WORK DONE BY A RECIPROCATING COMPRESSOR WITH CONSIDERING THE CLEARANCE VOLUME (PROBLEMS)

Dr. K. Pratheesh

Associate Professor

Mechanical Engineering Department
Marian Engineering College

Trivandrum

A single acting air compressor compresses the air from 1.2 bar to 8 bar. The clearance volume is 2.5 liters. The compression and expansion follow the law $pV^{1.35} = C$. If the volumetric efficiency of a compressor is 80%, find the stroke volume and the cylinder dimensions. Assume that the diameter of piston is equal to 0.8 times the stroke of the piston.

Given data:

$$p_1 = 1.2\ bar = 120\ kPa$$

$$p_2 = 8\ bar = 800\ kPa$$

$$V_c = 2.5\text{litre} = \frac{2.5}{1000} m^3 = 0.0025 m^3$$

$$n = 1.35$$

$$\eta_{vol} = 0.8$$

$$D = 0.8L$$

☺ ***Solution*:**

Volumetric efficiency, $\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

where C = clearance ratio = $\frac{V_c}{V_s}$

$$0.8 = 1 + C - C\left[\frac{800}{120}\right]^{\frac{1}{1.35}}$$

$$0.8 = 1 + C - C\,[4.077]$$

$$0.8 = 1 + C\,[1 - 4.077]$$

$$\therefore \quad C = \frac{0.8 - 1}{1 - 4.077}$$

$$C = 0.065$$

We know that $C = \frac{V_c}{V_s}$

$$= \frac{V_c}{\frac{\pi}{4} D^2 \times L}$$

$$0.065 = \frac{0.0025}{\frac{\pi}{4} \times (0.8L)^2 \times L}$$

$\therefore$ $L = \mathbf{0.4245}\ \boldsymbol{m}$ **Ans.**

$\therefore$ $D = 0.8\ L = 0.8 \times 0.4245 = \mathbf{0.34}\ \boldsymbol{m}$ **Ans.**

Stroke volume, $V_s = \frac{\pi}{4} D^2 L$

$$= \frac{\pi}{4} \times 0.34^2 \times 0.4245$$

$V_s = \mathbf{0.0385}\ \boldsymbol{m^3}$ **Ans.**

A single stage reciprocating compressor receives air at 25 m³/min at 1 bar, 15° C and discharges it at 15 bar. Assume the value of n for compression as 1.35 and volumetric efficiency as 0.75. Determine the: (1) theoretical power required (2) piston displacement per min. and (3) maximum air temperature.

Given data:

$V_a = 25\ m^3/min$

$p_1 = 1\ bar = 100\ kPa$

$T_1 = 15°\ C + 273 = 288\ K$

$p_2 = 15\ bar = 1500\ kPa$

$n = 1.35$

$\eta_{vol} = 0.75$

Solution:

Work done on the single stage compressor with clearance volume,

$$W = \frac{n}{n-1} p_1 V_a \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{1.35}{1.35-1} \times 100 \times 25 \times \left[\left(\frac{1500}{100} \right)^{\frac{1.35-1}{1.35}} - 1 \right]$$

$$W = 9816.04\ kJ/min = 163.6\ kJ/s$$

$$P = 163.6\ kW$$ **Ans.**

We know that volumetric efficiency,

$$\eta_{vol} = \frac{V_a}{V_s}$$

$$0.75 = \frac{25}{V_s}$$

$$V_s = 33.33\ m^3/min$$ **Ans.**

We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \times T_1 = \left(\frac{1500}{100}\right)^{\frac{1.35-1}{1.35}} \times 288 = \mathbf{581.17\ K}$$ **Ans.**

Air enters a single stage double acting air compressor at 100 kPa and 29° C. The compression ratio is 6:1. The index of compression and expansion is 1.3. The speed of compression is 550 rpm. The volume rate measured at suction condition is 5 m³/min. Find the motor power required if the mechanical efficiency is 90%. If the volumetric efficiency is 80%, find the swept volume of cylinder.

