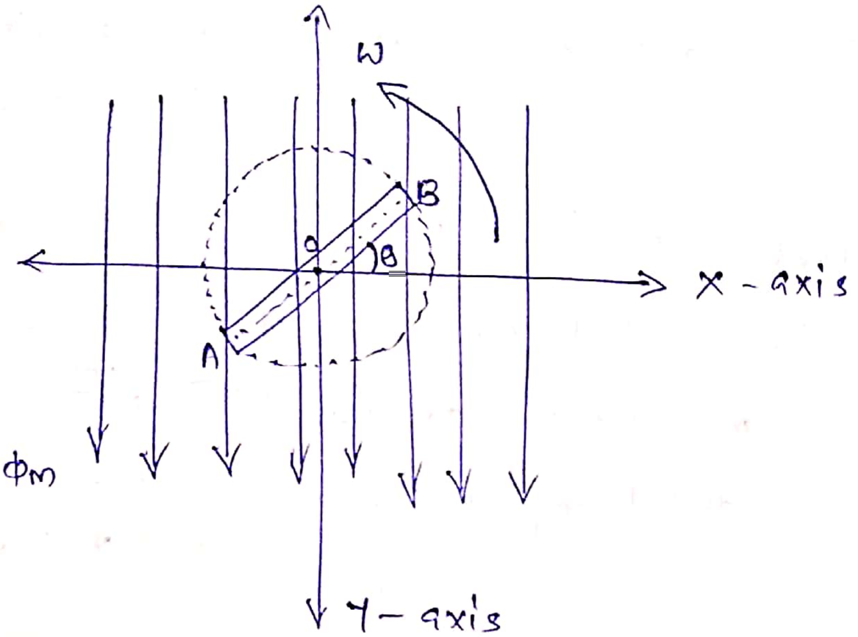
BASICS OF ELECTRICAL ENGINEERING

Module II

FUNDAMENTALS OF ALTERNATING CURRENT

Production of alternating Emf



$\theta = wt$

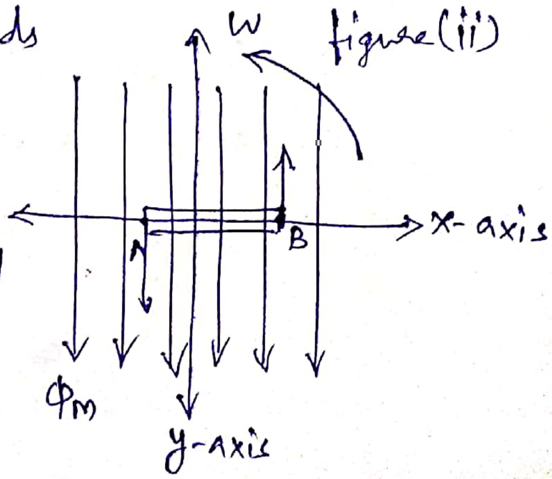
figure (i)

Consider a conductor AB rotating with a constant angular velocity ω radians/sec in a uniform magnetic field. The axis of conductor rotation is ⊥lar to the magnetic lines of force. The angle θ swept by the coil in a time t seconds is given by $\theta = wt$.

Let the conductor position at instant 1 be parallel to x-axis as in figure (ii)

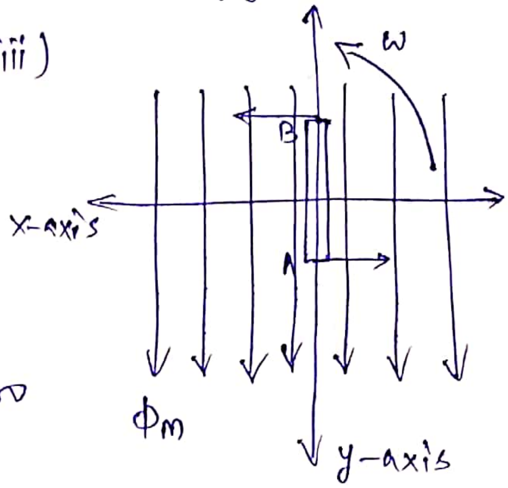
At this instant, point B moves upwards and point A moves downwards, both being parallel to magnetic field. Hence flux cut does not occur and thus Induced emf is zero.

figure (ii)



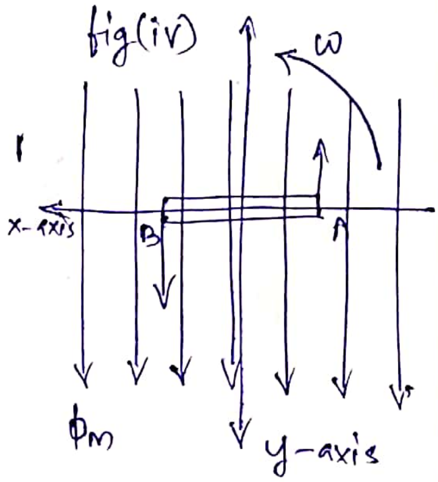
After 90°, at instant 2, the conductor position is parallel to y-axis as in fig(iii). Point B moves left and point A moves towards right, both being $\perp$lar to magnetic field. Hence flux cut is maximum and thus induced emf is also maximum.

fig(iii)



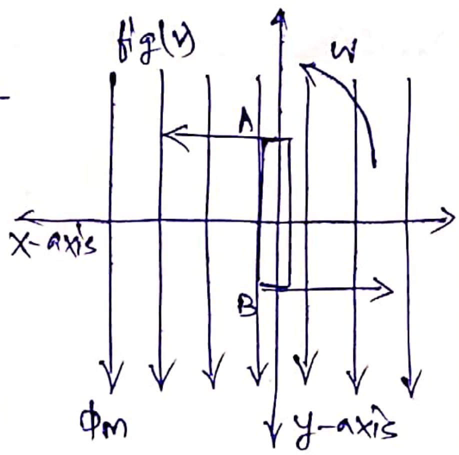
After 180° at instant 3, the conductor position falls similar to that at instant 1 as shown in figure (iv). Flux cut does not occur and hence induced emf is zero.

fig(iv)



After 270° at instant 4, the conductor position falls similar to that at instant 2 as in figure (v). Point B moves towards right and point A towards left; both being $\perp$lar to magnetic field. Hence flux cut is maximum and thus induced emf is maximum in the negative direction since at this instant, the conductor position is opposite to that at instant 2 eventhough similar.

