WORK DONE BY A RECIPROCATING COMPRESSOR WITH OUT CONSIDERING THE CLEARANCE VOLUME (PROBLEMS)

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A single cylinder single acting air compressor compresses 30 m^3 of air at a pressure of 1 bar and 27° C to 700 kPa. Calculate the power required for the compressor if the compression is (i) isothermal, (ii) polytropic and (iii) adiabatic. Take n = 1.25.

Given data:

$V_1 = 30\ m^3$

$p_1 = 1\ bar = 100\ kPa$

$T_1 = 27°\ C + 273 = 300\ K$

$p_2 = 700\ kPa$

$n = 1.25$

Solution:

Work done during isothermal compression [$pV = C$]

$$W = p_1V_1 \ln\left[\frac{p_2}{p_1}\right]$$

$$W = 100 \times 30 \times \ln\left[\frac{700}{100}\right]$$

$$W = 5837.73\ kJ$$ **Ans.**

Work done during polytropic compression [$pV^n = C$]

$$W = \frac{n}{n-1} p_1V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{1.25}{1.25-1} \times 100 \times 30 \times \left[\left(\frac{700}{100}\right)^{\frac{1.25-1}{1.25}} - 1\right]$$

$$W = 7136.6\ kJ$$ **Ans.**

Work done during isentropic compression [$pV^\gamma = C$]

$$W = \frac{\gamma}{\gamma-1} p_1V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$W = \frac{1.4}{1.4-1} \times 100 \times 30 \times \left[\left(\frac{700}{100}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$W = 7808.21\ kJ$$ **Ans.**

) ***A single stage single acting air compressor has a cylinder bore of 175 mm and a stroke of 225 mm. The compressor sucks air at 100 kN/m² and 27° C and delivers at 8 bar. Find the power required to drive a compressor if its speed is 120 rpm. Also calculate the mass of air compressed per minute and delivery temperature. The compression follows the law $pV^{1.3} = C$.***

Given data:

$D = 175\ mm = 0.175\ m$

$L = 225\ mm = 0.225\ m$

$p_1 = 100\ kN/m^2 = 100\ kPa$

$T_1 = 27°\ C = 27 + 273 = 300\ K$

$p_2 = 8\ bar = 800\ kPa$

$N = 120\ rpm$

$pV^{1.3} = C$

Solution:

Stroke or swept volume, $V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4}(0.175)^2 \times 0.225 = 5.41\times10^{-3}\ m^3$

$$V_s = V_1 \qquad [\because \text{Clearance volume is neglected}]$$

$$V_1 = 5.41\times10^{-3}\ m^3$$

Work done during polytropic compression,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{1.3}{1.3-1} \times 100 \times 5.41\times10^{-3} \times \left[\left(\frac{800}{100}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$W = 1.44\ kJ$$

Indicated power of the compressor,

$$P = \frac{W \times N}{60} = \frac{1.44\times120}{60} = \mathbf{2.88\ kW} \qquad \textbf{Ans.}$$

We know that $p_1 V_1 = mRT_1$

$$m = \frac{p_1 V_1}{RT_1} = \frac{100\times 5.41\times10^{-3}}{0.287\times300} = 6.28\times10^{-3}\ kg$$

Mass of air delivered per minute,

$$m = 6.28\times10^{-3} \times N$$

$$= 6.28\times10^{-3} \times 120 = \mathbf{0.754\ kg/min} \qquad \textbf{Ans.}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$= 300\left(\frac{800}{100}\right)^{\frac{1.3-1}{1.3}} = \mathbf{484.76\ K}$$

2) ***A single stage double acting air compressor of 150 kW power takes air in at 1 bar and delivers at 6 bar. The compression follows the law $pV^{1.35} = C$. The compressor runs at 160rpm with the average piston speed of 150 m/min. Determine the size of the cylinder.***

Given data:

$P = 150\ kW$

$p_1 = 1\ bar = 100\ kPa$

$p_2 = 6\ bar = 600\ kPa$

$n = 1.35$

$N = 160\ rpm$

Piston speed, $S = 150\ m/min$

Solution:

We know that piston speed $= 2LN$

$$150 = 2 \times L \times 160$$

∴ Piston stroke length, $L = 0.469\ m$ **Ans.**

Stroke volume, $V_s = \frac{\pi}{4} D^2 L = V_1$ [∵ Clearance volume is neglected]

$$V_1 = \frac{\pi}{4} D^2 \times 0.469 = 0.368 D^2$$

Work done during polytropic process,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{1.35}{1.35-1} \times 100 \times 0.368 D^2 \left[\left(\frac{600}{100} \right)^{\frac{1.35-1}{1.35}} - 1 \right] = 83.93\, D^2$$

Indicated power, $P = \frac{W \times (2N)}{60}$ [∵ For double acting]

$$150 = \frac{83.93 D^2 \times 2 \times 160}{60}$$

∴ $D = \mathbf{0.579\ m}$ **Ans.**

Air is to be compressed in a single stage reciprocating compressor from 1.013 bar and 15° C to 7 bar. Calculate the indicated power required for a free air delivery of 0.3 m³/min when the compression process is (i) isentropic and (ii) polytropic with n = 1.25.

