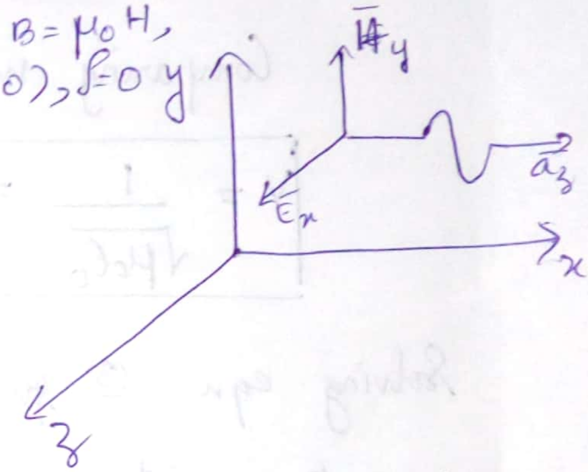
Module VI:

Electromagnetic waves in free space (loss less)

Free space is a lossless medium. $D = \epsilon_0 E$, $B = \mu_0 H$, $J = 0 (\sigma = 0)$, $\rho = 0$

From general wave eqn;



$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

in free space; $\sigma = 0$, $\rho = 0$ (also for a perfect dielectric lossless medium)

$$\Rightarrow \boxed{\begin{aligned} \nabla^2 E &= \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \rightarrow (1) \\ \nabla^2 H &= \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \quad \rightarrow (2) \end{aligned}}$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

Since wave is travelling in z dirn;

$$\nabla^2 E = \frac{\partial^2 E}{\partial z^2}$$

from (1);

$$\frac{\partial^2 E}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \rightarrow (3)$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E}{\partial z^2} \rightarrow (4)$$

velocity of light, $c = 3 \times 10^8$ m/s

Classical wave eqn, $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$

Comparing with eqn ④;

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c} \rightarrow \text{velocity of light in free space}$$

Solving eqn ③ gives;

$$E_x = E_m^+ \cos(\omega t - \beta z) + E_m^- \cos(\omega t + \beta z)$$

$$H_y = H_m^+ \cos(\omega t - \beta z) + H_m^- \cos(\omega t + \beta z)$$

A perfect dielectric (loss less) medium ($\sigma = 0$) is a medium where there is no ohmic heating & hence no loss of energy for the electromagnetic medium.

Hence $D = \epsilon E$, $B = \mu H$, $\sigma = 0$, $\rho = 0$.

$$\boxed{\begin{aligned} \nabla^2 E &= \mu \epsilon \frac{\partial^2 E}{\partial t^2} \\ \nabla^2 H &= \mu \epsilon \frac{\partial^2 H}{\partial t^2} \end{aligned}}$$

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}} \Rightarrow \boxed{v = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$

where, $\sqrt{\mu_r \epsilon_r} = n \rightarrow$ is called refractive index of the medium

~~Wave propagation in a good dielectric (lossy dielectric)~~

Relation b/w $\bar{E}$ & $\bar{H}$:

From Maxwell's eqn,

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} \rightarrow ①$$

Solving eqn ①

$$E_x = \sqrt{\frac{\mu}{\epsilon}}\, H_y \qquad \left(E^2 = E_x^2 + E_y^2 + E_z^2\right)$$

$$E_y = -\sqrt{\frac{\mu}{\epsilon}}\, H_x$$

$$\Rightarrow \boxed{\frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}}.$$ where $\bar{E}$ & $\bar{H}$ are in phase

Intrinsic Impedence : (or characteristic impedance)

The ratio of $\bar{E}$ to $\bar{H}$ of the waves in either direction is called intrinsic impedance. It is denoted by η & is given by,

$$\boxed{\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}}$$

Intrinsic impedence in free space, $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$$\eta_0 = \sqrt{\frac{4\pi \times 10^{-7}}{8.87 \times 10^{-12}}} \simeq 377\ \Omega$$