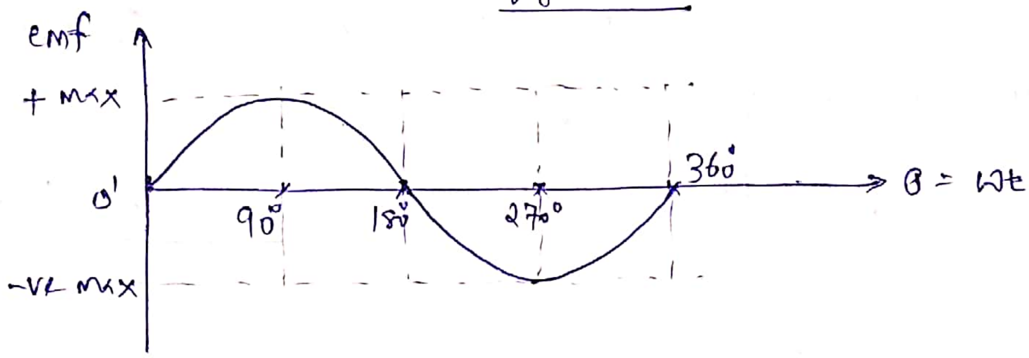
fig(v)



At any instant in between, the induced emf ranges between zero and +ve or −ve maximum values.

When these induced emf points are traced on a 360° scale, we get a sine wave as in figure (vi).

figure (vi)



The instantaneous value of emf generated at any time t is

$$e = E_m \sin(\omega t)$$

Similarly, induced alternating current is given by,

$$i = I_m \sin(\omega t)$$

Important terms:-

① cycle:- One complete set of positive and negative values of an alternating quantity is called a cycle.

② Time period:- The time taken for completing one cycle is called time period or periodic time (T). It is given by,

$$T = \frac{1}{f}, \text{ where } f \text{ is the frequency}$$

③ Frequency:- The number of cycles completed in one second is called the frequency of an alternating quantity. It is expressed in cycles per second or Hertz.

④ Amplitude:- It is the magnitude of the maximum positive or maximum negative value of alternating quantity. It is often referred to as peak value.

(5) Instantaneous value:- The value of alternating quantity at any particular instant is known as instantaneous value.

(6) Average value:- The average value of an alternating quantity is the arithmetic mean of values at equal intervals over a half cycle of the wave.

It is found by, average value = $\frac{\text{area over half cycle}}{\text{Base}}$

For the above sine wave, average value is given by,

$$\text{average value} = \frac{\text{area over half cycle}}{\text{Base}}$$

$$\text{area} = \int_0^{\pi} e.d\theta = \int_0^{\pi} E_m \sin\theta . d\theta$$

$$= E_m(-\cos\theta)_0^{\pi} = E_m(-\cos\pi + \cos 0)$$

$$= \underline{\underline{2E_m}}$$

$$\text{Base} = \pi - 0 = \underline{\underline{\pi}}$$

$$\therefore \text{average value } (E_{avg}) = \underline{\underline{\frac{2E_m}{\pi}}} = \underline{\underline{0.637\ E_m}}$$

(7) Root Mean Square value (RMS value)!

RMS value of an alternating quantity may be defined as that value of dc current which when flowing through a given resistance produces the same amount of heat as that produced by the alternating current passing through the same resistance for the same time. RMS value is also called effective or virtual value.

For the above sine wave, the rms value is given by,

$$E_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} e^2 \cdot d\theta}$$

(Root of mean of the squares of different instantaneous values)

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (E_m \sin\theta)^2 \cdot d\theta} = \sqrt{\frac{E_m^2}{2\pi}\int_0^{2\pi} (\sin^2\theta) \cdot d\theta}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right) \cdot d\theta} = \sqrt{\frac{E_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}}$$

$$= \sqrt{\frac{E_m^2}{4\pi}\left[2\pi - 0 - \cancel{\frac{\sin 4\pi}{2}} + \cancel{\frac{\sin 2(0)}{2}}\right]}$$

$$= \sqrt{\frac{E_m^2}{\cancel{4\pi}\, 2} \times \cancel{2\pi}} = \frac{E_m}{\sqrt{2}} = 0.707\, E_m.$$

$$\therefore \text{RMS value} = \frac{\text{Max. value}}{\sqrt{2}}$$

(8) Form factor: Form factor of an alternating wave is defined as the ratio of its rms value to average value.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{average value}}$$

For a sine wave, form factor $= \dfrac{0.707 \times \text{maxi. value}}{0.637 \times \text{max. value}}$

$= 1.11$

(9) Peak factor:- Peak factor of an alternating wave is defined as the ratio of its maximum value to rms value.

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{rms value}}$$

For sine wave, $\text{peak factor} = \frac{\text{Max. value}}{0.707 \times \text{max. value}}$

$$= \underline{\underline{1.414}}$$

Module III

AC CIRCUITS

Phasor representation

Any alternating quantity can be represented by a rotating phasor. Phasors can be expressed mathematically in the following forms.

(i) Rectangular form
(ii) Trigonometric form
(iii) Exponential form
(iv) Polar form

Note

A phasor is a rotating vector, rotating in the anti clockwise direction. The magnitude of the quantity is proportional to length of vector and angle represents the degree of advancement

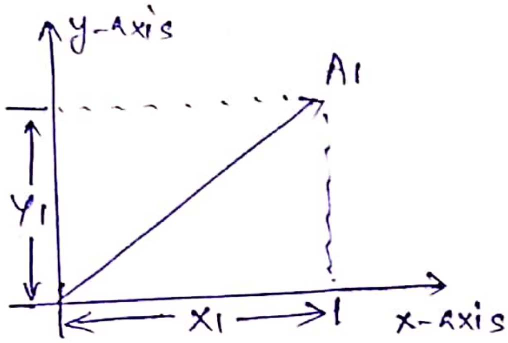
The 'j' operator

Letter 'j' is used to express operation of counter clockwise rotation of a vector through 90°. If this operation is done twice on a vector, it gets rotated counter clockwise through 180° and hence reverses its sign.

Thus $j^2A = -A \Rightarrow j^2 = -1 \Rightarrow \underline{\underline{j = \sqrt{-1}}}$

(i) Rectangular form:

Any vector may be resolved into X-component and Y-component.



For a vector A1, X-Component = X1

Y-Component = Y1

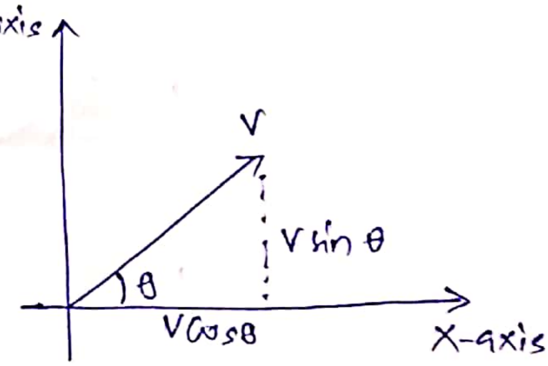
$\therefore \vec{A_1} = X_1 + jY_1$

X_1 is called real component and Y_1 is called imaginary component. This is the rectangular form of phasor representation.

Numerical value of A1 is $\sqrt{X_1^2 + Y_1^2}$

angle of $\vec{A_1}$ w.r.to x axis $= \tan^{-1}\left(\frac{Y_1}{X_1}\right)$

(ii) Trigonometric form:



For a vector, $\vec{V}$ at an angle θ w.r.to x-axis as in figure,

horizontal component = V cos θ.

Vertical component = V sin θ.

