Numerical problems:

Q: ① A current vector of magnitude 100 A is

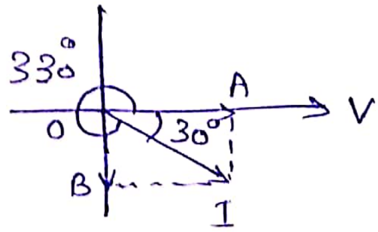
ⓐ lagging the voltage vector by 30°.

ⓑ leading the voltage vector by 30°.

ⓒ in phase with voltage vector.

Represent the current in different forms of phasor representation.

ⓐ



$$OA = I\cos 30 = 100 \cdot \cos 30 = \underline{\underline{86.6}}$$

$$OB = I\sin 30 = 100\sin 30 = \underline{\underline{50}}$$

angle of I w.r.t to V in the anticlockwise direction $= 330^\circ$.

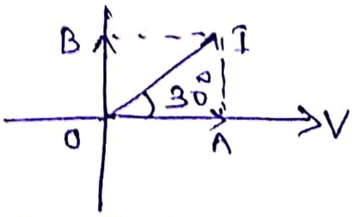
rectangular form $\rightarrow (86.6 - j50)$ A

trigonometric form $\rightarrow 100(\cos 30 - j\sin 30)$ A

Polar form $\rightarrow$ ~~~~ $100\angle 330^\circ$ A or $100\angle -30^\circ$ A

exponential form $\rightarrow 100e^{j330^\circ}$ A or $100e^{-j30}$ A.

ⓑ



$$OA = I\cos 30 = 100\cos 30 = \underline{\underline{86.6}}$$

$$OB = I\sin 30 = 100 \cdot \sin 30 = \underline{\underline{50}}.$$

angle of I w.r.t to V in the anticlockwise direction $= 30^\circ$.

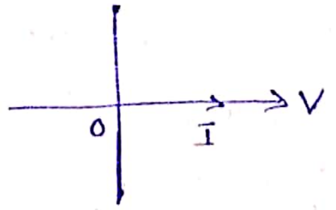
rectangular form $= (86.6 + j50)$ A

trigonometric form $= 100(\cos 30 + j\sin 30)$ A

Polar form $= 100\angle 30^\circ$ A or $100\angle -330^\circ$ A

exponential form $= 100e^{j30^\circ}$ A or $100e^{-j330}$ A

(c)



angle of I w.r.t. to V = 0°.

$\therefore$ trigonometric form $= I(\cos 0 + j\sin 0)$

$= I \cdot \cos 0 = I$

$= 100\ A$

rectangular form $= 100 + j0\ A$

polar form $= 100 \angle 0^\circ A$

exponential form $= 100\, e^{j0^\circ}\ A.$

Q (2) An alternating voltage $(160 + j120)$ V is applied to a circuit and the current in the circuit is found to be $(6 + j8)$ A. Find (a) the impedance of the circuit (b) the phase angle (c) the power consumed.

$\vec{V} = (160 + j120)\,V, \quad \vec{I} = (6 + j8)\,A$

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{160 + j120}{6 + j8}$$

Now, $160 + j120$ → Magnitude $= \sqrt{160^2 + 120^2} = 200$

→ angle $= \tan^{-1}\left(\frac{120}{160}\right) = 36.87^\circ$.

$\therefore 160 + j120 \equiv 200 \angle 36.87^\circ$.

Similarly, $6 + j8 \equiv 10 \angle 53.13^\circ$.

(a) $\therefore \vec{Z} = \frac{200 \angle 36.87}{10 \angle 53.13} = 20 \angle (36.87 - 53.13) = 20 \angle -16.26$

$= 19.2 - j5.6$

(b) phase angle $= 16.26^\circ$ leading ($\because$ I leads voltage by angle)

(c) Power consumed $= VI\cos\phi$

$= 200 \times 10 \times \cos(16.26)$

$= 1920\,W = 1.92\,kW$

Q: ③ An alternating current is given by $I = 50\sin(314t)$ (30)

Find ⓐ maximum value ⓑ frequency ⓒ time period ⓓ Value of current after $\frac{1}{80}$ second from zero?