Electromagnetic waves in a conducting (or lossy dielectric) medium :

For a conducting medium $\sigma \neq 0$. Hence, the electromagnetic field give rise to a conduction current, $J_c = \sigma E$ in the medium. This leads to dissipation of power & hence attenuation of electromagnetic waves.

$$B = \mu H, \quad D = \epsilon E, \quad \rho = 0, \quad \sigma \neq 0$$

Hence, Maxwell's eqns are :

$$\nabla \cdot D = \rho = 0 \qquad \nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

$$\nabla \cdot B = 0 \qquad \nabla \times H = \boxed{J + \frac{\partial D}{\partial t}} = \sigma E + \epsilon \frac{\partial E}{\partial t} \rightarrow (2)$$

Also, $\nabla \cdot E = 0$, $\nabla \cdot H = 0$

Taking curl of eq (1)

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} (\nabla \times H)$$

$$\underbrace{\nabla(\nabla \cdot E)}_{0} - \nabla^2 E = -\mu \frac{\partial}{\partial t} \left[\sigma E + \epsilon \frac{\partial E}{\partial t}\right]$$

$$-\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \epsilon\mu \frac{\partial^2 E}{\partial t^2}} \rightarrow (3)$$

Illy,

$$\boxed{\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \epsilon\mu \frac{\partial^2 H}{\partial t^2}} \rightarrow (4)$$

The terms responsible for energy loss are $\mu\sigma \frac{\partial E}{\partial t}$ and $\mu\sigma \frac{\partial H}{\partial t}$.

Electromagnetic wave eqn in phasor form:

from eqn ③ $\nabla \times E = -\mu \frac{\partial H}{\partial t}$

Taking curl & using vector identity

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} (\nabla \times H)$$

$$= -j\omega\mu (\nabla \times H)$$

$\left(\frac{\partial}{\partial t} = j\omega\right)$

from eqn ④:

$$\cancel{\nabla \times \nabla \quad \nabla(\nabla \cdot E) - \nabla^2 E = -j\omega\mu \frac{\partial}{\partial t}}$$

$$\nabla \times (\nabla \times E) = -j\omega\mu \left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \nabla(\nabla \cdot E) - \nabla^2 E = -j\omega\mu (\sigma + j\omega\epsilon) E$$

$$\nabla^2 E = j\omega\mu (\sigma + j\omega\epsilon) E$$

$$\Rightarrow \boxed{\nabla^2 E = \gamma^2 E}$$

Illy, $\boxed{\nabla^2 H = \gamma^2 H}$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

Propagation constant:

The propagation constant expressed in terms of properties of the medium is,

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}} \rightarrow (1)$$

It is a complex qty made of real & imaginary terms.

Hence,

$$\gamma = \alpha + j\beta \rightarrow (2)$$

When a wave travels through the medium, it gets attenuated ie, the amplitude of the wave reduces. It is represented by the real part of the propagation constant. The attenuation constant α is measured in Neper per metre (Np/m). Smaller the value of α, better is the transmission & the value of α for lossless transmission is zero.

When a wave travels through a medium, its phase changes. This phase change is expressed by the imaginary part of the propagation constant. It is referred to as phase shift β & is measured in rad/m.

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}, \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

Phase velocity :

Phase velocity of a uniform plane wave is defined as the velocity with which the phase of the wave propagates. It is denoted by V_p or V.

$$V = \frac{\omega}{\beta}$$ where ω - angular velocity
β - phase shift (rad/m)

$$\boxed{\beta = \omega\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.87 \times 10^{-12}}}$$

$$= \underline{3 \times 10^8 \ m/s}$$

Wave length (λ) :

The distance travelled by the wave to change the phase by 2π radians is known as wave length, λ.

$$\lambda = \frac{2\pi}{\beta} \quad (m)$$

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{\left(\frac{1}{\sqrt{\mu\epsilon}}\right)2\pi}{2\pi f}$$