$\therefore \vec{V} = V\cos\theta + jV\sin\theta = V(\cos\theta + j\sin\theta)$

This is the trigonometric form of phasor representation

(iii) Exponential form:

According to Euler equation,

$$e^{+j\theta} = \cos\theta + j\sin\theta$$

and

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$\therefore \vec{V}$ can be expressed in exponential form as:

$$\vec{V} = Ve^{+j\theta} \quad \text{(for the above case)}$$

(iv) Polar form:

Consider the above vector $\vec{V}$ making an angle θ with the x-axis and has magnitude V. This vector can be written as: $\vec{V} = V\angle\theta$.

Where θ is the angle with x-axis measured counter clockwise.

Addition and Subtraction of phasors:

Rectangular form of phasors can be added and subtracted as follows.

Consider two phasors $\vec{A} = a_1 + ja_2$

$\vec{B} = b_1 + jb_2$

$$\vec{A}+\vec{B} = (a_1 + ja_2) + (b_1 + jb_2)$$
$$= (a_1 + b_1) + j(a_2 + b_2)$$

$$\vec{A}-\vec{B} = (a_1 + ja_2) - (b_1 + jb_2)$$
$$= (a_1 - b_1) + j(a_2 - b_2)$$

Multiplication and division of phasors:

Polar form of phasors can be multiplied and divided as follows.

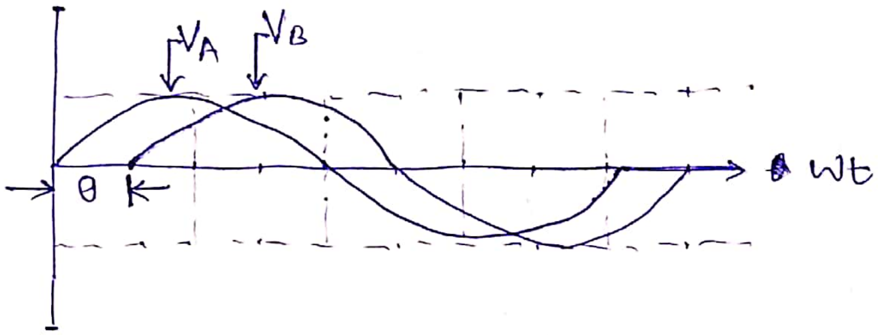
Consider two phasors $\vec{A} = A\angle\theta_1$ and $\vec{B} = B\angle\theta_2$

$$\vec{A}\times\vec{B} = AB\angle(\theta_1 + \theta_2)$$

$$\frac{\vec{A}}{\vec{B}} = \frac{A}{B}\angle(\theta_1 - \theta_2)$$

Phase difference:-

Consider two voltages A and B represented by sine waves.

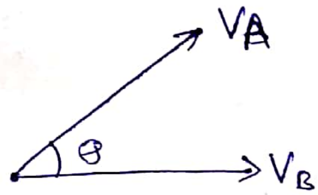


But voltage B starts after an interval of θ.

$\therefore$ If $V_A = V_m \sin wt$

then, $V_B = V_m \cdot \sin(wt - \theta)$

Minus sign indicates that voltage B starts after $\theta°$. It is said to be lagging.



We can say, V_A leads V_B by θ

Or

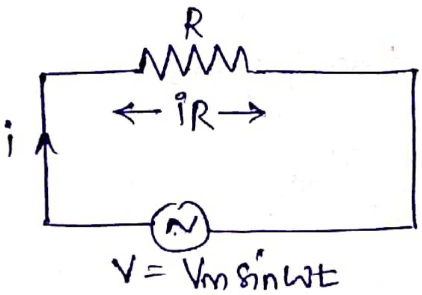
V_B lags V_A by θ

The difference in angle between two voltages is known as phase difference.

If two alternating quantities reach their maximum and zero values at the same time, then they are said to be in phase.

Analysis of AC circuits:-

① AC circuit containing resistance only:-



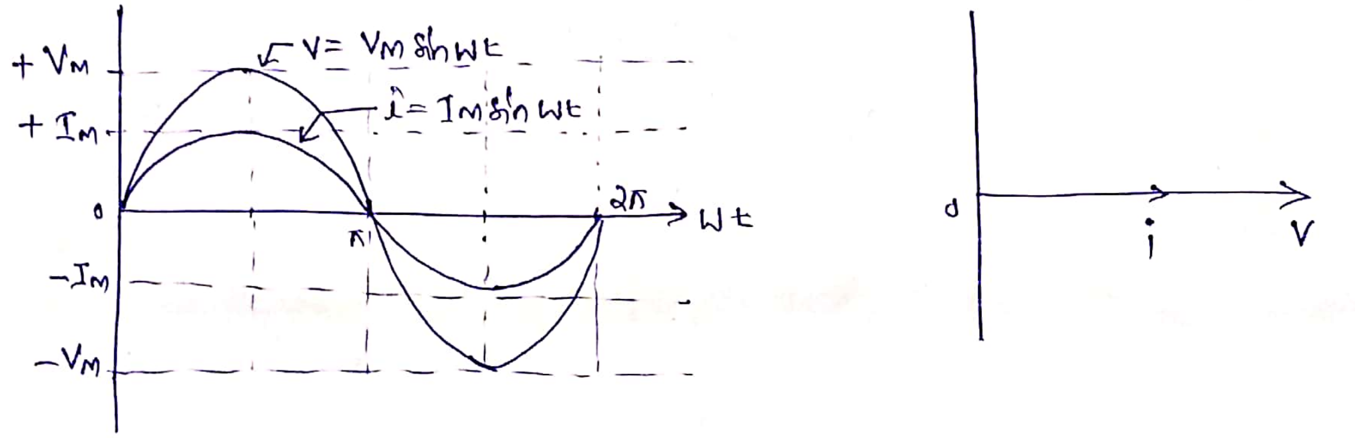
Consider an ac circuit consisting of pure resistance R Ω, to which an ac voltage $v = V_m \sin \omega t$ is applied.

The instantaneous value of current is,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where $I_m = \frac{V_m}{R}$

We can see that voltage and current are in phase.



Power in resistive circuit

Power drawn by the circuit at any instant is the product of instantaneous voltage and instantaneous current.

$$p = v \times i = V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t = V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}$$

Total power consists of two parts,

(i) a constant power $= \frac{V_m I_m}{2}$

(ii) a fluctuating power $= \frac{V_m I_m}{2} \cos 2\omega t$

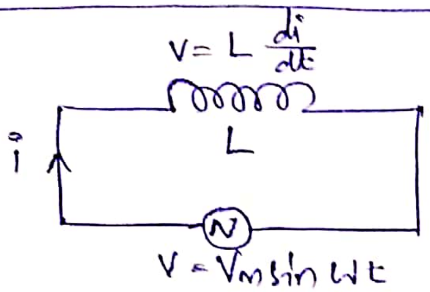
Over a complete cycle; the average value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero. Hence power over a complete cycle is;

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms}.$$

$P = V \times I$ watts

$V \rightarrow$ RMS value of voltage in volts.
$I \rightarrow$ RMS value of current in Amperes.

