# Module II

Reference Text: C.L. Wadhwa

## Economic Load Dispatch.

The Economic load dispatch involves the solution of two different problems. The first of these is the unit commitment or pre-dispatch problem, where in it is required to select optimally out of the available generating sources to operate, to meet the expected load and provide a specified margin of operating reserve over a specified period of time. The second aspect of economic dispatch is the on-line economic dispatch where in it is required to distribute the load among the generating units actually paralleled with the system, in such a manner as to minimize the total cost of supplying the minute to minute requirements of the system.

In case of economic dispatch, the generations are not fixed, but they are allowed to take values again within certain limits, so as to meet a particular load demand with minimal fuel consumption. Economic load dispatch is a solution of a large number of load flow problems and choosing the one which is optimal in the sense that it needs minimum cost of generation. Since the total cost of generation is a function of individual generation of the sources which can take values within certain constraints, the cost of generation will depend upon the system constraint for a particular load demand.

## System Constraints.

There are two types of constraints :- (i) Equality constraints and (ii). Inequality constraints. Inequality constraints are of two types : (i) Hard type (ii) Soft type. The hard type are those which are definite and specific like the tapping range of an on-load tap changing transformer. Soft type are those which have some flexibility associated with them like the nodal voltages and phase angles between the nodal voltages etc. Soft inequality constraints have been very efficiently handled by penalty function methods.

## Equality Constraints:

The equality constraints are the basic load flow equations, given by.

$$P_i =$$

$$Q_i =$$

$$i = 1, 2, \ldots\ldots n$$

## Inequality Constraints.

**(a) Generator Constraints:** The kVA loading on a generator is given by $S_i = \sqrt{P_i^2 + Q_i^2}$ and this should not exceed a pre specified value $C_i$, because of the temperature rise condition.

$$\therefore \quad P_i^2 + Q_i^2 \leq C_i^2$$

The maximum active power generation of a source is again limited by thermal consideration and also minimum power generation limited by flame instability of boiler.

$$P_i \min \leq P_i \leq P_i \max .$$

Similarly max and min reactive power generation of the source is limited

$$\therefore Q_i \min \leq Q_i \leq Q_i \max .$$

(b) Voltage Constraints: It is essential that the voltage magnitudes and phase angles at various nodes should vary within certain limits. The voltage magnitudes should vary within certain limits, because otherwise most of the equipments connected to the system will not operate satisfactorily or additional use of voltage regulating devices will make the system uneconomical, Thus

$$|v_i \min| \leq |v_i| \leq |v_i \max|$$

$$\delta_i \min \leq \delta_i \leq \delta_i \max .$$

(c) Running Spare Capacity Constraints: These constraints are required to meet (i) forced outages of one or more alternators (ii) Unexpected load on the system.

$$\therefore P_G \geq P_i + P_{so}$$ ; where $P_{so}$ spare capacity min.

(d) Transformer tap settings: If an auto transformer is used, the min tap setting is zero and maximum one

$$\therefore 0 \leq t \leq 1$$

Similarly for two winding transformer; $0 \le t \le n$ ; $n \rightarrow$ ratio of transformation

Phase shift limits of phase shifting transformer $\theta_{i\,min} \le \theta_i \le \theta_{i\,max}$.

**(e) Transmission line constraints:** The flow of active and reactive power through the transmission line circuit is limited by thermal capacity of the circuit.

$$C_i \le C_{i\,max}$$

$C_{i\,max} \rightarrow$ max loading on $i^{th}$ line.

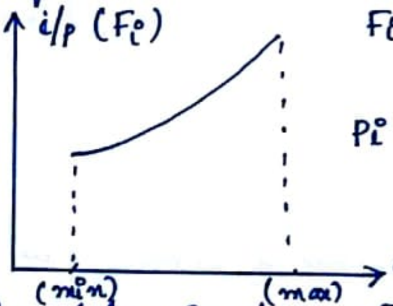
**(f) Network Security Constraints:**

If initially a system is operating satisfactorily and there is an outage, may be scheduled or forced one, it is natural that some of the constraints of the system will be violated.

## Characteristics of Thermal Power Plant:

**(i). Cost Curve:** The curve drawn between input of the plant on Y axis (in Rs/hr or Kcal/hr) and net output power on X axis (MW).

