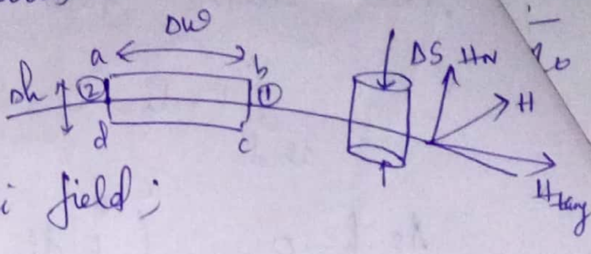
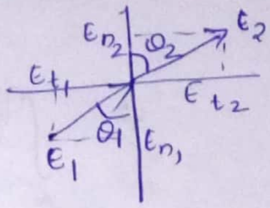
Magnetic Boundary condition



(Same fig)

By Gauss law in magnetic field;

$$\oint B \cdot ds = 0$$



Assuming Gaussian cylindrical surface;

$$\int_{top} B \cdot ds + \int_{bottom} B \cdot ds + \int_{lateral} B \cdot ds = 0$$

lateral surface; $\int_{lat} B \cdot ds = 0 \Rightarrow B_{N_1} \Delta S - B_{N_2} \Delta S = 0$

$$\Rightarrow \boxed{B_{N_1} = B_{N_2}} \quad \Rightarrow \mu_1 H_{N_1} = \mu_2 H_{N_2}$$

↳ ie, normal component of B is continuous

$$\boxed{\frac{H_{N_1}}{H_{N_2}} = \frac{\mu_2}{\mu_1}}$$

normal comp.

↳ normal component of H is discontinuous by the ratio $\frac{\mu_1}{\mu_2}$

For tangential comp;

By Ampere Circuital law, $\oint H \cdot dl = I$

$$\int_a^b H \cdot dl + \int_b^c H \cdot dl + \int_c^d H \cdot dl + \int_d^a H \cdot dl = I$$

$$H_{t_1} \Delta w + H_{N_1} \frac{\Delta h}{2} + H_{N_2} \frac{\Delta h}{2} - H_{N_2} \frac{\Delta h}{2} - H_{N_1} \frac{\Delta h}{2} - H_{t_2} \Delta w = I$$

$$\Rightarrow H_{t_1} \Delta w - H_{t_2} \Delta w = I \quad (\text{assume } I = 0 \text{ in free space})$$

$$\boxed{H_{t_1} = H_{t_2}}$$

$$B = \mu H \Rightarrow \frac{B_{t_1}}{\mu_1} = \frac{B_{t_2}}{\mu_2} \Rightarrow \boxed{\frac{B_{t_1}}{B_{t_2}} = \frac{\mu_1}{\mu_2}}$$

Similiar to the case in dielectrics;

$$\frac{\mu_1}{\mu_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

% Maxwell's Equations: — Maxwell's eqns links the time varying electric field & magnetic fields.

Maxwell's eqns are based on;

a) Faraday's law
b) Ampere Circuital law
c) Gauss's law for electric field
d) Gauss's law for magnetic field.

⇒ Based on Faraday's law:

$$e = N \frac{d\phi}{dt}$$

Assuming $N=1$ ⇒ $e = \frac{d\phi}{dt}$ → ①

For a closed path, $e = -\oint E \cdot dl$ → ②

$$\Rightarrow -\oint E \cdot dl = \frac{d\phi}{dt}$$

$(\phi = \oint_S B \cdot ds)$ ⇒ $-\oint E \cdot dl = \frac{d}{dt}\left(\oint_S B \cdot ds\right)$

$$+\oint E \cdot dl = \oint_S -\frac{\partial}{\partial t}(B \cdot ds) \rightarrow ③$$

By Stoke's law; $\oint E \cdot dl = \int_S (\nabla \times E) \cdot ds$

$$\Rightarrow \oint_S (\nabla \times E) \cdot ds = \oint_S -\frac{\partial}{\partial t}(B \cdot ds)$$

$\boxed{\nabla \times E = \frac{-\partial B}{\partial t}}$ → Point form

Integral form of Maxwell's eqn ⇒ $\boxed{\oint E \cdot dl = \oint -\frac{\partial (B \cdot ds)}{\partial t}}$

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Based on Ampere Circuital law:
- to include time varying electric field & linked it with magnetic field.

$I = \oint H \cdot dl$

$I = \oint_S J \cdot ds \Rightarrow \boxed{\oint H \cdot dl = \oint_S J_t \cdot ds}$ Integral form

$\oint H \cdot dl = \oint (J_1 + J_2) \cdot ds$

$J_1 = \sigma E$ [conduction current density]

$J_2 = \epsilon \frac{d}{dt}(E) = \frac{d(D)}{dt}$ (displacement current density)

$\Rightarrow \oint H \cdot dl = \oint_S \left(\sigma E + \frac{dD}{dt}\right) \cdot ds$

By Stokes theorem

$\oint_S \left(\sigma E + \frac{d}{dt} D\right) \cdot ds = \oint_S (\nabla \times H) \cdot ds$

$\Rightarrow \boxed{\nabla \times H = \sigma E + \frac{dD}{dt}}$ Point form.

