the materials conductivity.



. a) the resulting Landau-levels after applying a magnetic field, b) highest Landau-level far away from Fermi-energy - no scattering, c) highest Landau-level near the Fermi-energy - scattering possible [3]

- As the intensity of *B* is varied, the energy and degeneracy of the Landau levels also varies.

- In many experimental conditions, the density in energy of the system is kept constant, while the magnitude of the magnetic field is varied.

- As *B* increases, the Landau levels move up in energy, Similarly the degeneracy *D* of each level also increases.

- A *filling factor v* is defined as $$\nu = \frac{n_{2\mathrm{D}}}{D} = \frac{2\pi\hbar n_{2\mathrm{D}}}{eB}$$

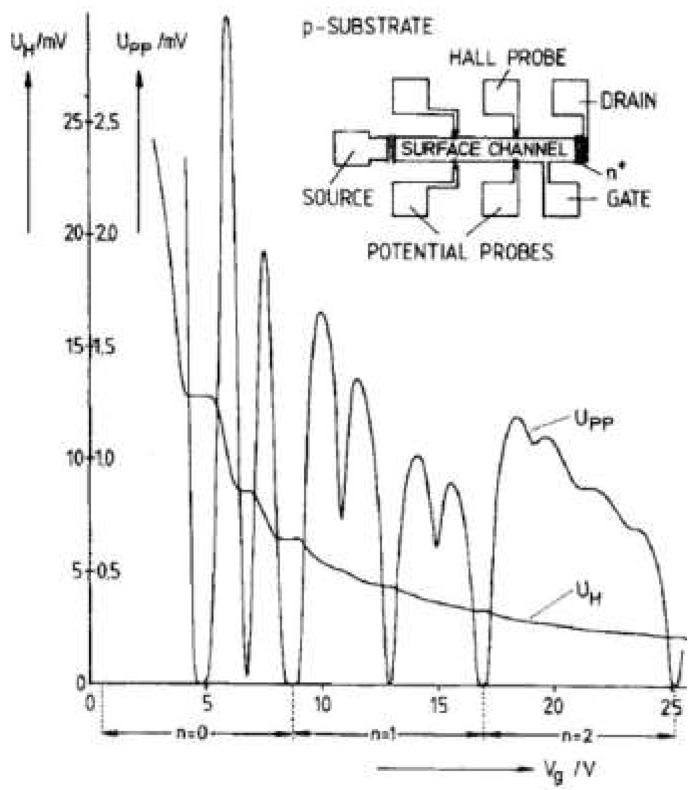
- It indicate the number *N* of Landau levels which are completely occupied.

- when the filling factor is an integer, all Landau levels are completely occupied.

- a small change in energy has a large effectand the conductivity of the sample should be large.

- Figure shows the oscillatory dependence on the gate voltage of the potential difference *U*PP between two probes situated along the length of the sample, like in the inset of the figure.

- In the case of figure, a magnetic field *B* of 18Tesla is applied perpendicularly to the
  2D structure.



- In previous experiment, the filling factor of the Landau levels is changed by the positive gate voltage, which controls the electron concentration .

- Shubnikov–de Haas oscillations in the 2D systems depend only on the component of *B* perpendicular to the interface.

- These oscillations were previously observed in bulk semiconductors, but they were much weaker and dependent on both components of *B* , the perpendicular and the in-plane ones.