By fitting a suitable 2 polynomial the expression for the operating cost can be written as $F_i = \frac{1}{2} a_i P_i^2 + b_i P_i + c_i$

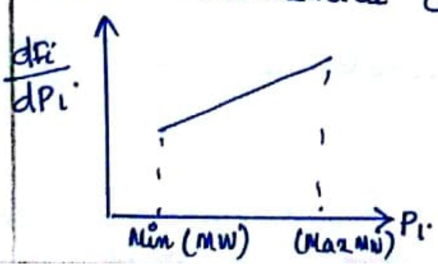



$F_i \rightarrow$ fuel input

$P_i \rightarrow$ Power o/p

where $a_i, b_i, c_i$ are constants and can be determined experimentally

**(ii) Incremental cost curve:** The slope of the cost curve $dF_i/dP_i$ is called incremental cost function or incremental production cost.



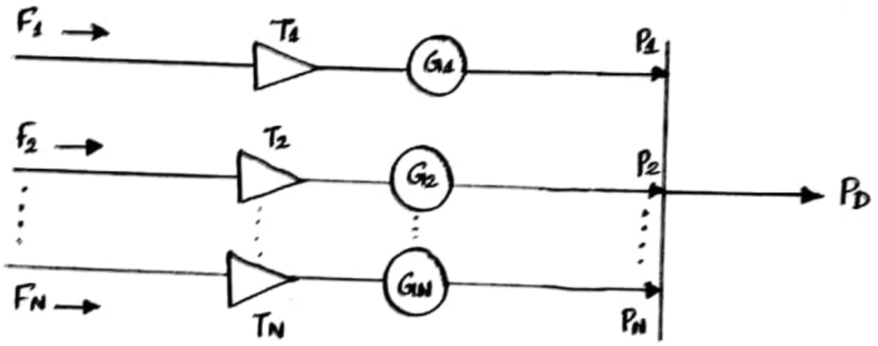

Represented in Rs/MWhr. ($I_c$)

$\therefore$ Incremental Cost $= \frac{dF_i}{dP_i}$

$= a_i P_i + b_i$

## Economic load dispatch neglecting transmission losses.

Consider, there are 'N' generation units. $F_1, F_2 \cdots F_N$ be the fuel inputs to the plants (fuel cost). $P_1, P_2 \cdots P_N$ be the power output from plants 1, 2 ... N. Let T → Thermal plant, G - generator and $P_D$ be the power demand.




Our objective is to minimize the cost of production of power subjected to some constrain

Total fuel (fuel cost) $\rightarrow F_T = F_1 + F_2 + \cdots + F_N$

$$= \sum_{i=1}^{N} F_i$$

$$\therefore \quad \text{Objective Function} = F_T = \sum_{i=1}^{N} F_i .$$

Constrain → Since losses are neglected, total power generated = demand

$$\text{or} \quad P_D = \sum_{i=1}^{N} P_i$$

$$P_D - \sum_{i=1}^{N} P_i = 0 \longrightarrow \text{equality constrain } (\phi) .$$

Hence, our problem is to minimize the objective function subjected to the equality constrain.

For solving the problem, define a Lagrange function '$\mathcal{L}$'.

This function is obtained by adding the O.F to the constrain, after multiplying the constrain by an undetermined multiplier $\lambda$ (Lagrange multiplier).

$$\therefore \quad \mathcal{L} = O.F + \lambda \cdot \phi$$

$$\therefore \quad \mathcal{L} = \sum_{i=1}^{N} F_i + \lambda \left( P_D - \sum_{i=1}^{N} P_i \right)$$

To minimize the function, differentiate $\mathcal{L}$ w.r.t. $P_i$.

$$\therefore \quad \frac{d\mathcal{L}}{dP_i} = \frac{dF_i}{dP_i} + \lambda(0-1)$$

now equate to zero

$$\therefore \quad \frac{dF_i}{dP_i} - \lambda = 0 \quad \text{or} \quad \boxed{\frac{dF_i}{dP_i} = \lambda}$$

ie $\quad \dfrac{dF_1}{dP_1} = \dfrac{dF_2}{dP_2} = \cdots\cdots = \dfrac{dF_N}{dP_N} = \lambda.$