Given data:

$p_1 = 100\ kPa$

$T_1 = 29°\ C + 273 = 302\ K$

Compression ratio, $\frac{V_1}{V_c} = 6:1$

$n = 1.3$

$N = 550\ rpm$

$V_1 = 5\ m^3/min$

$\eta_{max} = 90\%$

$\eta_{vol} = 80\%$

☺ ***Solution:***

Compression ratio, $r = \frac{\text{Total cylinder volume}}{\text{Clearance volume}} = \frac{V_1}{V_c}$

$$\frac{V_1}{V_c} = 6$$

$$\frac{5}{V_c} = 6$$

$$V_c = 0.833\ m^3/min$$

We know that $V_1 = V_c + V_s$

$$5 = 0.833 + V_s$$

$$V_s = \mathbf{4.167\ m^3/kg}$$ **Ans.**

Work done on the single stage double acting compressor with clearance volume,

$$W = 2 \times \frac{n}{n-1} p_1 V_a \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \qquad \dots (5.17)$$

Volumetric efficiency, $\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

$$C = \frac{V_c}{V_s}$$

$$\eta_{vol} = 1 + \frac{V_c}{V_s} - \frac{V_c}{V_s}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$0.8 = 1 + \frac{0.833}{4.167} - \frac{0.833}{4.167} \times \left(\frac{p_2}{100}\right)^{\frac{1}{1.3}}$$

$$p_2 = 246.31\ kPa$$

We know that $\eta_{vol} = \frac{V_a}{V_s}$

$$0.8 = \frac{V_a}{4.167}$$

$$V_a = 3.33\ m^3/min$$

Applying V_a, p_2 values in equation (5.17),

$$W = 2 \times \frac{1.3}{1.3-1} \times 100 \times 3.33 \times \left[\left(\frac{247.03}{100}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$W = 669.75\ kJ/min = 11.16\ kJ/s$$

$$P = 11.16\ kW$$

Mechanical efficiency, $\eta_{mech} = \frac{\text{Power output of the compressor}}{\text{Power supplied to the compressor}}$

$$0.9 = \frac{11.16}{\text{Power supplied to the compressor}}$$

Power supplied to compressor,

$= \mathbf{12.4\ kW}$ **Ans.**

A single acting single stage compressor is belt driven from an electric motor at 400rpm. The cylinder diameter is 15 cm and the stroke is 17.5 cm. The air is compressed from 1 bar to 7 bar and the law of compression $pV^{1.35}$ = Constant. Find the power of the motor if the transmission efficiency is 97% and the mechanical efficiency of the compressor is 90%. Neglect clearance effects. [Anna Univ. May'06, Dec'07 & Dec'08]

Given data:

$$N = 400\ rpm$$

$$D = 15\ cm = 0.15\ m$$

$$L = 17.5\ cm = 0.175\ m$$

$$p_1 = 1\ bar = 100\ kPa$$

$$p_2 = 7\ bar = 700\ kPa$$

$$pV^{1.3} = \text{Constant}$$

$$\eta_{vol} = 97\%$$

$$\eta_{Mech} = 90\%$$

Solution:

Stroke volume, $V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.15)^2 \times 0.175 = 3.09 \times 10^{-3}\ m^3$

Volumetric efficiency, $\eta_{vol} = \frac{V_a}{V_s}$

$$0.97 = \frac{V_a}{3.09 \times 10^{-3}}$$

$$V_a = 3 \times 10^{-3}\ m^3$$

Work done, $W = \frac{n}{n-1} p_1 V_a \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$

$$W = \frac{1.3}{1.3-1} \times 100 \times 3 \times 10^{-3} \times \left[\left(\frac{700}{100} \right)^{\frac{1.3-1}{1.3}} - 1 \right] = 0.737\ kJ$$

Power output, $P = \frac{W \times N}{60} = \frac{0.737 \times 400}{60} = 4.91\ kW$

$$\eta_{mech} = \frac{P_{output}}{P_{input}}$$

$$0.9 = \frac{4.91}{P_{input}}$$

$\therefore$ Power input $= \frac{4.91}{0.9} = 5.46\ kW$ **Ans.**

A single stage double acting compressor has a free air delivery (FAD) of 14 m^3/min measured at 1.013 bar and 15° C. The pressure and temperature in the cylinder during induction are 0.95 bar and 32° C respectively. The delivery pressure is 7 bar and index of compression and expansion, n = 1.3. The clearance volume is 5% of the swept volume. Calculate the indicated power required and the volumetric efficiency.