(2) AC circuit containing Inductance only



Consider an ac circuit consisting of pure Inductance L Henry, to which an AC voltage $v = V_m \sin \omega t$ is applied.

The inductor does not contain any resistive element. Hence the entire applied voltage has to overcome the self induced emf alone.

$$e = -L\frac{di}{dt}. \quad \text{and} \quad e + v = 0.$$

$$\Rightarrow -L\cdot\frac{di}{dt} + V_m \sin \omega t = 0 \quad \Rightarrow \quad V_m \sin \omega t = L\cdot\frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t \cdot dt$$

Integrating on both sides,

$$i = \frac{V_m}{L}\int \sin \omega t \, dt = \frac{V_m}{L} \cdot \frac{-\cos \omega t}{\omega}$$

$$= \frac{-V_m}{L\omega} \cos \omega t = \frac{-V_m}{L\omega} \cdot \sin(\pi/2 - \omega t)$$

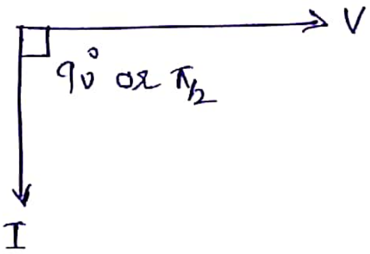
$$= \frac{V_m}{L\omega} \sin(\omega t - \pi/2)$$

when $\sin(\omega t - \pi/2)$ is unity, current is maximum and is denoted by I_m.

Then, $$I_m = \frac{V_m}{L\omega} = \frac{V_m}{X_L}$$

where X_L is the opposition offered to current by an inductive circuit called inductive reactance. It plays the same role as resistance in a resistive circuit. It is expressed in Ω.

$$X_L = L\omega = (2\pi f L)\ \Omega$$



Hence $i = I_m \sin(\omega t - \pi/2)$

From above eqns it is clear that 'I' lags behind 'V' by 90° or $\pi/2$.

<u>Power in purely inductive circuits</u>

Instantaneous power is given by,

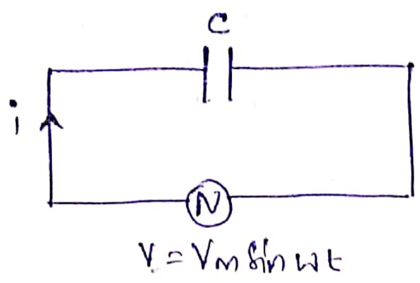
$$P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$$
$$= -V_m I_m \cdot \sin\omega t \cdot \cos\omega t$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Average power for one complete cycle,

$$P = \frac{-V_m I_m}{2} \times \text{avg. of } (\sin 2\omega t) = 0.$$

Hence total power consumed by a purely inductive circuit is zero.

AC circuit containing capacitance only



Consider an ac circuit consisting of pure capacitance C Farad, to which an AC voltage $v = V_m \sin \omega t$ is applied.

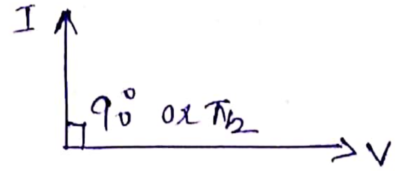
Charging current in the capacitive circuit is given by,

$i = C \times$ rate of change of potential difference

$$= C \times \frac{dv}{dt} = C \times \frac{d}{dt}(V_m \sin \omega t)$$

$$= C \omega V_m \cos \omega t = \omega C V_m \sin(\omega t + \pi/2)$$

$$= \frac{V_m}{1/C\omega} \cdot \sin(\omega t + \pi/2)$$



$$= I_m \sin(\omega t + \pi/2)$$

where $I_m = \frac{V_m}{1/C\omega}$ is the maximum current.

The term $\frac{1}{C\omega}$ is called capacitive reactance and is denoted by X_C. It is expressed in ohms. It can be seen that current leads the applied voltage by an angle $\pi/2$ or $90°$. $X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C}$

Power in a purely capacitive circuit

Instantaneous power is given by,

$$P = V \times i = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin \omega t \cdot \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

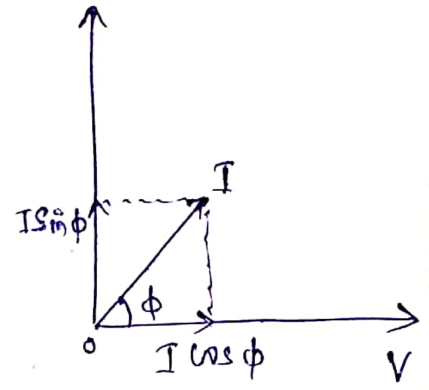
Average power over one complete cycle

$$P = \frac{V_m I_m}{2} \text{ avg. of} (\sin 2\omega t) = 0.$$

Hence total power consumed in a purely capacitive circuit is zero.

Power in an AC circuit

A typical AC circuit contains combinations of R, L and C elements. For a reference voltage phasor (taken along x-axis), current can either lead or lag the voltage by an angle ϕ. We can resolve I into two mutually $\perp$lar components, namely $I\cos\phi$ along V and $I\sin\phi$ $\perp$lar to V as in figure. The component $I\cos\phi$ is in phase with the applied voltage and is therefore called in-phase component or active component. The component $I\sin\phi$ is in quadrature with the applied voltage and is therefore called quadrature component or reactive component.



Active power or real power (P)

This is the actual power dissipated in the circuit resistance. It is given by the product of voltage V and the active component of current through the circuit. Its unit is watts. Thus active power is given by,

$$P = (VI\cos\phi) \text{ watts}.$$

Reactive power (Q)

This is the power developed in the inductive or capacitive reactance of the circuit. It is given by the product of voltage V and the reactive component of current through the circuit. Its unit is volt ampere reactive (VAR). Reactive power is given by, $Q = (VI\sin\phi)$ VAR.

Apparent power (S)

It is the product of rms value of applied voltage and current. It's unit is Volt ampere (VA). Thus apparent power is given by, $S = (VI)$ VA. It is the total power consumed by a circuit.

Power factor:

It is defined as the cosine of the angle between voltage and current in a circuit.

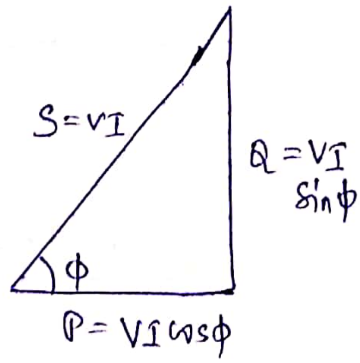
power factor $= \cos\phi$.

Greater the power factor of an ac circuit, greater is the amount of useful power available for doing work.

Power triangle

A right angled triangle can be defined with apparent power (S) as the hypotenuse, active power (P) as the base and reactive power (Q) as the altitude.

It is seen that the angle between P and S $= \phi$.