$\therefore$ To minimize the cost of production; $\dfrac{dF_i}{dP_i} = \lambda.$

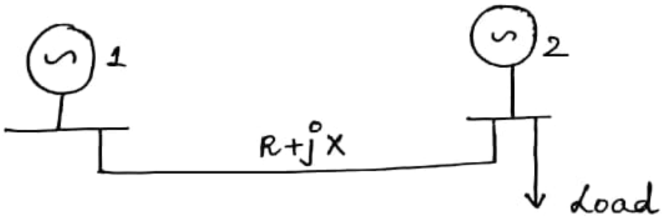
where $\dfrac{dF_i}{dP_i}$ is called Incremental production cost.

also we have $\boxed{\dfrac{dF_i}{dP_i} = a_i P_i + b_i}$

where $a_i$ and $b_i$ are constants.

## Economic load dispatch (Optimal load dispatch) including Transmission Losses.

When the Energy is to be transported over a large distance, transmission losses in some cases may amount to 20 to 30% of total load, and it then become necessary to take the losses also into account, when developing our economic dispatch strategy.




Here, 2 generators have equal incremental production cost. If generator 2 has a local load, according to equal production cost criteria, total load must be shared by the two. But commonsense tells us that it is more economical to let Gen 2 supply most of the local load, because gen 1 has to supply in addition to the load, the transmission losses also. So here, equal incremental cost does not hold good, so we have to consider transmission losses also.

∴ Objective function $F_T = \sum_{i=1}^{N} F_i$.

constraint $P_D = \sum_{i=1}^{N} P_i - P_L$ ; where $P_L \rightarrow$ total transmission loss.

or $P_D - \sum_{i=1}^{N} P_i + P_L = 0$

$\therefore$ Lagrange Function $\mathcal{L} = O.F + \lambda \phi$

$$\therefore \quad \mathcal{L} = \sum_{i=1}^{N} F_i + \left(P_D - \sum_{i=1}^{N} P_i + P_L\right) \cdot \lambda$$

now differentiate $\mathcal{L}$ w.r.t $P_i$ and equate to zero.

$$\frac{d\mathcal{L}}{dP_i} = \frac{dF_i}{dP_i} + \lambda\left(0 - 1 + \frac{dP_L}{dP_i}\right) = 0$$

$$\therefore \quad \frac{dF_i}{dP_i} + \lambda\left(\frac{dP_L}{dP_i} - 1\right) = 0$$

$P_L$ is a function of $P_i$.

or $\boxed{\frac{dF_i}{dP_i} + \lambda \cdot \frac{dP_L}{dP_i} = \lambda}$ $\rightarrow$ co-ordination equation.

To solve the equation, There are 2 methods.

(1). Optimal load flow or Penalty factor method

(2) Loss formula using $\beta$- Coefficient.

<u>Penalty Factor Method.</u>

$$\mathcal{L}_i = \frac{1}{1 - \frac{dP_L}{dP_i}} \simeq \left(1 + \frac{dP_L}{dP_i}\right)$$

we have $\frac{dF_i}{dP_i} + \lambda \frac{dP_L}{dP_i} = \lambda$

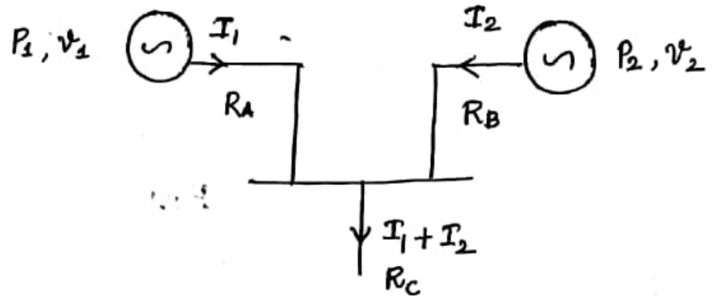
$$\frac{dF_i}{dP_i} = \lambda\left(1 - \frac{dP_L}{dP_i}\right)$$

$$\frac{dF_i}{dP_i} \times \left[\frac{1}{\left(1 - \frac{dP_L}{dP_i}\right)}\right] = \lambda \quad \text{or} \quad \boxed{\frac{dF_i}{dP_i} \cdot \mathcal{L}_i = \lambda}$$

where

## Loss formula using B coefficients.

Consider the single line diagrams of a two generating s/m.