$\because (\omega = 2\pi f)$
$V = \frac{1}{\sqrt{\mu\epsilon}}$

$$\Rightarrow \boxed{\lambda = \frac{V}{f}}$$ where V - velocity
f - frequency

① In a lossless dielectric (or free space)

$\sigma = 0$, $\mu = \mu_0 \mu_r$, $\epsilon = \epsilon_0 \epsilon_r$ [if free space $\mu = \mu_0$, $\epsilon = \epsilon_0$]

$\Rightarrow v = \frac{1}{\sqrt{\mu \epsilon}}$

$\Rightarrow$ Propagation constant, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$= \sqrt{j\omega\mu(j\omega\epsilon)}$

$$\boxed{\gamma = \pm j\omega\sqrt{\mu\epsilon}}$$

→ For a perfect dielectric; there is no attenuation, $\alpha = 0$

→ $\beta = \omega\sqrt{\mu\epsilon}$

$\Rightarrow$ The intrinsic impedance, $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$

$$\boxed{\eta = 377\sqrt{\frac{\mu_r}{\epsilon_r}}}$$

II In a good conductor

For a perfect conductor, $\frac{\sigma}{\omega\epsilon} >> 1$ ie, $\sigma >> \omega\epsilon$

ie, the material have high conductivity

$\Rightarrow$ Propagation constant, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

since $\sigma >> \omega\epsilon$, $j\omega\epsilon$ can be neglected.

$$\gamma = \sqrt{j\omega\mu\sigma} \qquad \left[\begin{array}{l} j = 1\angle 90^\circ \\ \sqrt{j} = 1\angle 45^\circ \\ \quad = \cos 45 + j\sin 45 \\ \quad = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \end{array}\right]$$

$$= \sqrt{j}\;\sqrt{\omega\mu\sigma}$$

$$= \frac{1}{\sqrt{2}}(1+j)\sqrt{2\pi f\mu\sigma}$$

$$\boxed{\gamma = \sqrt{\pi f\mu\sigma} + j\sqrt{\pi f\mu\sigma}}$$

$\Rightarrow$ $\alpha = \sqrt{\pi f\mu\sigma}$ Np/m

$\beta = \sqrt{\pi f\mu\sigma}$ rad/m

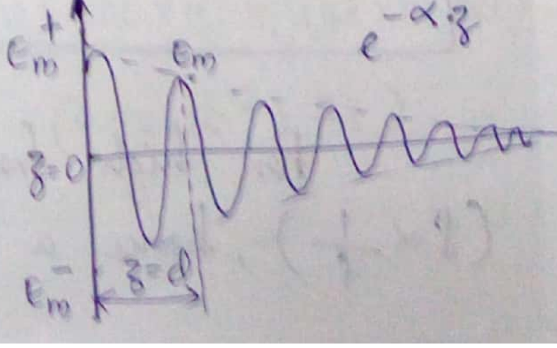
$\Rightarrow$ Intrinsic impedance is given by: $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{2\pi f\mu}{\sigma}}\;\frac{1}{\sqrt{2}}(1+j)$$

$$\boxed{\eta = \sqrt{\frac{\pi f\mu}{\sigma}}\;(1+j)}$$

Skin depth: ~~and~~ (or depth of penetration), δ

~~At z=0~~ Consider a conducting medium that extends to the dim of z axis with a plane surface at $z=0$.



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Let a plane electromagnetic wave be incident norm (1)
to the surface. As the wave propagates through the medium, its amplitude decreases exponentially ($e^{-\alpha z}$). The fall in amplitude in the wave propagation through the medium is called attenuation.

At $z = 0$, the amplitude of $E_x = E_m$ while at $z = d$, amplitude is $E_m e^{-zd}$.

Skin depth or depth of penetration, δ is a quantity defined to compare the relative fall in amplitude for various conducting medium.

