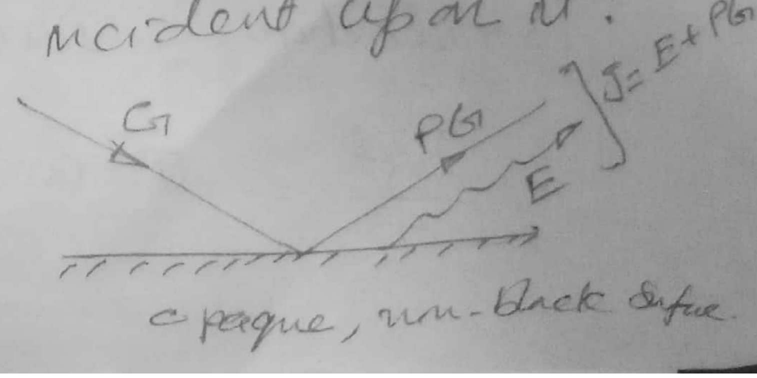
Electric Network analogy for Thermal Radiation Systems.

An electric network analogy is an alternative approach for analysing radiation heat exchange b/w gray or black surfaces. In this approach two terms commonly used are irradiation and radiosity (J).

Irradiation (G): It is defined as the total radiation incident upon a surface per unit time per unit area. unit is W/m^2

Radiosity (J): This is the total radiation leaving a surface per unit time per unit area. Unit is W/m^2.

This comprises of the original emittance 'E' from the surface plus the reflected portion of any radiation incident upon it.



opaque, non-black surface.

ie $J = E + \rho G$

or $\boxed{J = \varepsilon E_b + \rho G}$ — ①, E_b is the emissive power of a black body at the same temp

Also, $\alpha + \rho + \tau = 1$

$\alpha + \rho = 1$ [for opaque body $\tau = 0$

$\rho = 1 - \alpha$

by Kirchoff's law: $\alpha = \varepsilon$,

N.B The total emissivity ε of a surface at temp T is always equal to its total absorptivity α for radiation coming from a black body at the same temperature.

ie $\boxed{\rho = 1 - \varepsilon}$ — ②

∴ Sub. eq ② in ① ⟹

$J = \varepsilon E_b + (1-\varepsilon) G$

ie $$\boxed{G = \frac{J - \varepsilon E_b}{(1-\varepsilon)}}$$ — ③

The net energy leaving a surface is the difference b/w its radiosity and irradiation. Thus

$$\frac{q_{net}}{A} = J - G$$ — ④

$$\therefore \frac{Q_{net}}{A} = J - \frac{J - \varepsilon E_b}{1-\varepsilon}$$

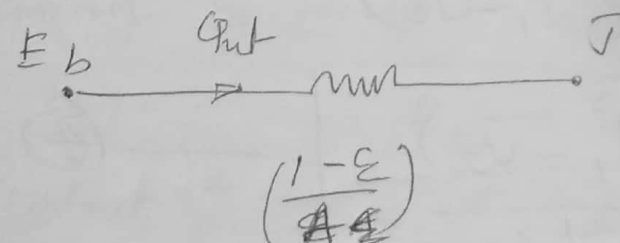
$$= \frac{J - J\varepsilon - J + \varepsilon E_b}{1-\varepsilon}$$

$$= \frac{\varepsilon(E_b - J)}{1-\varepsilon}$$

$$\boxed{Q_{net} = \frac{A\varepsilon(E_b - J)}{1-\varepsilon}} \quad \text{---} \ (5)$$

The representation of this eq. in the form of electric network is as follows.

$$Q_{net} = \frac{E_b - J}{\left(\frac{1-\varepsilon}{A\varepsilon}\right)}$$



The quantity $\left(\frac{1-\varepsilon}{A\varepsilon}\right)$ is known as the surface resistance, as it is related to surface properties of the radiating body.

Consider the exchange of radiation b/w two surfaces (non-black) 1 and 2.