Let $P_1, V_1, I_1, R_A, P_2, V_2, I_2, R_B, R_C$, be the powers, voltage, current and Resistances of generator 1 and 2 respectively. From the single line diagram, the total transmission loss: $P_L$ is:

$$P_L = 3\left\{ I_1^2 R_A + I_2^2 \cdot R_B + (I_1 + I_2)^2 \cdot R_C \right\} \longrightarrow (1)$$

Since this is 3 phase s/m;

$$P_1 = \sqrt{3}\, V_1 I_1 \cos\phi_1 \quad ; \quad P_2 = \sqrt{3}\, V_2 I_2 \cos\phi_2$$

$$\therefore I_1 = \frac{P_1}{\sqrt{3}\, V_1 \cos\phi_1} \quad ; \quad I_2 = \frac{P_2}{\sqrt{3}\, V_2 \cos\phi_2}$$

On substituting the values of $I_1$ and $I_2$ in eqn (1).

$$P_L = 3\left\{ \frac{P_1^2 \cdot R_A}{3 V_1^2 \cos^2\phi_1} + \frac{P_2^2 \cdot R_B}{3 V_2^2 \cos^2\phi_2} + \frac{P_1^2 \cdot R_C}{3 V_1^2 \cos^2\phi_1} + \frac{P_2^2 \cdot R_C}{3 V_2^2 \cos^2\phi_2} + \frac{2 P_1 P_2 R_C}{3 V_1 V_2 \cos\phi_1 \cos\phi_2} \right\}$$

$$\Rightarrow P_L = (R_A + R_C) \cdot \frac{P_1^2}{V_1^2 \cos^2\phi_1} + (R_B + R_C) \cdot \frac{P_2^2}{V_2^2 \cos^2\phi_2} + \frac{2\, P_1 \cdot P_2 \cdot R_C}{V_1 \cdot V_2 \cdot \cos\phi_1 \cdot \cos\phi_2}.$$

on re arranging,

$$P_L = P_1 \cdot \underbrace{\left[\frac{R_A + R_C}{V_1^2 \cos^2\phi_1}\right]}_{B_{11}} P_1 + P_2 \underbrace{\left[\frac{R_B + R_C}{V_2^2 \cos^2\phi_2}\right]}_{B_{22}} P_2 + P_1 \underbrace{\left[\frac{R_C}{V_1 V_2 \cos\phi_1 \cos\phi_2}\right]}_{B_{12}} P_2$$

$$+ P_2 \underbrace{\left[\frac{R_C}{V_1 V_2 \cos\phi_1 \cos\phi_2}\right]}_{B_{21}} P_1 .$$

$\therefore$ $P_L = P_1 B_{11} P_1 + P_2 B_{22} P_2 + P_1 B_{12} P_2 + P_2 B_{21} P_1$

where $B_{11}$, $B_{22}$, $B_{12}$ and $B_{21}$ are $\beta$ coefficients

$\therefore$ In general; $P_L = \sum_m \sum_n P_m B_{mn} P_n$ ; which is the loss formula.

$\therefore$ For a two generator s/m.

$P_L = \sum_m \sum_n P_m B_{mn} P_n$ : where m and n varies from 1 to 2.

$\therefore$ $P_L = P_1 B_{11} P_1 + P_1 B_{12} \cdot P_2 + P_2 B_{22} P_2 + P_2 B_{21} P_1$

to find $\frac{dP_L}{dP_i}$ ; if $i = 1$.

$$\frac{dP_L}{dP_1} = 2 P_1 B_{11} + B_{12} \cdot P_2 + B_{21} \cdot P_2 \qquad \{ \text{Since } B_{12} = B_{21} \}$$

$$= 2 P_1 B_{11} + 2 P_2 \cdot B_{12}$$

$$= 2 ( P_1 B_{11} + P_2 B_{12} )$$

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$\therefore$ In general,

$$\frac{dP_L}{dP_i} = 2 \sum_m B_{im} \cdot P_m .$$

we know the co-ordinate equation is $\frac{dF_i}{dP_i} + \lambda \cdot \frac{dP_L}{dP_i} = \lambda$

ie $$\frac{dF_i}{dP_i} + \lambda \left\{ 2 \sum_m B_{im} \cdot P_m \right\} = \lambda$$

