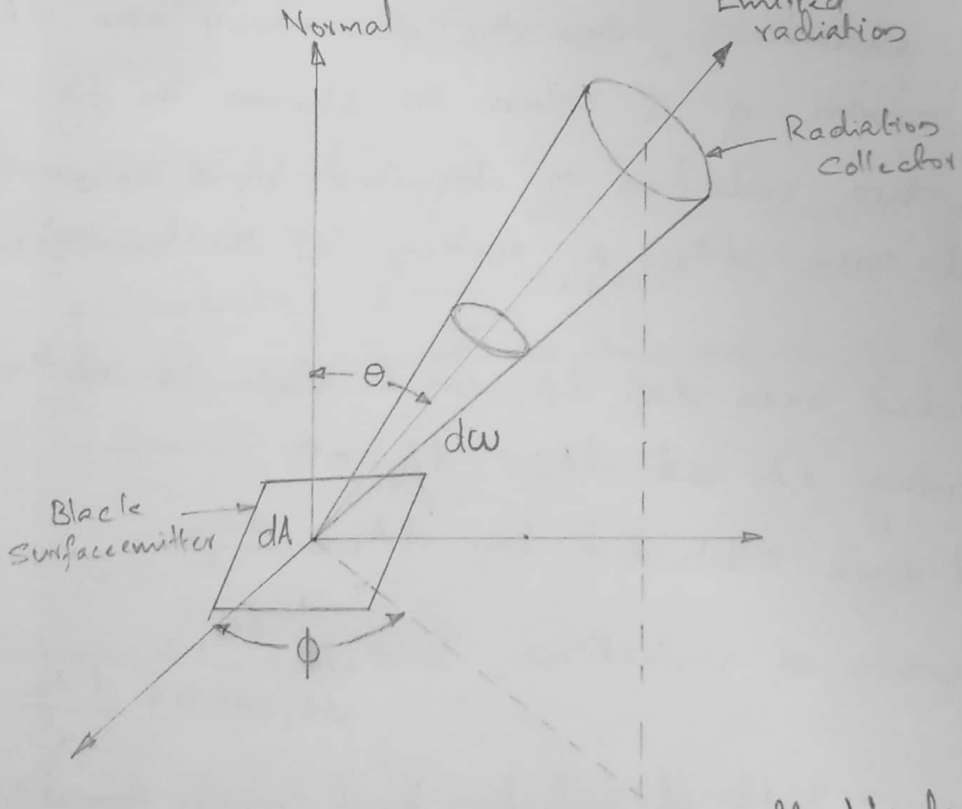
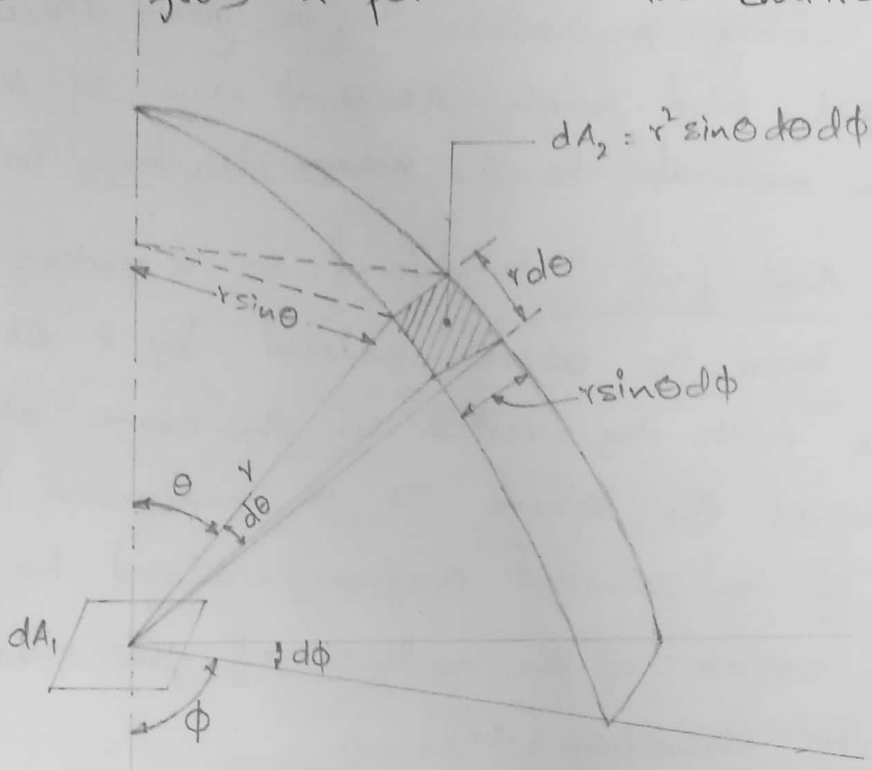
Intensity of Radiation (I) :- is defined as the rate of energy leaving a surface in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