From power triangle

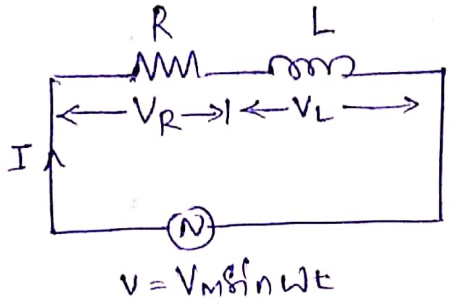
$$\text{power factor} = \cos\phi = \frac{P}{S}\left[\frac{\text{real power}}{\text{apparent power}}\right]$$

$P = VI\cos\phi$; $Q = VI\sin\phi$

$$\Rightarrow P^2 + Q^2 = V^2I^2\cos^2\phi + V^2I^2\sin^2\phi$$
$$= V^2I^2(\cos^2\phi + \sin^2\phi)$$
$$= V^2I^2 = \underline{\underline{S^2}}$$

$$\Rightarrow S = \sqrt{P^2 + Q^2}$$

AC circuit containing resistance and inductance (RL circuit)



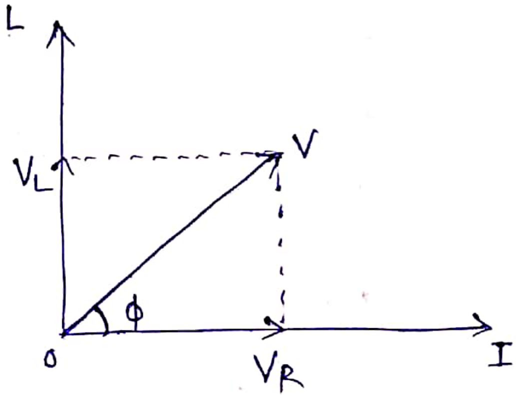
Consider an AC circuit consisting of a series combination of R and L.

Applied voltage $v = V_m \sin \omega t$

Let I be the rms value of resulting current. V_R is the voltage across R and V_L is the voltage across L.

$$V_R = I \cdot R$$

$$V_L = jIX_L \quad \text{where } X_L = L\omega = 2\pi f L$$



$$\text{Applied voltage, } \vec{V} = \vec{V_R} + \vec{V_L}$$

$$= \vec{IR} + j\,\vec{I X_L}$$

$$= I(R + jX_L)$$

$$\Rightarrow I = \frac{V}{R + jX_L} = \frac{V}{Z}$$

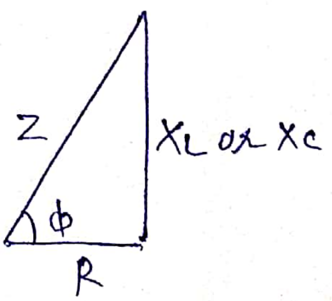
where $Z = R + jX_L$ is called impedance of series RL circuit. It is the total opposition offered by all elements in a circuit.

Magnitude of Z, $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (L\omega)^2}$

Impedance triangle:

Similar to power triangle a right angled triangle can be defined with impedance (Z) as the hypotenuse, resistance (R) as the base and inductive or capacitive reactance (X_L or X_C) as the altitude.

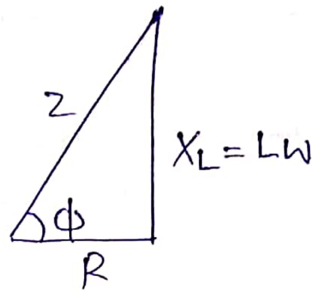
It is seen that the angle between R and Z is ϕ (power factor angle).



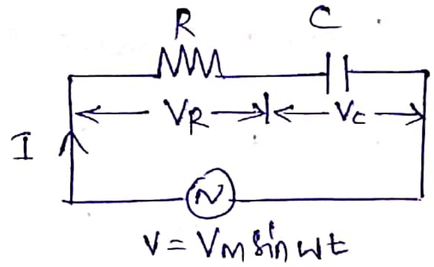
From impedance triangle,

power factor = $\cos\phi = \frac{R}{Z} \left(\frac{\text{resistance}}{\text{impedance}}\right)$.

For an RL circuit, impedance △le is given by

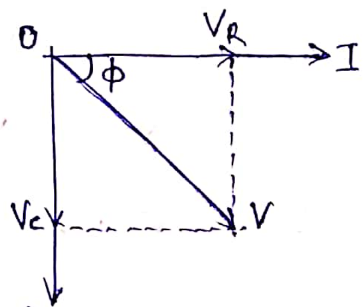


AC circuit containing resistance and capacitance (RC circuit)



Consider an AC circuit consisting of a series combination of R and C. Applied voltage, $V = V_m \sin\omega t$

Let I be the rms value of resulting current. V_R is the voltage across R and V_C is the voltage across C.



$V_R = I \cdot R$

$V_C = -j\, I X_C$ where $X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C}$

Applied voltage $\vec{V} = \vec{V_R} + \vec{V_C}$

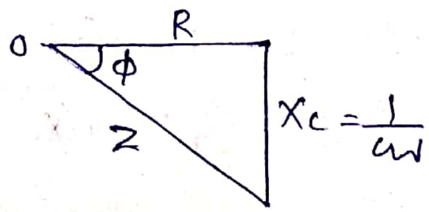
$= IR - j\, I X_C = I(R - jX_C)$

$\Rightarrow I = \frac{V}{R - jX_C} = \frac{V}{\left(R + \frac{1}{j\omega C}\right)} = \frac{V}{Z}$

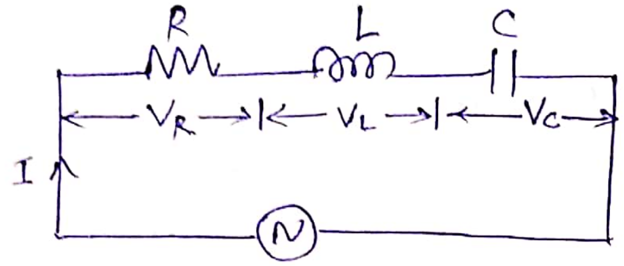
where $Z = \left(R + \frac{1}{j\omega C}\right)$ is the impedance of series RC circuit.

Magnitude of Z, $|Z| = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$

For an RC circuit, impedance △le is given by.



AC circuit containing resistance, inductance & capacitance (R, L and C)



$V = V_m \sin \omega t$

Consider an AC circuit consisting of a series combination of R, L and C elements. The behaviour of RLC circuits depend on the dominance of L and C elements.

RLC circuit with $X_L > X_C$ behaves as an RL circuit.

RLC circuit with $X_C > X_L$ behaves as an RC circuit

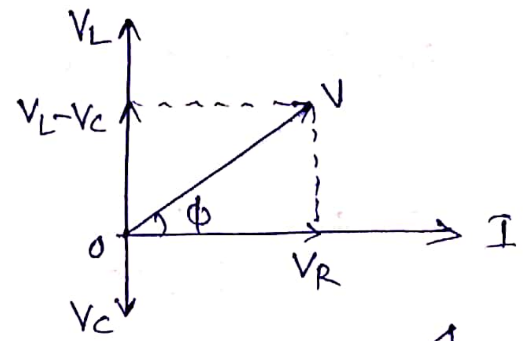
Let I be the rms value of current.