Since $\frac{dF_i}{dP_i} = a_i P_i + b_i$

$$a_i P_i + b_i + \lambda \left\{ 2 \sum_m B_{im} \cdot P_m \right\} = \lambda$$

$$a_i P_i + b_i + \lambda\, 2 B_{ii} P_i + \lambda \left\{ 2 \sum_{m \neq i} B_{im} \cdot P_m \right\} = \lambda$$

$$P_i (a_i + 2\lambda B_{ii}) = \lambda - b_i - \lambda \left\{ 2 \sum_{m \neq i} B_{im} P_m \right\}$$

$$P_i = \frac{\lambda - b_i - \lambda \left\{ 2 \sum_{m \neq i} B_{im} \cdot P_m \right\}}{a_i + 2 \lambda B_{ii}}$$

$$\boxed{P_i = \frac{1 - \frac{b_i}{\lambda} - 2 \sum_{m \neq i} B_{im} \cdot P_m}{\frac{a_i}{\lambda} + 2 B_{ii}}}$$

The formula for power loss using $\beta$ coefficients is derived using the following assumptions.

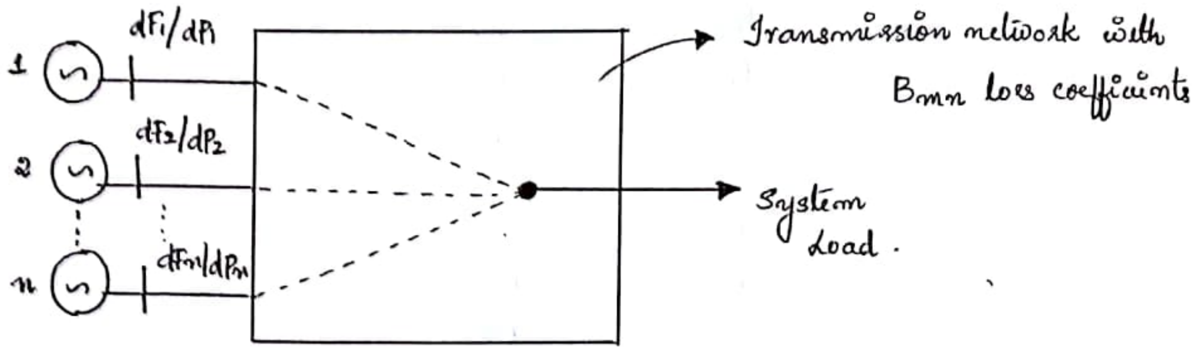
(1). The equivalent load current at any bus is a constant complex fraction of the total equivalent current.

(2). Generator bus voltage magnitude and angles are constant.

(3). Power factor of each source is constant.

## Physical Interpretation of Co-ordination Equation.

The physical interpretation of the co-ordination equation can be understood with the help of figure shown below.




Let there are 'n' number of plants connected to a hypothetical load through a transmission network. The incremental cost of production at the $n^{th}$ bus bar is $dF_n/dP_n$. Let there is an increase in power demand (load) $\Delta P_D$. Let this increase in demand is met by plant 'n' alone.

$\therefore$ Let $\Delta P_n$ be the increase in power generation at plant 'n' to meet the increase in demand $\Delta P_D$ and increase in transmission loss $\Delta P_L$.

$$\therefore \quad \Delta P_n = \Delta P_D + \Delta P_L.$$

Since the incremental cost of power at plant $n = dF_n/dP_n$ Rs/MW hr,

Cost of power at the plant bus for an additional generation of $\Delta P_n$ is

$$\frac{dF_n}{dP_n} \cdot \Delta P_n \quad \text{Rs/hr}.$$

Since the power at the receiving end is only $\Delta P_D$ (Since loss present, $\Delta P_n \neq \Delta P_D$), the cost of received power is,

$$\lambda = \frac{dF_n}{dP_n} \cdot \frac{\Delta P_n}{\Delta P_D}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{\Delta P_n}{\Delta P_n - \Delta P_L} \qquad \left\{ \text{since } \Delta P_n = \Delta P_D + \Delta P_L \right\}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{1}{1 - \frac{\Delta P_L}{\Delta P_n}}$$

$$= \frac{dF_n}{dP_n} \cdot \frac{1}{1 - \frac{dP_L}{dP_n}}$$

$$\therefore \quad \lambda = \frac{dF_n}{dP_n} \cdot L_n \quad ; \text{ where } L_n = \frac{1}{1 - \frac{dP_L}{dP_n}}.$$

which is same as the co-ordination equation and from this treatment, penalty factor for plant n can be defined as the ratio of small change in power at plant 'n' to the small change in received power when generation at plant 'n' alone is changed to meet the load.