If distance $d = \frac{1}{\alpha}$ then, amplitude becomes $\frac{1}{e}$ of the value at surface [ie, $e^{-\alpha z} = e^{-\alpha \cdot \frac{1}{\alpha}} = e^{-1}$]

For a good conductor $\frac{\sigma}{\omega \epsilon} >> 1$,

$$\alpha = \sqrt{\frac{\omega \sigma \mu}{2}} = \sqrt{\pi f \mu \sigma}$$

$$\Rightarrow \boxed{\text{Skin depth, } \delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

For high frequencies skin depth is very small ($\delta \propto \frac{1}{f}$), hence a conductor of negligible thickness can be considered as a conductor of infinite depth.

The phenomenon by which high frequency currents are confined to the skin of a conductor is called skin effect.

Q1. A 300 MHz ~~uni~~ uniform plane wave propagates through fresh water for which $\sigma = 0$, $\mu_r = 1$, $\epsilon_r = 78$. Calculate
i) attenuation constant
ii) phase constant
iii) wave length
iv) intrinsic impedance.

[note : for a dielectric medium: $\sigma = 0$, $\epsilon_r \neq 1$]

i) Given $\sigma = 0$ ie, conductivity zero means it is a lossless medium, hence attenuation constant, $\alpha = 0$

ii) phase constant, $\beta = \omega\sqrt{\mu\epsilon}$

$$\beta = 2\pi f \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r} = 2\pi \times 300 \times 10^6 \times \sqrt{(4\pi \times 10^{-7} \times 1)(8.87 \times 10^{-12} \times 78)}$$

$= 55.53$ rad/m

iii) wavelength, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{55.53} = 0.113\ m$

iv) intrinsic impedance, $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7} \times 1}{8.87 \times 10^{-12} \times 78}}$

$= 42.62\ \Omega$

Tutorial Q2:-

A 9375 MHz uniform plane wave is propagating in polystyrene. If the amplitude of the electric field intensity is 20 V/m and the material is assumed to be lossless, find i) attenuation constant
ii) phase constant iii) wavelength in polystyrene
iv) velocity of propagation v) intrinsic impedance
vi) propagation constant vii) amplitude of magnetic field
For polystyrene $\mu_r = 1$ & $\epsilon_r = 2.56$.

Ans: i) $\alpha = 0$ (lossless)

ii) $\beta = \omega\sqrt{\mu\epsilon} = 2\pi \times 9375 \times 10^{6} \sqrt{4\pi \times 10^{-7} \times 1 \times 8.87 \times 10^{-12} \times 2.56}$

$= 314.37\ rad/m$

iii) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{314.37} = 0.02\ m$

v) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7} \times 1}{8.87 \times 10^{-12} \times 2.56}} = 235.46\ \Omega$

vi) $\gamma = \alpha + j\beta = 0 + j\,314.37 = j\,314.37\ m^{-1}$

iv) velocity of propagation $v = \lambda f = 0.02 \times 9375 \times 10^{6}$
$= 187.5 \times 10^{6}$ m/s

$\left[\lambda = \frac{v}{f}\right]$

vii) Amplitude of magnetic field intensity, $\bar{H}$

$\left[\eta = \frac{E}{H}\right]$

$\bar{H} = \frac{\bar{E}}{\eta} = \frac{20}{235.46} = 0.085$ A/m

Tutorial

Q3. A lossy dielectric is characterised by $\epsilon_r = 2.5$, $\mu_r = 4$ & $\sigma = 10^{-3}$ ℧/m at a freq 10 MHz. Find; i) α ii) β iii) velocity of propagation iv) λ v) η

i) $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$= \sqrt{(j\,2\pi \times 10 \times 10^{6} \times 4\pi \times 10^{-7} \times 4)(10^{-3} + (j\,2\pi \times 10 \times 10^{6} \times 8.87 \times 10^{-12} \times 2.5))}$

$\gamma = 0.22 + j0.7$

$\Rightarrow \alpha = 0.22$ Np/m

ii) $\beta = 0.7$ rad/m

iii) $v = f\lambda = 10 \times 10^6 \times 8.97 = 89.7 \times 10^6$ m/s

iv) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.7} = 8.97$ m

v) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7} \times 4}{8.87 \times 10^{-12} \times 2.5}} = 476.54\ \Omega$

Q4. A uniform plane wave is travelling at a velocity of 2.5×10^5 m/s having wavelength $\lambda = 0.25$ mm in a non magnetic good conductor. Calculate the freq of wave & conductivity of the medium.