ⓐ $i = I_m \sin \omega t$

$i = 50\sin(314t)$

$\Rightarrow I_m = 50A$ (Maximum value)

ⓑ <u>frequency</u> $\omega = 314$

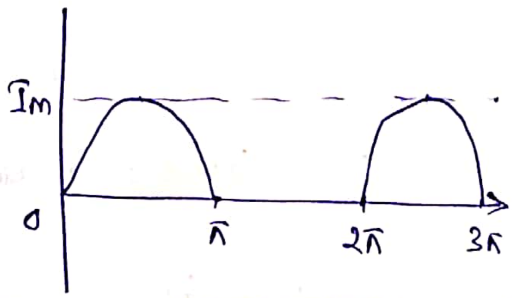
$\Rightarrow 2\pi f = 314 \Rightarrow f = \frac{314}{2\pi} = \frac{314}{2\times 3.14} = \underline{\underline{50 Hz}}.$

ⓒ time period, $T = \frac{1}{f} = \frac{1}{50} = \underline{\underline{0.02\ sec.}}$

ⓓ Value of current after $\frac{1}{80}$ sec $= 50\sin(100\pi t)$

$= 50\sin(100\times 180\times \frac{1}{80}) = \underline{\underline{-35.35A}}$

Q: ④ Find the rms and average value of half rectified sine wave.



$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \cdot d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} i^2 \cdot d\theta + \int_{\pi}^{2\pi} i^2 \cdot d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2 \cdot d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi}(I_m \sin\theta)^2 \cdot d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\sin^2\theta \cdot d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} = \sqrt{\frac{I_m^2}{4\pi}\left[\pi - 0 - \frac{\sin 2\pi}{2} + \frac{\sin 2(0)}{2}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\times\pi} = \underline{\underline{\frac{I_m}{2}}}$$

$$I_{avg} = \frac{1}{2\pi}\int_0^{\pi} i \cdot d\theta = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \cdot d\theta = \frac{I_m}{2\pi}\int_0^{\pi}\sin\theta \cdot d\theta = \frac{I_m}{2\pi}(-\cos\theta)_0^{\pi}$$

$$= \frac{I_m}{2\pi}(-\cos\pi + \cos 0) = \frac{I_m}{2\pi}\times 2 = \underline{\underline{\frac{I_m}{\pi}}}$$

Q! (5) A resistor of resistance $10\,\Omega$, an inductance of $0.3H$ and a capacitance of $100\,\mu F$ are connected in series across 230V, 50Hz mains. Calculate impedance, current, voltage across R, L and C, power in watts, VAR and VA, and also power factor.

$R = 10\,\Omega$; $L = 0.3H$ and $C = 100 \times 10^{-6} F$

$V = 230V$, $f = 50Hz$.

Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 0.3$
$= 94.24\,\Omega$

Capacitive reactance, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (94.24 - 31.83)^2}$
$= 63.2\,\Omega$

current, $I = \frac{V}{Z} = \frac{230}{63.2} = 3.64A$

Voltage across R, $V_R = IR = 3.64 \times 10 = 36.4V$

Voltage across L, $V_L = IX_L = 3.64 \times 94.24 = 343.02V$

Voltage across C, $V_C = IX_C = 3.64 \times 31.83 = 115.86V$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{10}{63.2} = 0.158$

Active power
Power in watts, $P = VI\cos\phi = 230 \times 3.64 \times 0.158 = 132.27W$

Reactive power
Power in VAR, $Q = VI\sin\phi = 230 \times 3.64 \times \sin(\cos^{-1}(0.158))$
$= 230 \times 3.64 \times 0.987$
$= 826.32$ Var

Apparent power
Power in VA, $S = VI = 230 \times 3.64 = 837.2$ VA

Q: (6) An alternating current is given by, $i = 14.14 \sin(377t)$
Find (i) rms value of current.
(ii) frequency
(iii) instantaneous value of current when $t = 3\,ms$.
(iv) time taken for the current to reach 10A for the first time after passing through zero value.

$i = 14.14 \sin 377t \equiv I_m \sin \omega t \Rightarrow I_m = 14.14\,A$ and $\omega = 377$

(i) $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{14.14}{\sqrt{2}} = \underline{10A}$

(ii) $\omega = 2\pi f = 377 \Rightarrow f = \dfrac{377}{2\pi} = \underline{60\,Hz}$.

(iii) After $t = 3\,ms$, $i = 14.14 \sin(2\pi \times 60 \times 3 \times 10^{-3})$
$= 14.14 \sin(64.8) = \underline{12.8A}$

(iv) $10 = 14.14 \sin(2\pi \times 60 \times t)$
$\Rightarrow \sin(21600t) = 0.707$
$\Rightarrow 21600t = \sin^{-1}(0.707) = 45$
$\Rightarrow t = \underline{\dfrac{1}{480}\ sec.}$

Q: (7) A coil of insulated wire of resistance $8\,\Omega$ and inductance $0.03H$ is connected to an ac supply at 240V, 50 Hz. Calculate
(i) the current, pf and power
(ii) the value of capacitance which when connected in series with the above coil and replacing the inductance causes no change in the values of current and power taken from the supply.