The total radiation leaves surface ① and reaches the surface ② is $= J_1 A_1 F_{12}$

|||ly.
total radiation leaves surface ② and reaches the surface ① $= J_2 A_2 F_{21}$

The net interchange of heat energy b/w the surfaces

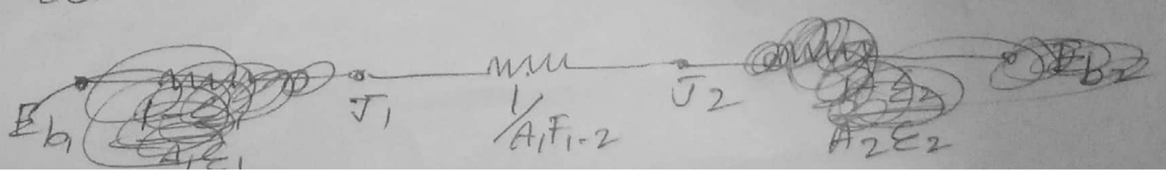
$$\boxed{Q_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{2-1}} \quad \text{———} ⑥$$

But $A_1 F_{1-2} = A_2 F_{2-1}$ — by reciprocity theorem

$\therefore \; Q_{12} = A_1 F_{1\text{-}2} (J_1 - J_2)$

$$\text{or} \quad \boxed{Q_{12} = \frac{(J_1 - J_2)}{1/A_1 F_{12}}} \quad \text{———} ⑦$$

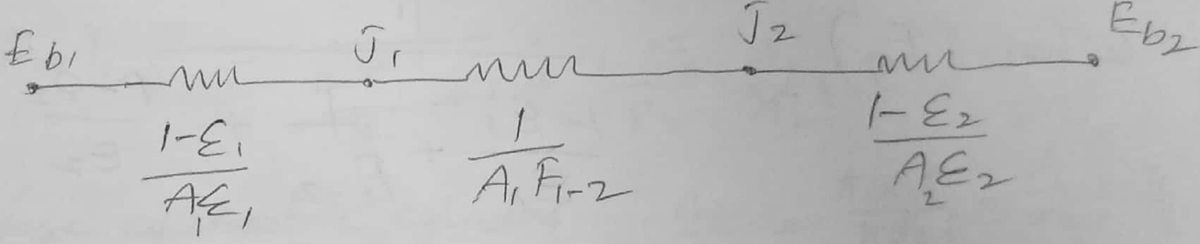
This can be represented in the form of electric network as



The quantity $\frac{1}{A_1 F_{1-2}}$ is called the space resistance because it is due to the distance and geometry of the radiating bodies.

In general.

If the surface resistances of two bodies and space resistance b/w them is considered then the net heat flow can be represented by an electric circuit as shown below,



The net heat exchange b/w the two grey surfaces is given by: $\frac{\text{overall Potential difference}}{\text{Total resistance}}$

$$(Q_{12})_{net} = \frac{E_{b_1} - E_{b_2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

E_{b_1}, E_{b_2} and $(Q_{12})_{net}$ all are fluxes w/m^2

Multiplying numerator and denominator by A_1

we get,

$$(Q_{12})_{net} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

or

$$(Q_{12})_{net} = (F_g)_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

where $$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

and is known as gray body factor

When heat exchange takes place b/w two black surfaces the surface resistance becomes zero as $\varepsilon_1 = \varepsilon_2 = 1$;

$(F_g)_{1-2} = F_{1-2}$ (i.e configuration factor)

and the above eq. reduces to,

$$\boxed{Q_{net} = F_{1-2} A \sigma (T_1^4 - T_2^4)}$$ for back surface

Let us consider the following cases.

1. When the radiating bodies are infinite parallel planes.

In this case $A_1 = A_2$ and $F_{1-2} = 1$

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

2. When radiating bodies are concentric cylinders or spheres, here

$$F_{1-2} = 1,$$

$$\therefore (F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

in case of coaxial cylinders $\frac{A_1}{A_2} = \frac{\pi d_1 l}{\pi d_2 l} = \frac{d_1}{d_2} = \frac{r_1}{r_2}$.