Given good conductor, hence assume $\epsilon_r = \mu_r = 1$.

$\Rightarrow \lambda = \frac{v}{f} \Rightarrow freq = \frac{2.5 \times 10^5}{0.25 \times 10^{-3}} = 1000$ MHz

$\beta = \frac{2\pi}{\lambda} = \sqrt{\pi f \mu \sigma}$

$\Rightarrow \frac{2\pi}{0.25 \times 10^{-3}} = \sqrt{\pi \times 1000 \times 10^6 \times 4\pi \times 10^{-7} \times 1 \times \sigma}$

Conductivity, $\sigma = 16 \times 10^{4}$ ℧/m

$E_x(z,t) = 10\cos(2\pi \times 10^{7} t - 0.1\pi z)$ V/m $\Rightarrow$ form: $E_x(z,t) = 10\cos(\omega t - \beta z)$

Transmission lines:

A pair of parallel wires & coaxial cables are the commonly employed transmission lines.

Circuit model:

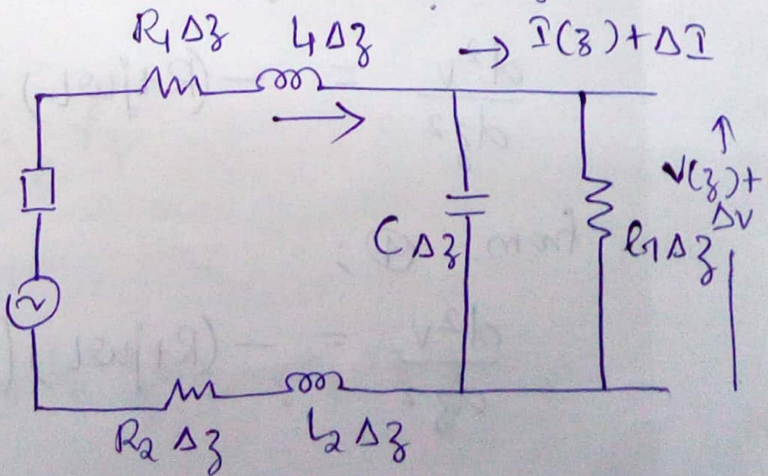
The transmission line is represented as a network of lumped elements like resistors, inductors & capacitors.

The finite conductivity & radiation loss is modelled as series resistance per unit length.

The associated magnetic field due to a current carrying conductor is represented by a series inductance.

The two conducting wires seperated by a distance situated in a dielectric medium gives rise to a capacitance in parallel.

The leakage current through the dielectric b/w the wires is represented by a shunt conductance per unit length.



Applying KVL;

$\cancel{V(z)} = (R_1 + j\omega L_1)\Delta z\, I(z) + V(z)$

$\cancel{V(z)} + \Delta V + (R_2 + j\omega L_2)\Delta z\, I(z)$

$$\Delta V = -\left[R_1 + R_2 + j\omega(L_1 + L_2)\right]\Delta z\, I(z)$$

$$\Rightarrow \frac{\Delta V}{\Delta z} = -\left[R + j\omega L\right] I(z) \longrightarrow ①$$

Applying KCL;

$$\cancel{I(z)} = I(z) + \Delta I + (G + j\omega C)\,\Delta z\,(V(z) + \Delta V)$$

$$\Rightarrow \frac{\Delta I}{\Delta z} = -(G + j\omega C)\left[V(z) + \Delta V\right] \longrightarrow ②$$