Solid Angle (ω) :- is defined as a portion of the space inside a sphere enclosed by a conical surface with the vertex of the cone at the centre of the sphere. It is measured by the ratio of the spherical surface enclosed by the cone to the square of the radius of the sphere. Its unit is steradian (sr).



The above fig. shows a small black surface of area dA (emitter) emitting radiation in different directions. A black body radiation collector through which the radiation pass is located at an angular position characterised by zenith angle θ towards the surface normal and angle ϕ of a spherical coordinate system. The collector subtends a solid angle '$d\omega$' when viewed from a point on the emitter.



Consider radiation from the elementary area dA_1 at the centre of a sphere as shown in fig. Suppose this radiation is absorbed by a second elemental area dA_2, a portion of the hemispherical surface.

The projected area of dA_1 on a plane $\perp$r to the line joining dA_1 and $dA_2 = dA_1 \cos\theta$.

The solid angle subtended by $dA_2 = \frac{dA_2}{r^2}$

$\therefore$ Intensity of radiation, $I = \dfrac{dQ_{1-2}}{dA_1 \cos\theta \times \frac{dA_2}{r^2}}$

$dQ_{1-2} \rightarrow$ Rate of radiation heat transfer from dA_1 to dA_2

From fig, $dA_2 = r^2 \sin\theta \, d\theta \, d\phi$

$$dQ_{1-2} = I \, dA_1 \sin\theta \cos\theta \, d\theta \, d\phi$$

The total radiation through the hemisphere is given by

$$Q = I\,dA_1 \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \sin\theta . \cos\theta . d\theta\, d\phi$$

$$= 2\pi I\,dA_1 \int_{\theta=0}^{\theta=\pi/2} \sin\theta \cos\theta\, d\theta$$

$$= \pi I\,dA_1 \int_{\theta=0}^{\theta=\pi/2} \sin 2\theta\, d\theta$$

$$Q = \pi I\,dA_1$$

$$Q = E . dA_1$$

$$E\,dA_1 = \pi I\,dA_1$$

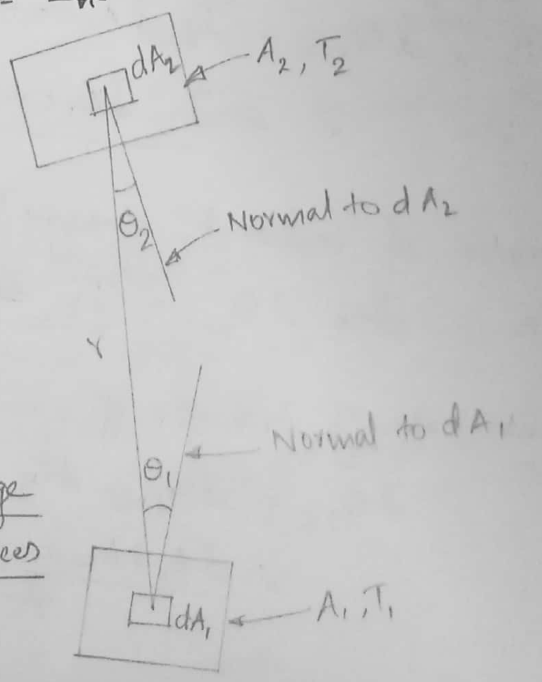
$$E = \pi I$$

The total emissive power of a diffuse surface is equal to π times its intensity of radiation.

