EFFECTIVENESS FOR THE PARALLEL FLOW HTX

The heat exchange dQ through an area dA of the heat exchanger is given by,

$$dQ = U.dA(T_h - T_c) \quad \text{——— (1)}$$

$$= -m_h c_{ph} \cdot dT_h = m_c C_{pc} \cdot dT_c$$

$$= -C_h \cdot dT_h = C_c \cdot dT_c \quad \text{——— (2)}$$

From eqn (2), $dT_h = \frac{-dQ}{C_h}$ and $dT_c = \frac{dQ}{C_c}$

$$dT_h - dT_c = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Substituting the value of dQ from eqn (1),

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

On integrating,

$$\ln\left(\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}}\right) = -U.A\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\left(\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}}\right) = \frac{-U.A}{C_h}\left[1 + \frac{C_h}{C_c}\right]$$

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \exp\left[-(UA/C_h)(1 + C_h/C_c)\right] \quad \text{— (3)}$$

$$\varepsilon = \frac{C_h(T_{h_1} - T_{h_2})}{C_{min}(T_{h_1} - T_{c_1})} = \frac{C_c(T_{c_2} - T_{c_1})}{C_{min}(T_{h_1} - T_{c_1})}$$

$$T_{h_2} = T_{h_1} - \frac{\varepsilon C_{min}(T_{h_1} - T_{c_1})}{C_h} \quad \text{——— (4)}$$

$$T_{c_2} = T_{c_1} + \frac{\varepsilon C_{min}(T_{h_1} - T_{c_1})}{C_c} \quad \text{——— (5)}$$

Substituting eqns ④ + ⑤ in eqn ③,

$$\frac{1}{(T_{h_1}-T_{c_1})}\left[(T_{h_1}-T_{c_1}) - \varepsilon C_{min}(T_{h_1}-T_{c_1})\left(\frac{1}{C_h}+\frac{1}{C_c}\right)\right] = \exp\left[-(UA/C_h)\left(1+\frac{C_h}{C_c}\right)\right]$$

$$1 - \varepsilon C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right) = \exp\left[-(UA/C_h)\left(1+\frac{C_h}{C_c}\right)\right]$$

$$\varepsilon = \frac{1-\exp\left[-(UA/C_h)\left(1+\frac{C_h}{C_c}\right)\right]}{C_{min}\left(\frac{1}{C_h}+\frac{1}{C_c}\right)} \quad \text{—— ⑥}$$

If $C_c > C_h$ then $C_{min} = C_h$ and $C_{max} = C_c$, then eqn ⑥ becomes,

$$\varepsilon = \frac{1-\exp\left[-(UA/C_{min})\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{1+C_{min}/C_{max}} \quad \text{—— ⑦}$$

If $C_c < C_h$ then $C_{min} = C_c$ and $C_{max} = C_h$. then eqn ⑥ becomes,

$$\varepsilon = \frac{1-\exp\left[-(UA/C_{max})\left(1+\frac{C_{max}}{C_{min}}\right)\right]}{1+C_{min}/C_{max}} \quad \text{—— ⑧}$$

By rearranging eqns ⑦ + ⑧, we get a common eqn

$$\varepsilon = \frac{1-\exp\left[-(UA/C_{min})\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{1+\frac{C_{min}}{C_{max}}}$$

Number of transfer units, $NTU = \frac{UA}{C_{min}}$

Capacity ratio, $R = \frac{C_{min}}{C_{max}}$

$$\varepsilon = \frac{1-\exp\left[-NTU(1+R)\right]}{1+R}$$

EFFECTIVENESS OF THE COUNTER FLOW HTX

The heat exchange dQ through an area dA of the heat exchanger is given by,

$$dQ = U.dA(T_h - T_c) \qquad \text{—— ①}$$

$$= -m_h.C_{ph}.dT_h = -m_c C_{pc} dT_c$$

$$= -C_h.dT_h = -C_c.dT_c \qquad \text{—— ②}$$

From eqn ②, $dT_h = -\frac{dQ}{C_h}$ and $dT_c = -\frac{dQ}{C_c}$

$$d(T_h - T_c) = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] = dQ\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