$V_R \rightarrow$ Voltage across resistance $= IR$

$V_L \rightarrow$ Voltage across inductance $= jIX_L$

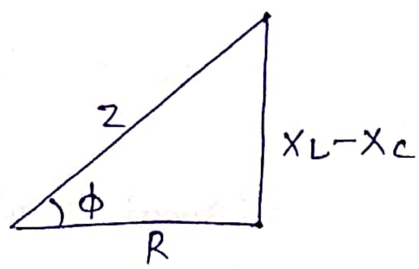
$V_C \rightarrow$ Voltage across capacitance $= -jIX_C$

Case (i) $X_L > X_C$.



$$\Rightarrow \vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$
$$= IR + jI(X_L - X_C)$$
$$= I[R + j(X_L - X_C)]$$

$$\Rightarrow I = \frac{V}{R + j(X_L - X_C)}$$
$$= \frac{V}{Z}$$



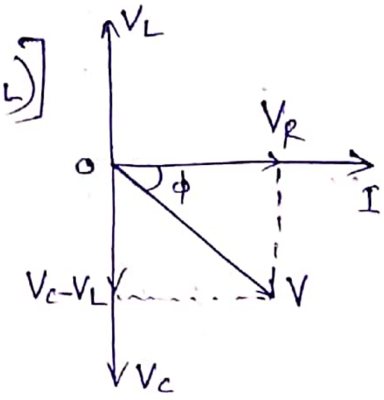
where $Z = R + j(X_L - X_C)$ is the impedance of series RLC circuit

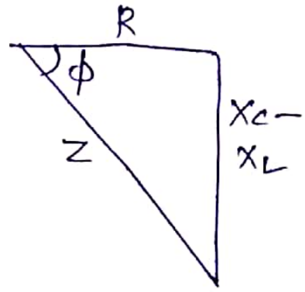
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Case (ii) $X_C > X_L$

$$\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$

$$= I\left[R + j(X_C - X_L)\right]$$





$$\Rightarrow I = \frac{V}{R + j(X_C - X_L)}$$

$$= \frac{V}{Z}$$

Where $Z = R + j(X_C - X_L)$ is the impedance of series RLC circuit

$$|Z| = \sqrt{(R^2 + (X_C - X_L)^2)}$$

Three phase systems:-

Advantages of ~~pd~~ three phase systems:-

① Generation of three phase power is cheaper compared to single phase power.

② Three phase machine has higher efficiency and higher pf compared to single phase machine.

③ For the same size, the output of a three phase machine is greater than that of a single phase machine. Hence it is lighter and cheaper.

④ Single phase motors are not self starting whereas polyphase motors are self starting.

⑤ Parallel operation of single phase alternators is not very smooth; whereas three phase alternators run in parallel without any difficulty.

Production of three phase voltages

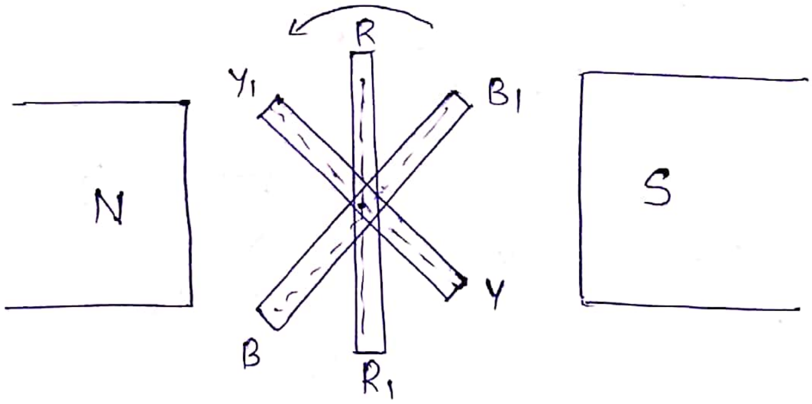
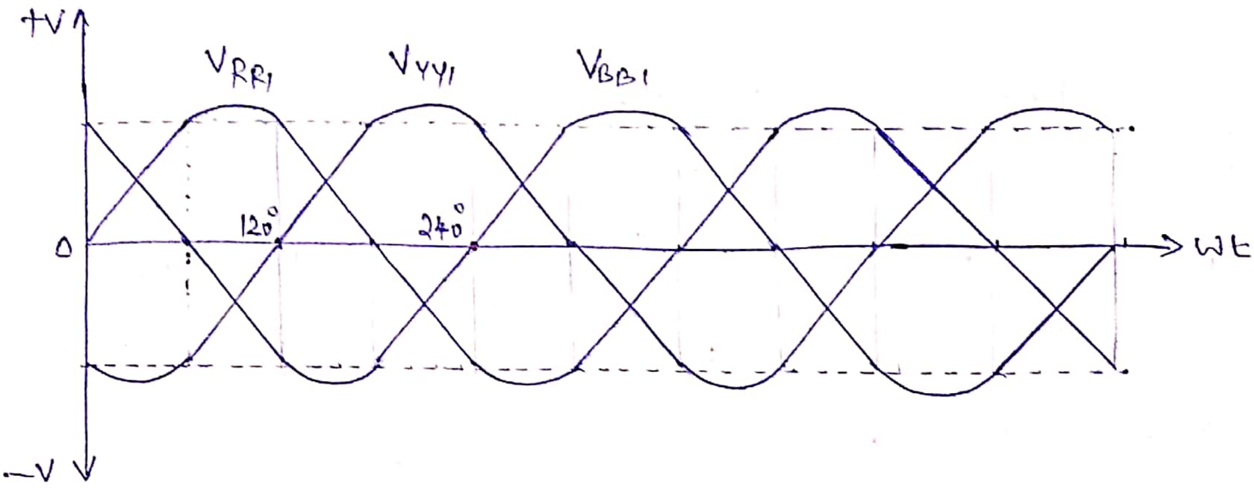
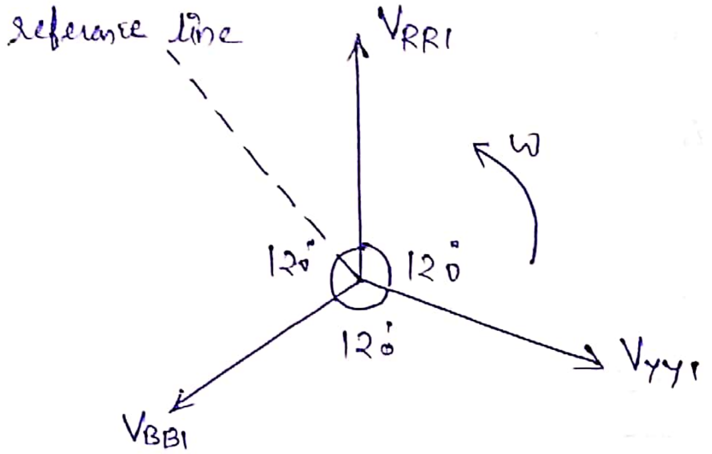