## Exact Transmission Loss Formula.

Here a formula for calculating transmission losses ($P_L$) by making use of bus powers and system parameters.

Let $S_i$ be the total power at bus 'i', which is equal to generated power at bus 'i' minus load power at bus 'i'. This means the net power at bus 'i' corresponds to losses. The summation of all such powers at all buses gives the total losses in the system.

ie,

$$P_L + jQ_L = \sum_{i=1}^{n} S_i$$

$$= \sum_{i=1}^{n} V_i I_i^*$$

{ where $P_L$ = active component of loss.

$Q_L$ = Reactive comp of loss.

$S_i$ = Complex power at $i^{th}$ bus

$= V_i . I_i^*$.

$$V_{bus} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad I_{bus} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \quad I_{bus}^* = \begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_n^* \end{bmatrix}$$

$$\sum_{i=1}^{n} V_i I_i^* = V_1 . I_1^* + V_2 I_2^* + \ldots . V_n . I_n^* .$$

$$= [V_{bus}]^T [I_{bus}^*]$$

$$\therefore \quad P_L + jQ_L = V_{bus}^T I_{bus}^*$$

$$= [Z_{bus} . I_{bus}]^T I_{bus}^*$$

{ $V = Z . I$

$\therefore V_{bus} = Z_{bus} . I_{bus}$.

$$= I_{bus}^T . Z_{bus}^T I_{bus}^*$$

$$P_L + jQ_L = I_{bus}^T . Z_{bus} I_{bus}^* \longrightarrow eq(1)$$

{ $Z = R + jX$

$\therefore Z_{bus} = R_{bus} + jX_{bus}$

$$= \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ \vdots & & & \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}$$

Pn

{ Since $Z_{bus}$ is symmetrical, $Z_{bus}^T = Z_{bus}$ }

The bus current vector $I_{bus}$, can also be written as the sum of a real and reactive component of current vector. ie $I = I_p + jI_q$

$$\therefore \quad I_{bus} = I_{bus\,p} + j\, I_{q\,bus}$$

$$= I_p + jI_q$$

$$\therefore \quad I_{bus} = \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

$\therefore$ On combining all this, in eqⁿ (1)

$$P_L + jQ_L = [I_p + jI_q]^T [R + jX][I_p - jI_q]$$

On expanding and separating real and reactive parts.

we get $P_L = I_p^T R I_p - I_q^T X I_p + I_p^T X I_q + I_q^T R . I_q$

mirror matrix

Since X is symmetric matrix $I_q^T X I_p = I_p^T X I_q$

$$\therefore P_L = I_p^T . R I_p + I_q^T . R . I_q$$

$$= \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ \vdots & & & \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}^T \begin{bmatrix} R \end{bmatrix} \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

$$\therefore P_L = \sum_{\substack{i=1 \\ j=1}}^{n} I_{pi} . R_{ij} . I_{pj} + I_{qi} R_{ij} . I_{qj}$$

dP

$$P_L = \sum_{i=1}^{n} R_{ij} ( I_{pi} . I_{pj} + I_{qi} . I_{qj} )$$

Transmission losses has been expressed in terms of bus current. In actual plant, the system operators usually know bus powers and voltages. Hence it is more practical to express $P_L$ in terms of power and voltage.

$$P_L = \sum_{\substack{i=1 \\ j=1}}^{n} \alpha_{ij} ( P_i P_j + Q_i Q_j ) + \beta_{ij} ( Q_i P_j - P_i Q_j )$$

→ Exact transmission loss formula

where $\alpha_{ij} = \frac{r_{ij}}{|v_i||v_j|} \cos(\delta_i - \delta_j)$

$$\beta_{ij} = \frac{r_{ij}}{|v_i||v_j|} \sin(\delta_i - \delta_j).$$

In this cases, even though the formulation for transmission loss is exact, the method requires the calculation of bus impedance matrix which is time consuming and needs more computer memory.

## Modified Coordination Equation.

These equations are derived as follows.