Given data:

$$p_1 = 1.013\ bar = 101.3\ kPa$$

$$T_1 = 15°\ C + 273 = 288\ K$$

$$p_2 = 7\ bar = 700\ kPa$$

$$V_o = 0.3\ m^3/min$$

$$n = 1.25$$

Solution:

We know that $\frac{p_o V_o}{T_o} = \frac{p_1 V_1}{T_1}$

$$\therefore \quad V_1 = \frac{p_o V_o}{T_o} \times \frac{T_1}{p_1} \qquad \dots (5.8)$$

We know that the pressure and temperature are at atmospheric condition

$$p_o = 101.3 \ kPa$$

$$T_o = 298 \ K$$

Substituting T_o, p_o, V_o, p_1, V_1 values in equation (5.8),

$$V_1 = \frac{101.3 \times 0.3}{298} \times \frac{288}{101.3} = 0.3 \ m^3/min$$

Work done during isentropic compression $[pV^{\gamma} = c]$

$$W = \frac{\gamma}{\gamma - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

here V_o in m^3/min

So $P = \frac{W}{60}$

$$= \frac{1.4}{1.4-1} \times 101.3 \times 0.289 \left[\left(\frac{700}{101.3} \right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

If V_o in m^3

$$W = 75.54 \ kJ/min$$

$P = \frac{W \times N}{60}$

$$P_{isen} = 75.54/60 = \mathbf{1.26 \ kW}$$ **Ans.**

Work done during polytropic compression,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{1.25}{1.25-1} \times 101.3 \times 0.289 \left[\left(\frac{700}{101.3} \right)^{\frac{1.25-1}{1.25}} - 1 \right]$$

$$W = 69.09\ kJ/min$$

$$P_{poly} = 69.09/60 = \mathbf{1.15\ kW}$$

A single stage reciprocating air compressor takes 1 m^3 of air per minute at 1 bar and 15° C and delivers it at 7 bar. The law of compression is $pV^{1.3}$ = constant. Calculate the indicated power. Neglect clearance. If the speed of compressor is 300 rpm and stroke to bore ratio is 1.5, calculate the cylinder dimensions. Find the power required if the mechanical efficiency of compressor is 85% and the motor transmission efficiency is 90%.

Given data:

$V_1 = 1\ m^3/min$

$p_1 = 1\ bar = 100\ kPa$

$T_1 = 15°C = 15 + 273 = 288\ K$

$p_2 = 7\ bar = 700\ kPa$

$pV^{1.3} = C$

$N = 300\ rpm$

$\frac{L}{D} = 1.5$

$\eta_{mech} = 85\%$

Motor transmission efficiency = 90%

Solution:

We know that work done during polytropic compression,

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times 100 \times 1 \left[\left(\frac{700}{100} \right)^{\frac{1.3-1}{1.3}} - 1 \right] = 245.63\ kJ/min$$

Indicated power $P = \frac{245.63}{60} = 4.09\ kJ/s = \mathbf{4.09\ kW}$ **Ans.**

Stroke volume, $V_s = V_1 = \frac{\pi}{4} D^2 L$

$$\frac{1}{300} = \frac{\pi}{4} D^2 L$$

$$\frac{1}{300} = \frac{\pi}{4}\left[D^2 \times 1.5D\right] \qquad \left[\because \frac{L}{D} = 1.5\right]$$

$$\frac{1}{300} = \frac{\pi}{4}\left(1.5D^3\right)$$

$$\therefore \quad D = 0.1414\ m$$

$$\frac{L}{D} = 1.5$$

$$\therefore \quad L = 1.5 \times D = 1.5 \times 0.1414 = 0.2121\ m = 212.1\ mm$$

We know that $\eta_{mech} = \dfrac{\text{Indicated power}}{\text{Power input}}$

Power input $= \dfrac{\text{Indicated power}}{\text{Mechanical efficiency}} = \dfrac{4.09}{0.85} = 4.81\ kW$

Motor efficiency $= \dfrac{\text{Power input}}{\text{Motor power}}$

Motor power $= \dfrac{4.81}{0.9}$

Motor power $= \mathbf{5.34\ kW}$ **Ans.**