THE HALL EFFECT

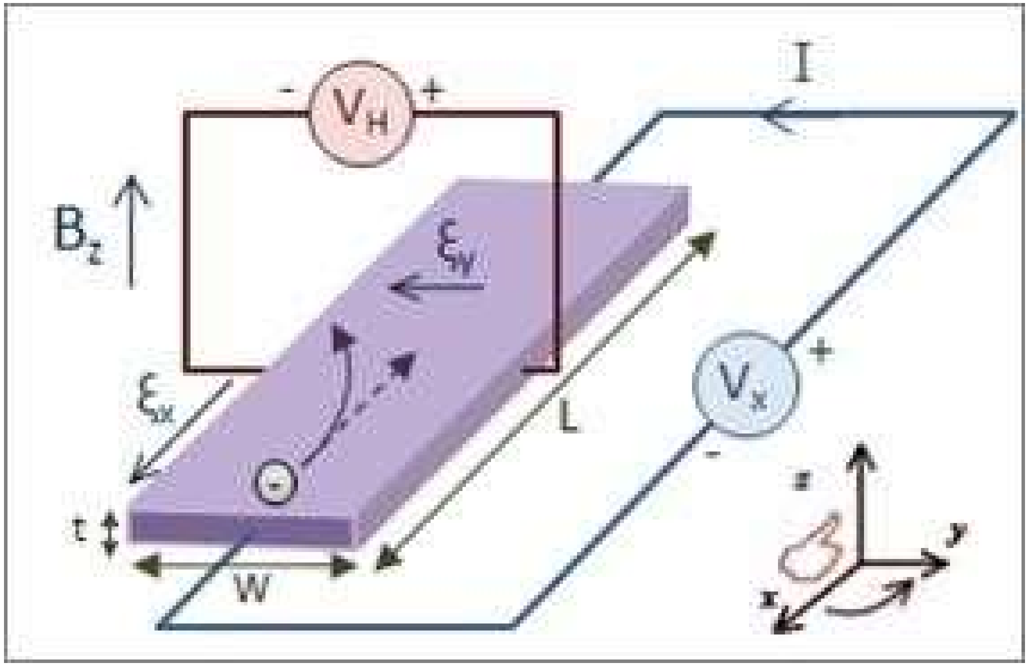
- If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force (Lorentz force) on the moving charge carriers which tends to push them to one side of the conductor.

- When such a magnetic field is absent, the charges follow approximately straight.

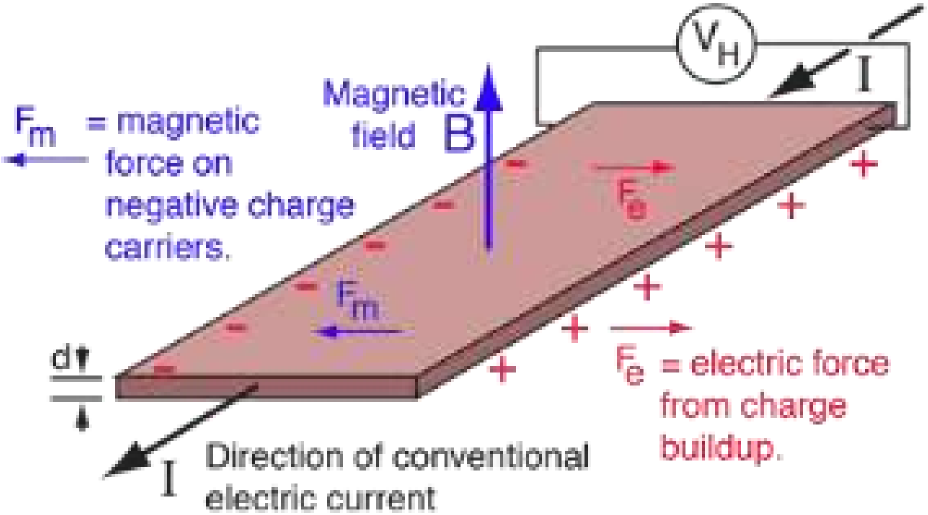
- The moving charges accumulate on one face of the material.

- A buildup of charge at the sides of the conductors will balance this magnetic influence, producing a measurable voltage between the two sides of the conductor.

- The presence of this measurable transverse voltage is called the Hall effect



I
-
V_H
+
B_z
ξ_y
ξ_x
L
V_x
+
-
t
W
z
y
x



V_H
I
Magnetic field
B
F_m = magnetic force on negative charge carriers.
F_e
F_m
F_e = electric force from charge buildup.
d
I
Direction of conventional electric current