Consider the limiting case, Δz, ΔV & $\Delta I \longrightarrow 0$

Then eqn ① & ② becomes;

$$\frac{dV}{dz} = -(R + j\omega L)\, I(z) \longrightarrow ③$$

$$\frac{dI}{dz} = -(G + j\omega C)\, V(z) \longrightarrow ④$$

Differentiating eqns ③ & ④;

$$\frac{d^2V}{dz^2} = -(R + j\omega L)\frac{dI}{dz}$$

from ④;

$$\frac{d^2V}{dz^2} = -(R + j\omega L)\left[-(G + j\omega C)\, V(z)\right]$$

$$\Rightarrow \frac{d^2V}{dz^2} = (R+j\omega L)(G+j\omega C)\,V(z)$$

$$\frac{d^2V}{dz^2} = \gamma^2 \cdot V(z) \longrightarrow (5)$$

IIrly;

$$\frac{d^2I}{dz^2} = \gamma^2 I \longrightarrow (6)$$

where $\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}}$ is called propagation constant

General solution of eqn (5) is;

$$\Rightarrow V(z) = \underbrace{V^+ e^{-\gamma z}}_{\text{incident wave}} + \underbrace{V^- e^{\gamma z}}_{\text{reflected wave}} \longrightarrow (7)$$

incident wave ↳ propagates is the dirn of z axis

reflected wave ↳ propagates in −ve dirn

Differentiating eqn (7);

$$\frac{dV}{dz} = -\gamma V^+ e^{-\gamma z} + \gamma V^- e^{\gamma z}$$

from (3);

$$-(R+j\omega L)\,I(z) = -\gamma\left[V^+ e^{-\gamma z} - V^- e^{\gamma z}\right]$$

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$$I(z) = \frac{\gamma\left[V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right]}{R+j\omega L}$$

$$= \frac{\sqrt{(R+j\omega L)(G+j\omega C)}}{R+j\omega L}\left(V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right)$$

$$I(z) = \sqrt{\frac{G+j\omega C}{R+j\omega L}}\left[V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right]$$

$$\Rightarrow I(z) = \frac{1}{Z_0}\left[V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right]$$

where Z_0 is the characteristic impedance of a transmission line.

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

$\Rightarrow$ For lossless lines, $R \approx 0$, $G \approx 0$ [Since $\omega L >> R$ & $\omega C >> G$]

Hence, $Z_0 = \sqrt{\frac{L}{C}}$

Characteristic impedance of a loss less line is a pure resistance, $\boxed{R_0 = \sqrt{\frac{L}{C}}}$

→ Propagation constant, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\boxed{\gamma \approx j\omega\sqrt{LC}}$$

→ $\alpha = 0$

→ $\beta = \omega\sqrt{LC}$

Since $\alpha = 0$, there is no attenuation for the wave that propagates along a lossless transmission line.

⇒ Group velocity, $v_g = \frac{d\omega}{d\beta} = \frac{1}{\sqrt{LC}}$

⇒ phase velocity, $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

Voltage Standing Wave Ratio (VSWR):

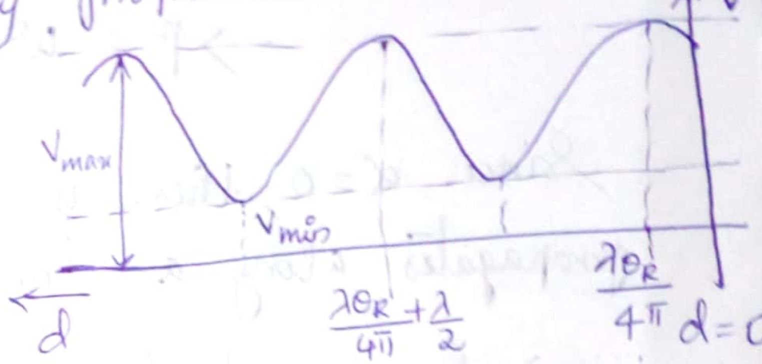
Line voltage shows a periodic variation with a minimum & a maximum if the line is terminated in a mismatched load.