In case of concentric spheres $\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$

3. When a small body lies inside a large enclosure:

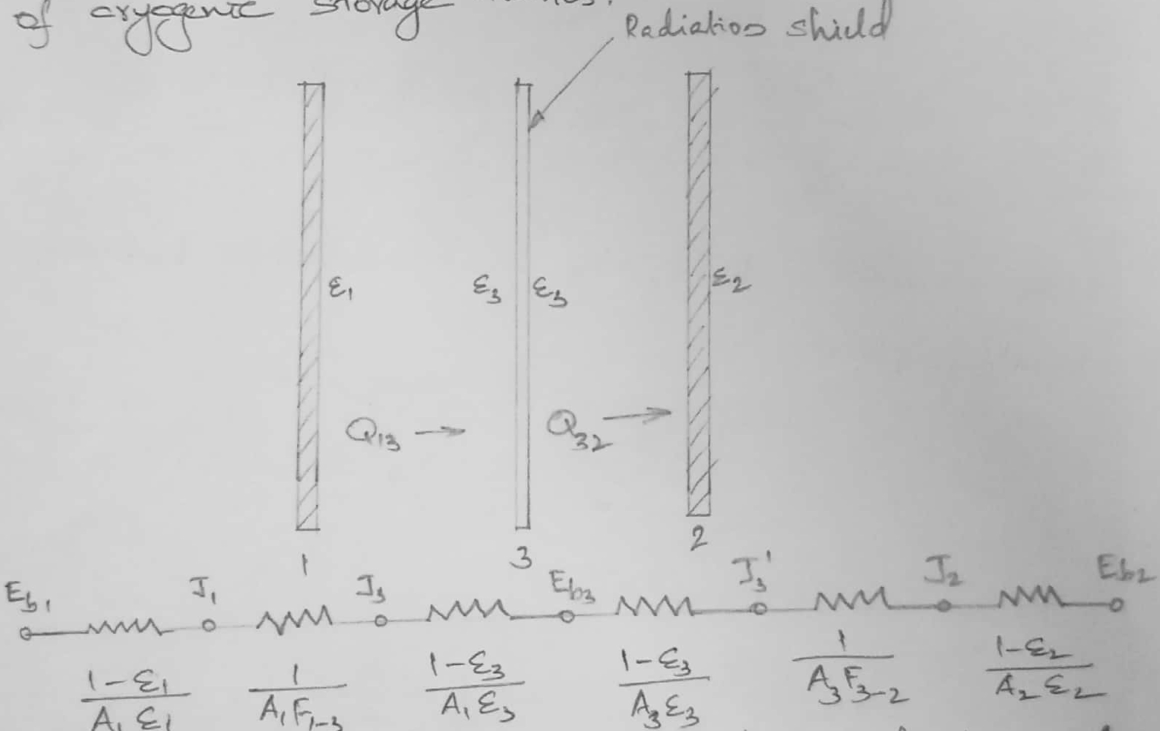
Here $F_{1-2} = 1$, $A_1 << A_2$ so that $\frac{A_1}{A_2} \to 0$

$$\therefore (F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1}$$

ex: (i) A pipe carrying steam in a large room and (ii) A thermocouple bead located inside a duct to measure temp. of the fluid.

RADIATION SHIELD

In certain situations it is required to reduce the overall heat transfer between two radiating surfaces. This is done by either using materials which are highly reflective or by using radiation shields between the heat exchanging surfaces. The radiation shields reduce the radiation heat transfer by increasing the surface resistances without actually removing any heat from the overall system. These are used for the insulation of cryogenic storage tanks.



Consider two parallel plates 1 and 2, each of area A at temperatures T_1 and T_2 respectively with a radiation shield placed between them.

With no radiation shield, the net heat exchange b/w the parallel plates,

$$Q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad \text{——— (1)}$$

If the emissivity of the radiation shield is ε_3, then the heat exchange btw the surfaces 1,3 and 3,2,

$$Q_{13} = \frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$Q_{32} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

Since the radiation shield does not deliver or remove heat from the system,

$$Q_{13} = Q_{32}$$

$$\frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \quad \text{——} \ (2)$$

With radiation shield, the heat exchange between the parallel plates is,

$$Q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)} \quad \text{——} \ (3)$$

Dividing eqn (3) by eqn (1),

$$\frac{(Q_{12})_{\text{with shield}}}{(Q_{12})_{\text{without shield}}} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)} \quad \text{——} \ (4)$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, then $\frac{(Q_{12})_{\text{with shield}}}{(Q_{12})_{\text{without shield}}} = \frac{1}{2}$