Integer quantum hall effect

- Experimental setup

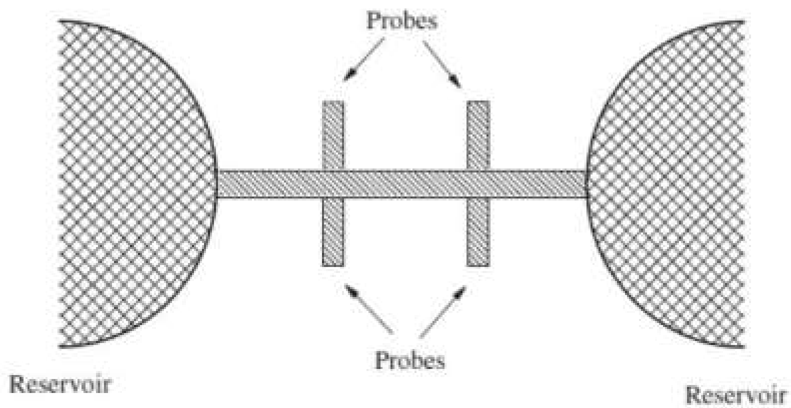


Diagram of a typical nanostructure used to make quantum Hall effect measurements

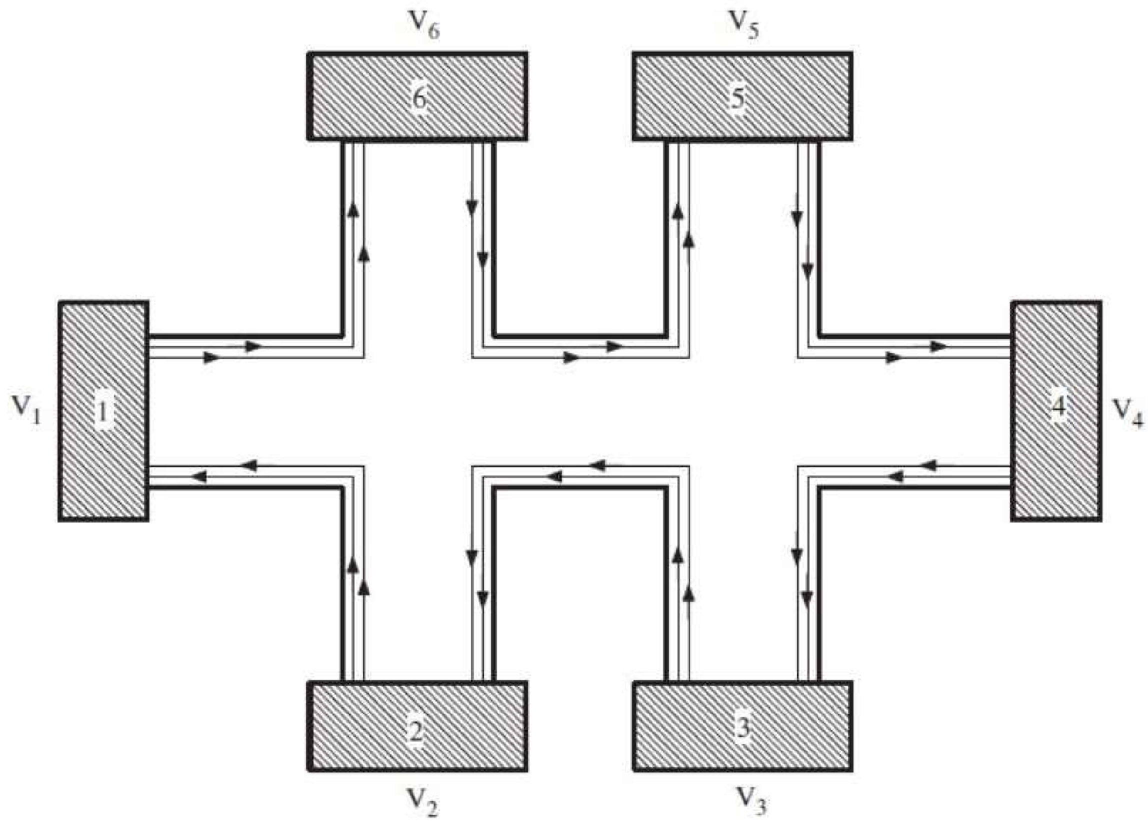
- System consist of two current leads connected to corresponding reservoirs and several voltage probes.

- The reservoirs serve as an infinite source and sink of electrons and are kept at constant temperature.

- Expression for total current is obtained as

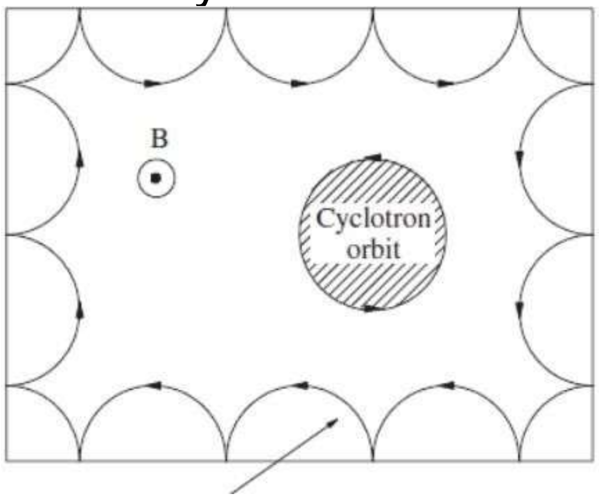
$$I_i = \frac{2e^2}{h}\left[(N_i - R_i)V_i - \sum_{j\neq i} T_{ij}V_j\right]$$