Given data:

$$V_o = 14\ m^3/min = 0.233\ m^3/s$$

$$P_o = 1.013\ bar = 101.3\ kPa$$

$$T_o = 15°C = 15 + 273 = 288\ K$$

$$p_1 = 0.95\ bar = 95\ kPa$$

$$T_1 = 32°\ C = 32 + 273 = 305\ K$$

$$p_2 = 7\ bar = 700\ kPa$$

$$V_c = 5\%\ V_s = 0.05\ V_s$$

$$n = 1.3$$

Solution:

Clearance ratio, $C = \frac{V_c}{V_s} = \frac{0.05V_s}{V_s} = 0.05$

Volumetric efficiency, $\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

$$= 1 + 0.05 - 0.05\left(\frac{700}{95}\right)^{\frac{1}{1.3}}$$

$$\eta_{vol} = 0.818 = \mathbf{81.8\%}$$

We know that $\frac{p_o V_o}{T_o} = \frac{p_1 V_a}{T_1}$

$$\frac{101.3\times0.233}{288}=\frac{95\times V_a}{305}$$

$$\therefore \qquad V_a=0.263\ m^3/s$$

Work done or power for double acting compressor,

$$P=2\times\frac{n}{n-1}p_1V_a\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$=2\times\frac{1.3}{1.3-1}\times95\times0.263\left[\left(\frac{700}{95}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

Indicated Power $= \mathbf{126.78}\ \boldsymbol{kW}$ **Ans.**

A single stage single acting reciprocating air compressor compresses 7×10^{-3} m^3 of air per second from 1.0132 bar to 14 bar. The index of polytropic compression is 1.3 and mechanical efficiency 82%. If clearance is 3% of the swept volume, determine the volumetric efficiency and power required to drive the compressor and show the process on p-V diagram. [June'17]

Given data:

$$V_a = 7\times10^{-3}\ m^3/s$$

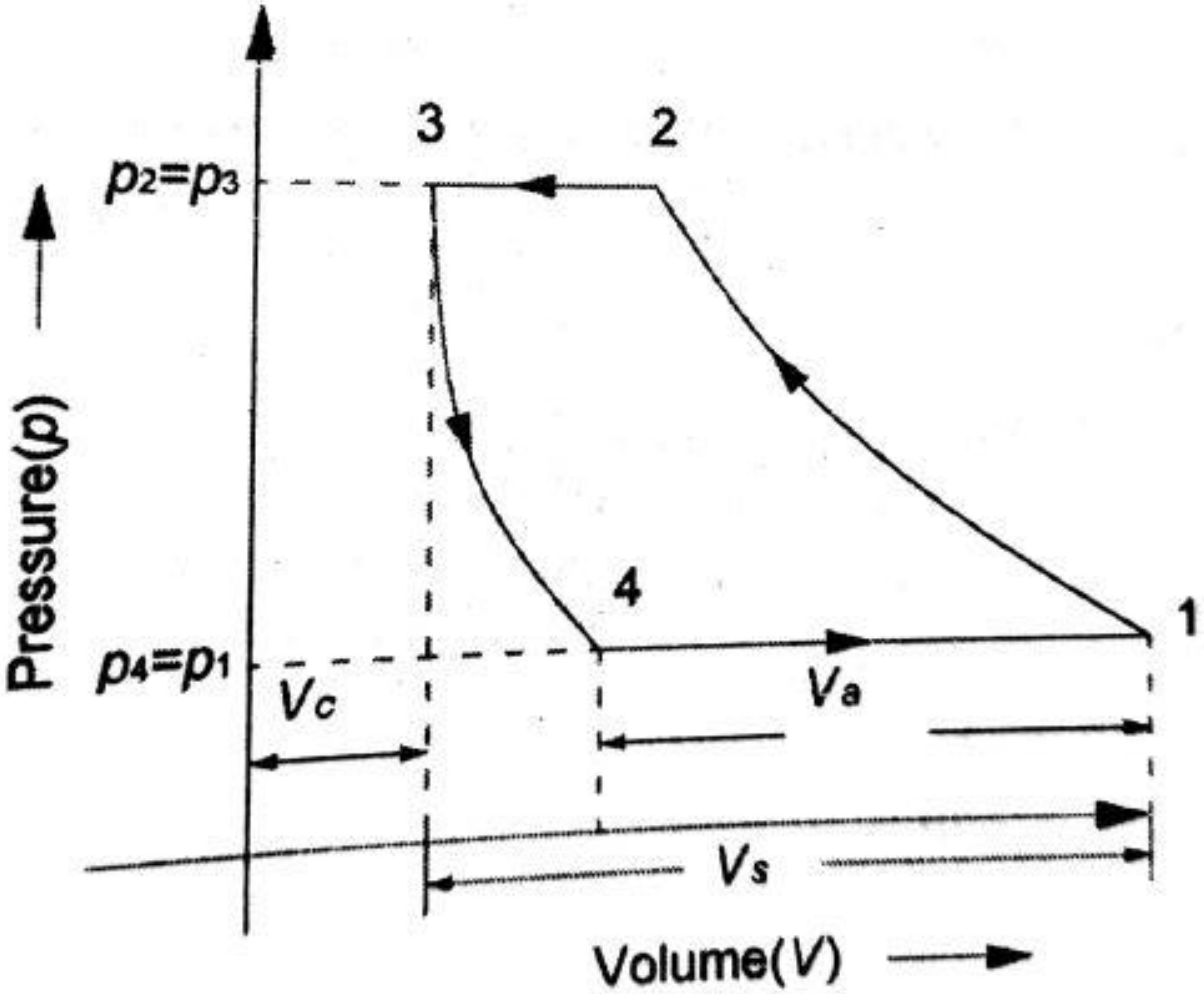
$$p_1 = 1.0132\ bar = 101.32\ kPa$$

$$p_2 = 14\ bar = 1400\ kPa$$

$$n = 1.3$$

$$\eta_{mech} = 82\% = 0.82$$

$$V_c = 3\%\ V_s = 0.03V_s$$


3
2
$p_2 = p_3$
Pressure(p)
4
1
$p_4 = p_1$
V_c
V_a
V_s
Volume(V)

Clearance ratio, $$C = \frac{V_c}{V_s} = \frac{0.03V_s}{V_s} = 0.03$$

Clearance volume, $$\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.03 - 0.03\left(\frac{1400}{101.32}\right)^{\frac{1}{1.3}}$$

$$\eta_{vol} = 0.8039 = \mathbf{80.39\%}$$ **Ans.**

Power supplied to the compressor with clearance volume,

$$P \text{ or } W = \frac{n}{n-1} p_1 V_a \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 101.32 \times 7 \times 10^{-3} \times \left[\left(\frac{1400}{101.32}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$= \mathbf{2.56\ kW}$$ **Ans.**

A single acting reciprocating air compressor with air entering at 1 bar and leaving at 7 bar following $pV^{1.3}$ = constant. Free air delivered is 5.6 m^3/min and mean piston speed is 150m/min. Take stroke to bore ratio of 1.3 and clearance volume to be 1/5th of swept volume per stroke. The suction temperature and pressure are atmospheric. Determine (1) the volumetric efficiency and (2) speed of rotation and (3) stroke and bore. *[June'17]*

Given data:

$p_1 = 1\ bar = 100\ kPa$

$T_1 = 27°C = 273+27 = 300\ K$ (Assumed)

$p_2 = 7\ bar = 700\ kPa$

$pV^{1.3} = C$

$V_o = 5.6\ m^3/min = 0.093\ m^3/s$

Piston speed, $LN = 150\ m/min$

$L = 1.3D$

$V_c = (1/5)\ V_s = 0.2\ V_s$

$p_o = 1.0132\ bar = 101.32\ kPa$ (Assumed)

$T_o = 15°C = 273+15 = 288\ K$ (Assumed)

Solution:

Clearance ratio, $C = \frac{V_c}{V_s} = \frac{0.2V_s}{V_s} = 0.2$

Since, $\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

$$= 1 + 0.2 - 0.2\left(\frac{700}{100}\right)^{\frac{1}{1.3}}$$

$$= 0.307 = \mathbf{30.7\ \%}$$

Then $\frac{p_o V_o}{T_o} = \frac{p_1 V_a}{T_1}$

$$\frac{101.32 \times 0.093}{288} = \frac{100 \times V_{a_1}}{300}$$

$\therefore$ $V_a = 0.098\ m^3/s$

Piston speed, $LN = 150\ m/min$

$$N = 150/L$$

We know that the free air delivery,

$$V_a = V_s \times \eta_{vol} \times N$$

$$0.098 = V_s \times 0.307 \times 150/L$$

$$\therefore \quad V_s = 0.00213L$$

We know that swept volume,

$$V_s = \frac{\pi}{4} D^2 \times L$$

$$0.00213L = \frac{\pi}{4} \times D^2 \times L$$

$$\therefore \quad D = \mathbf{0.052\ m} \quad \textbf{Ans.}$$

$$L = 1.3D = 1.3 \times 0.052 = \mathbf{0.068\ m} \quad \textbf{Ans.}$$

$\therefore$ Speed of rotation, $N = 150/L = 150/0.068 = \mathbf{2205.88\ rpm}$ **Ans.**