$R = 8\,\Omega$, $L = 0.03H$

$X_L = 2\pi f L = 2\pi \times 50 \times 0.03 = \underline{9.42\,\Omega}$

Impedance, $Z = \sqrt{R^2 + X_L^2} = \underline{12.36\,\Omega}$

current, $I = \frac{240}{12.36} = 19.41\,A$

Power factor, $= \frac{R}{Z} = \frac{8}{12.36} = 0.648$ lagging

Power, $P = 240 \times 19.41 \times \cos(0.648) = 3.01\,kW$

If capacitor is placed in circuit, replacing inductor:

then $Z = \sqrt{R^2 + X_C^2} = 12.36\,\Omega$

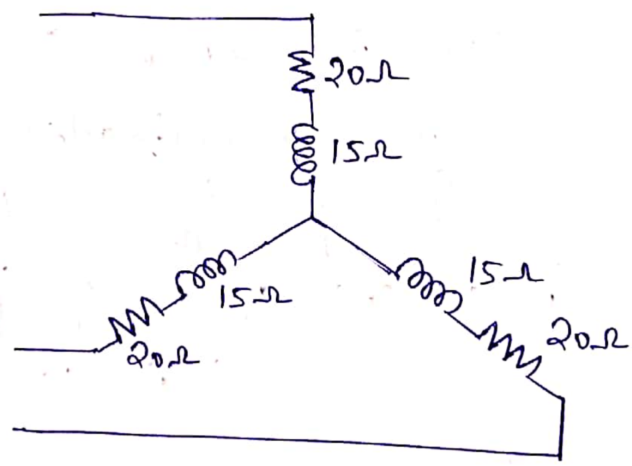
$\Rightarrow X_C = 9.42\,\Omega$

$\Rightarrow \frac{1}{2\pi f C} = 9.42 \Rightarrow \frac{1}{C} = 9.42 \times 2\pi f$

$C = 3.38 \times 10^{-4}\,F$

Q!(8) A balanced star connected load of $(20+j15)\,\Omega$ are connected in star across a 400 V, 3 phase AC supply. Calculate the line current, power factor, total power and reactive volt ampere?

$V_L = 400V$, $R = 20\Omega$, $X_L = 15\Omega$



$Z_P = \sqrt{R^2 + X_L^2}$
$= \sqrt{20^2 + 15^2} = 25\,\Omega$

phase voltage, $V_P = \frac{400}{\sqrt{3}} = 231\,V$

phase current, $I_P = \frac{V_P}{Z_P} = \frac{231}{25}$
$= 9.24\,A$

line current = phase current for star connected system.

$\therefore I_L = 9.24\,A$

$pf = \cos\phi = \frac{R}{Z} = \frac{20}{25} = 0.8$

total power, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 = 5121.3\,W$

reactive volt ampere, $Q = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 400 \times 9.24 \times \sin(\cos^{-1}(0.8))$
$= 3840.9$ VAr.

Q:(9) A star connected, 3 phase load consists of three identical impedances. When the load is connected to a 3 phase 400 V supply, the line current is 23.09A and pf is 0.8 lagging. Calculate the total power taken by the load. If the load were reconnected in delta and supplied from the same three phase supply, calculate the current flowing in each line.

$V_L = 400V$, $I_L = 23.09A$, $pf = 0.8$

$P = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 23.09 \times 0.8 = 12797.7W$

$I_p = I_L$ (for star connection) $= 23.09A$

$V_p = \frac{V_L}{\sqrt{3}}$ (for star connection) $= \frac{400}{\sqrt{3}} = \underline{\underline{230.9V}}$

Now, impedance per phase $Z_p = \frac{V_p}{I_p} = \frac{230.9}{23.09} = \underline{\underline{10\Omega}}$

On connecting the load in delta,

$V_p = V_L = 400V$ (for delta connection)

$I_p = \frac{V_p}{Z_p} = \frac{400}{10} = \underline{\underline{40A}}$

Line current, $I_L = \sqrt{3}\, I_p$ (for delta connection)

$= \sqrt{3} \times 40 = \underline{\underline{69.28A}}$

Note:

Differentiate between zero power factor and unity power factor load.

→ Zero power factor (ZPF) load ⇒ The phase difference between current and voltage is 90°. There is no active (useful) power, total (apparent) power completely has only reactive component.

→ Unity power factor (UPF) load ⇒ The voltage and current are in phase or the phase difference between V & I is 0°. There is no reactive power, total (apparent) power completely has active or useful component.

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