VSWR is defined as the ratio of maximum voltage (V_{max}) to the minimum voltage (V_{min}) in the standing wave pattern on the line. (It is a measure of power transfer to the load)

It is denoted by S.

$$\boxed{VSWR = S = \frac{V_{max}}{V_{min}}}$$

S indicates how well a load is matched to transmission line. For maximum power transfer to the load from the line, it is desirable to have VSWR as near to 1. The average power that can be transmitted along the line is inversely proportional to VSWR.



Impedance matching

A transmission line transfers maximum power when the line is terminated in its characteristic impedance.

The condition for impedance matching is that the load impedance must be equal to the characteristic impedance i.e, $Z_R = Z_0$. A transmission line satisfying this condition is called a matched line. For such a line $|\tau_R| = 0$, $S = 1$ and there is no reflection at the load end. Reflected line is absent & line carrys travelling wave. The line impedance is equal to Z_0. ~~The chara~~

$$V_{max} = |V^+|[1+|\tau_R|]$$

$\tau_R = \frac{V^+}{V^-}$ [voltage reflection coefficient at receiving or load end]

$$V_{min} = |V^+|[1-|\tau_R|]$$

Two commonly employed impedance matching methods are

i) Quarterwave transformer

ii) Stub line

By selecting a quarter wave line (length $d = \frac{\lambda}{4}$) of suitable characteristic impedance, a resistive load can be matched to the line. The quarter wave line inverts the load impedance.

The method of impedance matching of a line by connecting stubs (sections of short circuited line) in parallel with the line at selective positions is called stub matching. Since stub is reactive, it will not absorb power.

Q5. An open wire transmission line of length 10.4 km has the following parameters; $R = 26.2\ \Omega/km$, $L = 0.62\ mH/km$, $G = 0.93\ \mu S/km$ & $C = 38.5\ nF/km$. Calculate the characteristic impedance, attenuation constant in dB/km & phase constant in deg/km at a freq of 1 kHz. Also calculate the velocity of propagation of electromagnetic waves along this line.

given $f = 1\,kHz$

angular freq, $\omega = 2\pi f = 2\pi \times 10^3 = 6.283 \times 10^3\ rad/s$ ~~kHz~~

$\Rightarrow$ Characteristic impedance, $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$R + j\omega L = 26.2 + j(0.62 \times 10^{-3} \times 6.283 \times 10^3) = 26.2 + j3.86$ ~~26.49~~

$= 26.49 \angle 8.47\ \Omega/km$

$G + j\omega C = 0.93\mu + (j\, 6.283 \times 10^{3} \times 38.5 \times 10^{-9})$

$= (0.93 + j241.9)\mu = 241.902 \angle 89.78\ \mu S/km$

$\Rightarrow Z_0 = \sqrt{\dfrac{26.49 \angle 8.47}{(241.902 \angle 89.78^\circ)\mu}} = 330.92 \angle -40.66^\circ\ \Omega/km$

$\Rightarrow$ Propagation constant, $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

$\gamma = \sqrt{(26.49 \angle 8.47)(241.902 \angle 89.78)\mu}$

$= 0.08 \angle 49.13^\circ = 0.0523 + j0.0605$

$\Rightarrow$ Attenuation constant, $\alpha = 0.0523\ Np/km$

$\boxed{1 Np = 8.686\ dB}$

$\alpha = 0.0523 \times 8.686 = 0.454\ dB/km$

$\Rightarrow$ Phase shift constant, $\beta = 0.0605\ rad/km$.

$\Rightarrow$ Phase velocity, $V_p = \dfrac{\omega}{\beta} = \dfrac{2\pi \times 1k}{0.0605}\ km/s$

$= 1.047 \times 10^{8}\ m/s$