(3)

$\Rightarrow$ Based on Gauss law:

$Q = \oint D \cdot ds$

$Q = \int_V \rho_v \cdot dv \Rightarrow \boxed{\int_V \rho_v \cdot dv = \int_S D \cdot ds}$ Integral form

By divergence theorem; $\oint D \cdot ds = \int_V (\nabla \cdot D) dv$

$\Rightarrow \int_V (\nabla \cdot D) dv = \int_V \rho_v dv$

$\boxed{\nabla \cdot D = \rho_v}$ Point form

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Based Gauss's law in magnetic field;

$$\oint B \cdot ds = Q = 0 \Rightarrow$$ total magnetic flux through closed surface is zero, absence of magnetic monopole

$$\oint B \cdot ds = \int_V \rho_v \cdot dv = 0 \Rightarrow \boxed{\oint B \cdot ds = 0}$$ Integral form

By divergence theorem;

$$\oint B \cdot ds = \int_V (\nabla \cdot B)\, dv$$

$$\Rightarrow \boxed{\nabla \cdot B = 0}$$ point form

Summary:

Differential form	Integral form
① $\nabla \times H = \sigma E + \frac{\partial D}{\partial t}$	$\oint H \cdot dl = \oint \left(\sigma E + \frac{\partial D}{\partial t}\right) \cdot ds$
② $\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = \oint -\frac{\partial B}{\partial t} \cdot ds$
③ $\nabla \cdot D = \rho$	$\oint D \cdot ds = \iiint_V \rho \cdot dv$
④ $\nabla \cdot B = 0$	$\oint B \cdot ds = 0$

Electrostatic Energy & Energy density:

If v is the P.D of capacitor having charge q at an instant $\Rightarrow q = Cv$.

Work done in transporting charge dq b/w plates is dw.

$dw = v \cdot dq$ [P.D here is the work done in displacing unit charge b/w the plates]

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$\Rightarrow$ Total work done, $W = \int_0^Q V \cdot dq = \int_0^Q \frac{q}{C} \cdot dq$ (3)

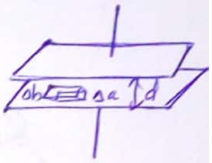
$\Rightarrow W = \left(\frac{q^2}{2C}\right)_0^Q = \frac{1}{2}\frac{Q2}{2C}$

$$\boxed{W = \frac{Q2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

Electrostatic energy resides in the dielectric medium.

Consider a small box situated in the Electric field produced by the charged capacitor

$\Delta C = \frac{\epsilon \, \Delta A}{\Delta d} = \frac{\epsilon \, \Delta a^2}{\Delta b}$



(a is side of small box considered & b is height)

P.D b/w top & bottom of box, $\Delta V = E \cdot \Delta b$

Energy stored, $\Delta W = \frac{1}{2}\Delta C (\Delta V)^2$

$$\Delta W = \frac{1}{2}\frac{\epsilon \, \Delta a^2}{\Delta b} E^2 (\Delta b)^2$$

$$= \frac{1}{2}\epsilon \, \underline{(\Delta a)^2 (\Delta b)} \, E^2$$

$$\Rightarrow \Delta W = \frac{1}{2}\epsilon E^2 (\Delta V) \rightarrow (1)$$

$\Rightarrow$ Electrostatic energy density $= \frac{\Delta W}{\Delta V} = \lim_{\Delta V \to 0} \frac{\Delta W}{\Delta V}$

$= \lim_{\Delta V \to 0} \frac{\frac{1}{2}\epsilon E^2 \Delta V}{\Delta V}$

$$\boxed{W_E = \frac{1}{2}\epsilon E^2}$$

Energy density $= \frac{\text{Energy}}{\text{Volume}}$

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Energy in Magnetic Fields:

An inductor carrying current stores energy in the form of magnetic field (similar to a capacitor that stores energy in an electrostatic field)

The induced emf, $\mathcal{E} = -L\frac{dI}{dt}$

Work done against the back emf in unit time,

$$\frac{dW_m}{dt} = -\mathcal{E}I$$

from $t=0$ to t

$I_{initial} = 0$

$I_t = I$

$$W_m = \int_0^I -\mathcal{E}I\,dt - \int_0^I \left(-L\frac{dI}{dt}\right)I\,dt$$

$$= \int_0^I LI\,dI = \frac{1}{2}LI^2$$

⇒ ie, Energy stored in magnetic field,

$$\boxed{W_m = \frac{1}{2}LI^2}$$

Inductance in terms of energy, $L = \frac{2W_m}{I^2}$

⇒ For a toroid, $L = \frac{\mu N^2 A}{2\pi r}$, $B = \frac{\mu N I}{2\pi r}$

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu N^2 I^2 A}{2\pi r}$$

Magnetostatic Energy Density

$$w_m = \frac{W_m}{\text{Volume}} = \frac{B^2}{2\mu}$$

$$\boxed{w_m = \frac{1}{2}\frac{B^2}{\mu}}$$

$$\boxed{w_m = \frac{1}{2}\mu H^2}$$

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Q. Find the total energy stored & energy density of 1000 ③ and having parallel plates of 40 × 40 cm surface area respectively in air by a distance of 5 mm.

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{8.85 \times 10^{-12} \times 1 \times (40 \times 40 \times 10^{-4})}{5 \times 10^{-3}}$$

$$= 28.32 \times 10^{-11} = 2.832 \text{ pF}$$

$$\text{Energy stored} = \frac{1}{2} CV^2 = \frac{1}{2} \times (2.832\text{p}) \times (1000)^2$$

$$= 1.416 \times 10^{-6} = 1.416 \ \mu\text{J}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{1.416 \mu}{40 \times 40 \times 10^{-4} \times 5 \times 10^{-3}}$$

$$= 0.0885 \times 10^{-1} = 8.85 \text{ mJ/m}^3$$

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