As explained before, the transmission loss is the algebraic sum of all powers in all buses.

ie $$P_L + jQ_L = \sum_{i=1}^{n} S_i$$

$$= \sum_{i=1}^{n} P_i + jQ_i$$

On separating real part,

$$P_L = \sum_{i=1}^{n} P_i$$

$P_L$ is the function of $P_1, P_2 \ldots\ldots$ upto $P_n$

if 'f' is a function of $x_1, x_2 \ldots\ldots x_n$

$$df = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} \cdot dx_2 + \ldots\ldots + \frac{\partial f}{\partial x_n} \cdot dx_n$$

$\therefore$ $\text{lly}$ $P_L$ is a function of $P_1, P_2, \ldots\ldots P_n$

$$\therefore \quad dP_L = \frac{\partial P_L}{\partial P_1} \cdot dP_1 + \frac{\partial P_L}{\partial P_2} \cdot dP_2 + \ldots\ldots + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

$$\therefore \quad dP_L = \sum_{i=1}^{n} \frac{\partial P_L}{\partial P_i} \cdot dP_i$$

Let in the interconnected system, bus powers of only two plants $j$ and $n$ be changed by small amounts, keeping the powers at all other buses fixed, then.

$P_L$ is function of '$j$' and '$n$'

$$\therefore \quad dP_{L\,j,n} = \frac{\partial P_L}{\partial P_j} dP_j + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

$\therefore$ change in $P_L$ = change in power at $j$ + change in power at $n$

$$\therefore \quad dP_{L\,j,n} = dP_{L\,j} + dP_{L\,n}$$

$$\therefore \quad dP_j + dP_n = \frac{\partial P_L}{\partial P_j} \cdot dP_j + \frac{\partial P_L}{\partial P_n} \cdot dP_n$$

ie

$$dP_j \left[1 - \frac{\partial P_L}{\partial P_j}\right] + dP_n \left[1 - \frac{\partial P_L}{\partial P_n}\right] = 0$$

$$\frac{dP_j}{dP_n} = - \frac{1 - \partial P_L / \partial P_n}{1 - \partial P_L / \partial P_j} \longrightarrow \text{Eq (1)}$$

Since the co-ordination equation is $\frac{dF_i^{\circ}}{dP_i^{\circ}} \cdot \frac{1}{1-\frac{dP_L}{dP_i^{\circ}}} = \lambda$

$\therefore$

$$\frac{dF_j^{\circ}}{dP_j^{\circ}} \cdot \frac{1}{1-\frac{dP_L}{dP_j^{\circ}}} = \lambda$$

$$\frac{dF_n}{dP_n} \cdot \frac{1}{1-\frac{dP_L}{dP_n}} = \lambda$$

$$\therefore \quad \frac{\frac{dF_n}{dP_n}}{\frac{dF_j^{\circ}}{dP_j^{\circ}}} = \frac{1-\frac{dP_L}{dP_n}}{1-\frac{dP_L}{dP_j^{\circ}}}$$

Substitue from eq (1)

$$\frac{dF_n/dP_n}{dF_j^{\circ}/dP_j^{\circ}} = -\frac{dP_j^{\circ}}{dP_n}$$

$$= -\frac{dP_j^{\circ}}{dP_{L\,j,n}^{\circ} - dP_j^{\circ}}$$

Since we have $dP_{L\,j,n}^{\circ} = dP_j^{\circ} + dP_n$

$$= -\frac{1}{\frac{dP_{L\,jn}}{dP_j^{\circ}} - 1}$$

$$\frac{dF_n/dP_n}{dF_j^{\circ}/dP_j^{\circ}} = \frac{1}{1-\frac{dP_{L\,j,n}}{dP_j^{\circ}}}$$

Modified co-ordination equation →

$$\boxed{\frac{dF_n}{dP_n} = \frac{1}{1-\frac{dP_{L\,j,n}}{dP_j^{\circ}}} \cdot \frac{dF_j^{\circ}}{dP_j^{\circ}}}$$

Here plant 'n' is taken as the reference plant.

From the above expression, it is clear that for economic load dispatch, the condition required is that the incremental cost of power at plant bus $n$ is equal to the incremental cost of power at plant bus $j$ corrected for the effect of the incremental transmission loss involved. These equations are known as modified co-ordination eq$^n$. The modified co-ordination equation can be rewritten as

$$\frac{dF_n}{dP_n} = -\frac{dF_j}{dP_j} \cdot \frac{\Delta P_j}{\Delta P_n} = \mu .$$