Figure shows 3 rectangular coils RR_1, YY_1 and BB_1 mounted on the same axis but displaced in space from each other by $120°$. Let the three coils be rotated anticlockwise with constant angular velocity in a uniform magnetic field between N and S poles. Let R, Y, B be the start terminals and R_1, Y_1 and B_1 be the finish terminals of these coils. Then V_{RR1} is the voltage induced in the coil RR_1. Similarly V_{YY1} and V_{BB1} are the voltages induced in the coils YY_1 and BB_1 respectively. When the complete coil system rotates, the emf induced in the three coils are all sinusoidal and equal in magnitude. However, since these coils are displaced $120°$ in space, the emfs V_{RR1}, V_{YY1} and V_{BB1} also have $120°$ phase difference. This system of voltages so obtained are called 3 phase voltages. The instantaneous values of generated emfs in coils RR_1 (phase R), YY_1 (phase Y) and BB_1 (phase B) are given by,

$V_{RR1} = V_m \sin\theta$.

$V_{YY1} = V_m \sin(\theta - 120°)$ and $V_{BB1} = V_m \sin(\theta - 240°)$





V_m is the maximum value of generated emf in each of the coils and θ is the position of the coil RR_1 from its initial position.

Phase sequence:

The order in which the phase voltage of three phase system attain their peak or maximum positive (or negative) value is called the phase sequence of the system. In the above phasor diagram, we can see that for a reference line, R phase cross it first, followed by Y phase and then B phase. Hence the phase sequence for the above system is RYB. This can also be written as YBR or BRY.

∴ RYB ≡ YBR ≡ BRY.

The other phase sequence is RBY.

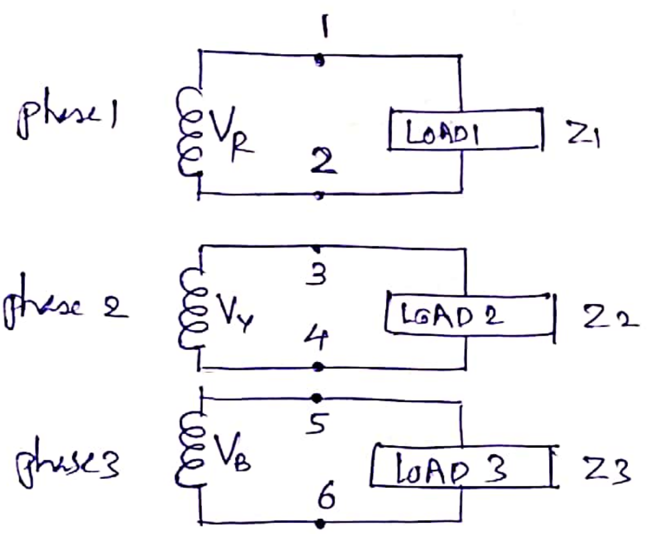
RBY ≡ BYR ≡ YRB.

Connection of 3 phase windings:-

Figure below shows connections of the three phases. If all the three phases are connected separately, it would require a total of six conductors (two for each). This system is complicated and costly. To reduce the number of conductors and thus cost, we use the following two interconnection methods.

(i) Star or Wye (Y) connection.

(ii) Mesh or delta (Δ) connection.



Star connected system:

In this method of connection the similar ends (either start or finish) of all the three coils are joined together at point N as shown in figure. The common point 'N' is called neutral point or star point.

V_R is the voltage between coil terminal R and neutral N.

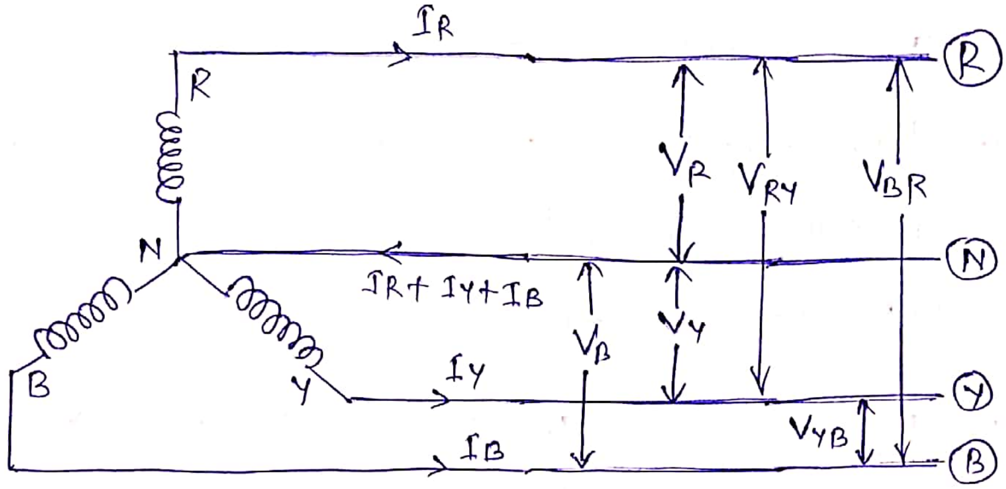
Similarly V_Y is the voltage of Y with respect to N.

V_B is the voltage of B with respect to N.

The three phases R, Y and B carry respective phase currents

I_R, I_Y and I_B. The current I_N through the neutral wire is the vector sum of I_R, I_Y and I_B.

$\therefore \vec{I_N} = \vec{I_R} + \vec{I_Y} + \vec{I_B}$.



<u>Balanced three phase system:</u>

A balanced three phase system is that system in which the currents through different phases are equal in magnitude and displaced from each other at an angle of 120°.

Hence for a balanced system,

If $I_R = I\sin\theta$, then $I_Y = I\sin(\theta - 120^\circ)$ and $I_B = I\sin(\theta - 240^\circ)$

Now, $I_N = \vec{I_R} + \vec{I_Y} + \vec{I_B}$

$= I\sin\theta + I\sin(\theta - 120) + I\sin(\theta - 240)$

$= I\sin\theta + I(\sin\theta\cos 120 - \cos\theta\sin 120) +$
$+ I(\sin\theta\cos 240 - \cos\theta\sin 240)$

$= I(\sin\theta - 0.5\sin\theta - \cancel{0.866\cos\theta} - 0.5\sin\theta + \cancel{0.866\cos\theta})$

$= I(\sin\theta - 0.5\sin\theta - 0.5\sin\theta)$

$= \underline{\underline{0}}$ Hence we can see that for a balanced three phase system, the current through neutral wire is zero.

Terms related with 3 phase systems:

(i) Phase voltage: The voltage between start and finish ends of any phase is called phase voltage. In the above star connected system, V_R, V_Y and V_B (or V_{RN}, V_{YN} and V_{BN}) are the phase voltages.