Lambert's Cosine law :-

The law states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.

$$E_\theta = E_n \cos\theta$$



Radiation heat exchange between two black surfaces

RADIATION EXCHANGE BETWEEN BLACK BODIES SEPARATED BY A NON-ABSORBING MEDIUM :-

Consider heat exchange between elementary areas dA_1 and dA_2 of two black radiating bodies, separated by a non-absorbing medium and having areas A_1 and A_2 and temperatures T_1 and T_2 respectively. The elementary areas are at a distance 'r' apart and the normals to these areas make angles θ_1 and θ_2 with the line joining them.

Let $d\omega_1$ be the solid angle subtended at dA_1 by dA_2 and $d\omega_2$ be the solid angle subtended at dA_2 by dA_1.

Then $d\omega_1 = \dfrac{dA_2 \cos\theta_2}{r^2}$ and $d\omega_2 = \dfrac{dA_1 \cos\theta_1}{r^2}$ — ①

The energy leaving dA_1 in the direction given by the angle per unit solid angle $= I_{b_1} dA_1 \cos\theta_1$.

I_{b_1} = Intensity of radiation

$dA_1 \cos\theta_1$ = Projection of dA_1 on the line b/w the centres.

The rate of radiant energy leaving dA_1 and striking on dA_2 is, $dQ_{1-2} = I_{b_1} dA_1 \cos\theta_1 . d\omega_1$

$$= \frac{I_{b_1} \cos\theta_1 . \cos\theta_2 . dA_1 dA_2}{r^2} \quad \text{— ②}$$

The rate of radiant energy leaving dA_2 and striking on dA_1 is, $dQ_{2-1} = \dfrac{I_{b_2} \cos\theta_2 . \cos\theta_1 . dA_2 . dA_1}{r^2}$ — ③

The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1}$$

$$= \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{r^2} (I_{b_1} - I_{b_2})$$

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$$dQ_{12} = \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}\left(E_{b_1} - E_{b_2}\right) \quad \left[I_{b_1} = \frac{E_{b_1}}{\pi},\ I_{b_2} = \frac{E_{b_2}}{\pi}\right]$$

$$dQ_{12} = \frac{\sigma\, dA_1 dA_2 \cos\theta_1 \cos\theta_2 \left(T_1^4 - T_2^4\right)}{\pi r^2} \quad \text{--- (4)}$$

Total rate of heat transfer between areas A_1 and A_2,

$$Q_{12} = \int dQ_{12} = \sigma\left(T_1^4 - T_2^4\right)\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1 dA_2}{\pi r^2} \quad \text{--- (5)}$$

The rate of radiant energy emitted by A_1 that falls on A_2, $Q_{1-2} = I_{b_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2} \quad \text{--- (6)}$$

The rate of total energy radiated by A_1, $Q_1 = A_1 \sigma T_1^4$

The fraction of the rate of energy leaving area A_1 and falling on area A_2,

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1 dA_2}{\pi r^2} \quad \text{--- (7)}$$

$$\frac{Q_{1-2}}{Q_1} = F_{1-2}$$

F_{1-2} is known as configuration factor or surface factor or view factor or shape factor between the two radiating surfaces.

The shape factor is defined as the fraction of radiative energy that is diffused from one surface and strikes the other surface. ~~directly with~~

$$Q_{1-2} = F_{1-2} A_1 \sigma T_1^4 \quad \text{--- (8)}$$

Similarly, the rate of radiant energy by A_2 that falls on A_1 is, $Q_{2-1} = \sigma T_2^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{\pi r^2}$

The rate of total energy radiated by A_2 is

$$Q_2 = A_2 \sigma T_2^4$$

The fraction of the rate of energy leaving area A_2 and falling on area A_1 is,

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{\pi r^2} \quad \text{--- (9)}$$

$$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

$$Q_{2-1} = F_{2-1} A_2 \sigma T_2^4 \quad \text{--- (10)}$$

From eqns (7) and (9),

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{--- (11)}$$

The above relation is known as reciprocity theorem.