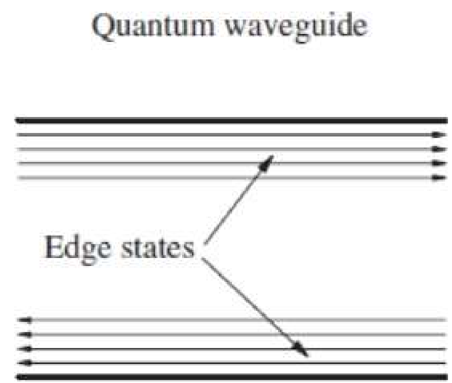
- Where *T*ij is transmission coefficient



Two-dimensional test sample for the measurement of the quantum Hall effect. The current goes from probe 1 to 4. The Hall voltage can be measured from probes 6 and 2 or, alternatively, 5 and 3. The voltage drop in the direction of the current is measured from probes 5 and 6 or 3 and 2. The edge currents (two in the figure) are also shown.

- Consider an electric current flows through the system in a magnetic field.
- cyclotron orbits are created to the various Landau levels by the perpendicular magnetic field.
- Let the cyclotron orbits are directed counterclockwise.





- The closed cyclotron orbits, which do not carry current on the average, are no longer possible near the edges of the sample.
- the electrons at the edges move with a net drift velocity and the orbits are called *skipping orbits*.
- Quantum mechanically, the states associated with the skipping orbits are called *edge states*.
- The upper edge states have a positive velocity, while the lower ones have negative velocity.

- Let us assume that the edge current contains *N* channels, although in Figure we have represented only two.

- Current only flows in or out of the sample through the contact leads 1 and 4 and the Hall voltage arises between probe contacts 6 and 2 or, alternatively, 5 and 3.'

- The longitudinal resistance of the sample can be measured between contacts 5 and 6 or 3 and 2.

- the current arising at contact 1 enters into probe 6, but since this is a voltage probe, it cannot take net current or $I_6 = 0$.

- The same argument can be applied to the other voltage probes 2, 3, and 5.

- Also observed that *N* states propagate from contact 1 to contact 6.
- $T32 = T43 = T54 = T65 = T16 = N$ and $T23 = T34 = T45 = T56 = T61 = 0$. All remaining *Tij are zero since current cannot jump contacts.*

$$I_i = \frac{2e^2}{h}\left[(N_i - R_i)\,V_i - \sum_{j\neq i} T_{ij}V_j\right]$$

- let us write above equation in matrix form
as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -I \\ 0 \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix} = \frac{Ne^2}{h}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

- We have written $I1 = -I$ and $I4 = I$ since the arrows in figure indicate the
electron motion.

- In order to calculate the Hall resistance we have to divide the voltage between probes 6 and 2 by the current I between contacts 1 and 4

$$R_{14,62} = \frac{V_6 - V_2}{I}$$

- Substituting $V6$ and $V2$ by their expressions obtained from matrix, we get for the Hall resistance:

$$R_H = \frac{h}{e^2}\frac{1}{N}$$

- The *quantification of the values of the Hall resistance* is given, with an outstanding precision, by the equation:

$$R_H = \frac{h}{e^2}\frac{1}{n} = 25812.807\,\Omega\left(\frac{1}{n}\right), \quad n = 1, 2, \ldots$$

- It means that, they remain constant for each L level. So VH also.
- Above expression with n=1 is known as the *von Klitzing constant* and is written as R_K.

- The above expression can also prove with classical physics.

- *VH=bBv (b* is the width of the sample and *v* the carrier drift velocity) and the current *I= bnev*.

$$R_{\mathrm{H}} = \frac{V_{\mathrm{H}}}{I} = \frac{B}{en}$$

- Expression for magnetic field *BN* when the levels are completely filled is $B_N = \frac{1}{N} \quad \frac{\hbar n}{e} \quad , \quad (N = 1, 2, \ldots)$

- Substituting above gives $R_{\mathrm{H}} = \frac{h}{e^2}\frac{1}{N}$