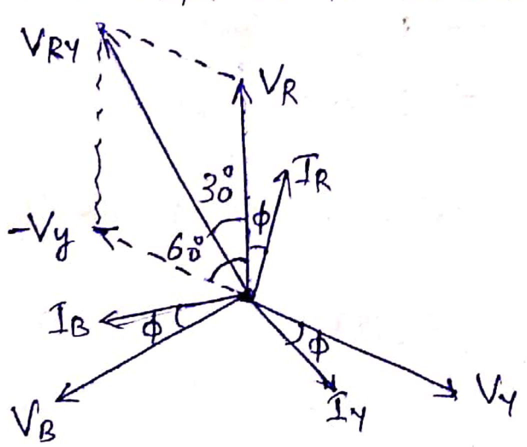
(ii) Line voltage: The voltage between any two phases is called line voltage. In the above system, V_{RY}, V_{YB} and V_{BR} are the line voltages.

(iii) Phase current: The current through any phase is called phase current. I_R, I_Y and I_B are the phase currents.

(iv) Line current: The current through any line is called line current. In the above system, it is seen that the ends of each phase is connected to individual line and hence line current is equal to phase current in a star connected system.

Relation between line voltage and phase voltage in a star connected system:

Let V_R, V_Y and V_B be the phase voltages equal to V_P.
Let V_{RY}, V_{YB} and V_{BR} be the line voltages equal to V_L.



Line voltage V_{RY} is the vector difference of V_R and V_Y.

$$\vec{V_{RY}} = \vec{V_R} - \vec{V_Y}$$
$$= \vec{V_R} + (-\vec{V_Y})$$
$$= \sqrt{V_p^2 + V_p^2 + 2 V_p \cdot V_p \cdot \cos 60}$$
$$= \sqrt{V_p^2 + V_p^2 + V_p^2} = \sqrt{3 V_p^2}$$
$$V_L = \sqrt{3} V_p$$

Hence for a star connected system,

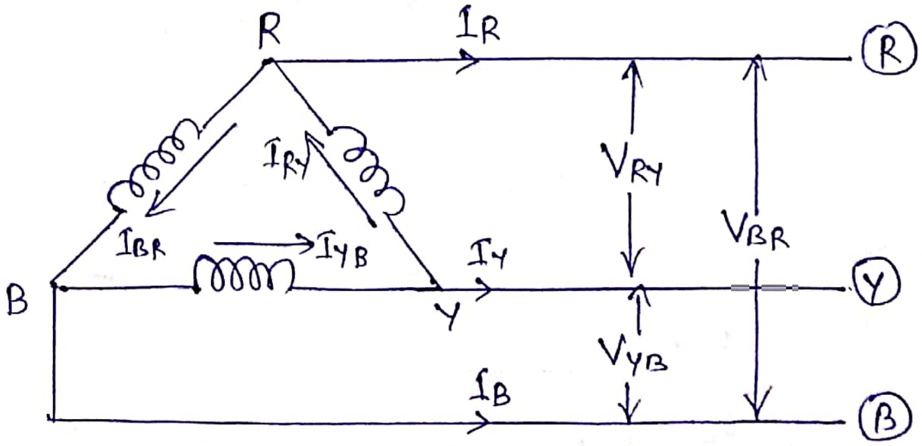
line voltage = $\sqrt{3}$ x phase voltage.

line current = phase current.

$$\boxed{\begin{array}{l} V_L = \sqrt{3} V_p \\ I_L = I_p \end{array}}$$

Delta Connected system

In a delta connected system, dissimilar ends of the three phase windings are joined to form a closed path or mesh. Thus the finishing end of one winding is joined to the starting end of the next winding and so on. Three leads are brought out from the three junctions as in the figure below.



V_{RY} is the voltage of terminal R w.r.t to Y. Similarly, V_{YB} is the voltage of Y w.r.t to B and V_{BR} is the voltage of B w.r.t to R.

We can see that there is only one phase winding completely included between any pair of terminals. Hence the voltage across any phase winding is same as that between any pair of terminals.

Thus for a delta connected system,

line voltage (V_L) = phase voltage (V_P).

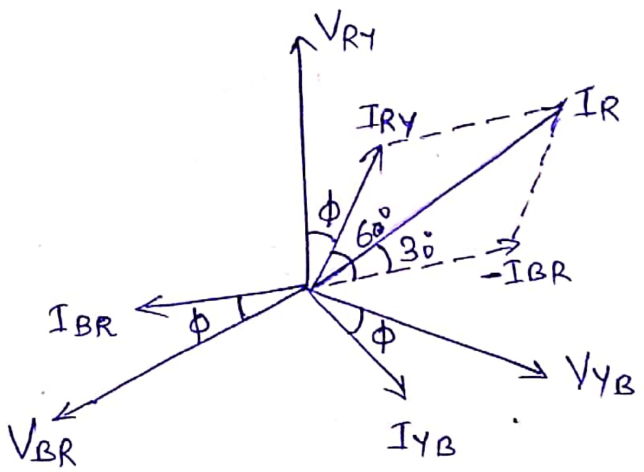
Relationship between line current and phase current

The current in each line is the vector difference between the two phase currents involved. Thus,

$$I_R = I_{RY} - I_{BR}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$



Let phase currents $I_{RY} = I_{YB} = I_{BR} = I_P$

Line currents $I_R = I_Y = I_B = I_L$

From phasor diagram,

$$I_L = \sqrt{I_P^2 + I_P^2 + 2 \cdot I_P \cdot I_P \cdot \cos 60}$$

$$= \sqrt{I_P^2 + I_P^2 + I_P^2}$$

$$I_L = \sqrt{3 I_P^2} = \underline{\underline{\sqrt{3}\, I_P}}$$

Hence for a delta connected system,

line voltage = phase voltage

line current = $\sqrt{3}$ x phase current

$$\boxed{\begin{array}{l} V_L = V_P \\ I_L = \sqrt{3} I_P \end{array}}$$

Power in a balanced 3 phase system :

Let I_P be the rms value of current in each phase and V_P be the rms value of voltage across each phase.

power in each phase = $V_P I_P \cos\phi$.

Total power (P) = 3 x power per phase

$= 3 V_P I_P \cos\phi$.

where ϕ = angle between phase voltage and phase current.

In a star connected circuit, $I_L = I_P$ and $V_L = \sqrt{3} V_P$

$\therefore$ Power (P) $= 3 V_P I_P \cos\phi$

$= 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cdot \cos\phi$

$= \sqrt{3} V_L I_L \cos\phi$.

In a delta connected circuit, $V_L = V_P$ and $I_L = \sqrt{3} I_P$.

$\therefore$ Power (P) $= 3 V_P I_P \cos\phi$

$= 3 \cdot V_L \cdot \frac{I_L}{\sqrt{3}} \cdot \cos\phi$

$= \sqrt{3} V_L I_L \cos\phi$

Hence we can see that the equation for total power remains the same whether the circuit is star or delta connected.

total power = $\sqrt{3}$ x line voltage x line current x powerfactor