Substituting the value of dQ from eqn ①,

$$\frac{d(T_h - T_c)}{T_h - T_c} = U.dA\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

On integrating,

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c2}}\right) = U.A\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

$$\ln\left(\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}}\right) = \frac{U.A}{C_c}\left[1 - \frac{C_c}{C_h}\right]$$

$$\frac{T_{h2} - T_{c1}}{T_{h1} - T_{c2}} = \exp\left[\left(UA/C_c\right)\left(1 - C_c/C_h\right)\right] \qquad \text{—— ③}$$

$$\varepsilon = \frac{C_h(T_{h1} - T_{h2})}{C_{min}(T_{h1} - T_{c1})} = \frac{C_c(T_{c2} - T_{c1})}{C_{min}(T_{h1} - T_{c1})}$$

$$T_{h2} = T_{h1} - \frac{\varepsilon\, C_{min}(T_{h1} - T_{c1})}{C_h} \qquad \text{—— ④}$$

$$T_{c2} = T_{c1} + \frac{\varepsilon\, C_{min}(T_{h1} - T_{c1})}{C_c} \qquad \text{—— ⑤}$$

Substituting eqn ④ & ⑤ in eqn ③,

$$\frac{\left[T_{h_1} - \frac{\varepsilon C_{min}(T_{h_1} - T_{c_1})}{C_h}\right] - T_{c_1}}{T_{h_1} - \left[T_{c_1} + \frac{\varepsilon C_{min}(T_{h_1} - T_{c_1})}{C_c}\right]} = \exp\left[(UA/C_c)(1 - C_c/C_h)\right]$$

$$\frac{(T_{h_1} - T_{c_1})\left[1 - \frac{\varepsilon . C_{min}}{C_h}\right]}{(T_{h_1} - T_{c_1})\left[1 - \frac{\varepsilon . C_{min}}{C_c}\right]} = \exp\left[(UA/C_c)(1 - C_c/C_h)\right]$$

$$\frac{1 - \frac{\varepsilon . C_{min}}{C_h}}{1 - \frac{\varepsilon . C_{min}}{C_c}} = \exp\left[(UA/C_c)(1 - C_c/C_h)\right] \quad \text{—⑥}$$

Assume $C_c < C_h$, thus $C_{min} = C_c$ and $C_{max} = C_h$,

Substituting these values in eqn ⑥,

$$\frac{1 - \frac{\varepsilon . C_{min}}{C_{max}}}{1 - \varepsilon} = \exp\left[(UA/C_{min})\left(1 - \frac{C_{min}}{C_{max}}\right)\right]$$

$$1 - \frac{\varepsilon \cdot C_{min}}{C_{max}} = \exp\left[(UA/C_{min})\left(1 - \frac{C_{min}}{C_{max}}\right)\right] - \varepsilon . \exp\left[(UA/C_{min})\left(1 - \frac{C_{min}}{C_{max}}\right)\right]$$

$$1 - \exp\left[(UA/C_{min})\left(1 - \frac{C_{min}}{C_{max}}\right)\right] = \varepsilon\left[\frac{C_{min}}{C_{max}} - \exp\left[(UA/C_{min})\left(1 - \frac{C_{min}}{C_{max}}\right)\right]\right]$$

$$\varepsilon = \frac{1 - \exp[NTU(1-R)]}{R - \exp[NTU(1-R)]}$$

$$\varepsilon = \frac{\exp[NTU(1-R)] - 1}{\exp[NTU(1-R)] - R}$$

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Dividing numerator and denominator by $\exp[NTU(1-R)]$,

$$\varepsilon = \frac{1 - \dfrac{1}{\exp[NTU(1-R)]}}{1 - \dfrac{R}{\exp[NTU(1-R)]}}$$

$$\varepsilon = \frac{1 - \exp[-NTU(1-R)]}{1 - R\exp[-NTU(1-R)]}$$

For condenser and evaporator :-

One fluid remains at constant temperature throughout the exchanger. Here $C_{max} = \infty$ and thus $R = \frac{C_{min}}{C_{max}} \approx 0$

Hence for parallel flow and counter flow heat exchangers, $\varepsilon = 1 - \exp(-NTU)$