Thus the net rate of heat transfer between two surfaces A_1 and A_2 is, $Q_{12} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$

$$= A_2 F_{2-1} \sigma (T_1^4 - T_2^4) \quad \text{--- (12)}$$

The above eqn is applicable to ~~X~~ black surfaces only and must ~~cannot~~ not be used for surfaces having emissivities different from unity.

SHAPE FACTOR ALGEBRA AND SALIENT FEATURES OF THE SHAPE FACTOR

1. When two bodies are exchanging radiant energy with each other, the shape factor relation is given by,

$$A_1 F_{1-2} = A_2 F_{2-1} \quad \text{(Reciprocity theorem)}$$

2. When all the radiation emanating from a convex surface 1 is intercepted by the enclosing surface 2, the shape factor of convex surface ~~wrt~~ w.r.t the enclosure F_{1-2} is unity. Thus in conformity with reciprocity theorem, the shape factor F_{2-1} is merely the ratio of areas.
i.e. when surface A_1 is entirely convex, say a sphere, completely enclosed by A_2, thus according to reciprocity theorem,

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$A_1 = A_2 F_{2-1} \quad (F_{1-2} = 1)$$

$$F_{2-1} = \frac{A_1}{A_2}$$

$$F_{2-1} + F_{2-2} = 1$$

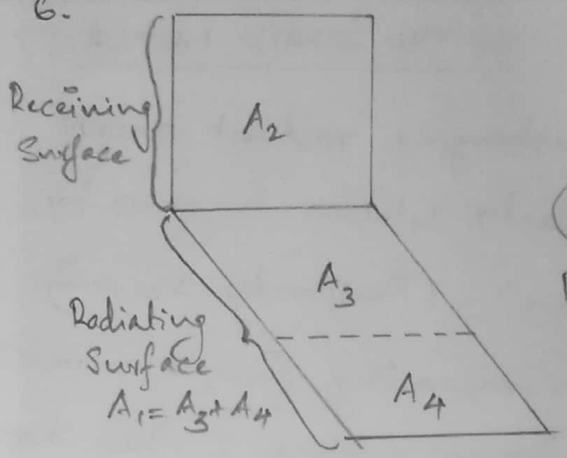
3. For a flat or convex surface, the shape factor with respect to itself is zero (i.e. $F_{1-1} = 0$)

4. For a concave surface, the shape factor w.r.t itself is not zero because the radiant energy coming from one part of the surface is intercepted by the another part of the surface.

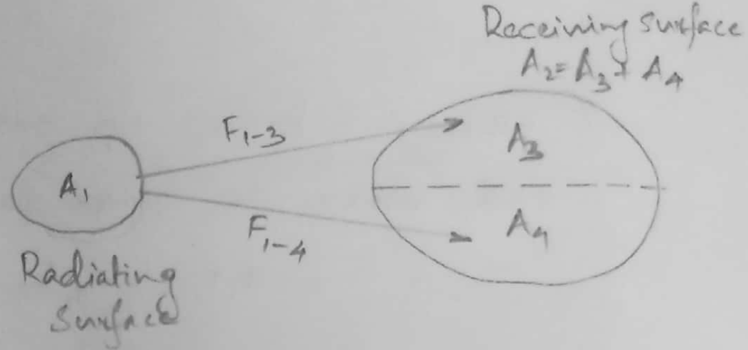
5. If two surfaces A_1 and A_2 are parallel and large, all radiation emitted by one surface falls on the other.

$$F_{1-2} = F_{2-1} = 1$$

6.



(a)



(b)

Fig (a): Radiating surface A_1 has been split into areas A_3 and A_4, $A_1 F_{1-2} = A_3 F_{3-2} + A_4 F_{4-2}$

$$F_{1-2} \neq F_{3-2} + F_{4-2}$$

Thus if the radiating surface is subdivided, the shape factor for the radiating surface w.r.t the receiving surface is not equal to the sum of the individual shape factors.

Fig (b): Receiving surface A_2 has been split into areas A_3 and A_4, $A_1 F_{1-2} = A_1 F_{1-3} + A_1 F_{1-4}$

$$F_{1-2} = F_{1-3} + F_{1-4}$$

The shape factor from a radiating surface to a subdivided receiving surface is the sum of individual shape factors.