Reference:
Modern PSA - Kothari
PSA : Nagoorkani .

# Module V

## Power System stability.

The stability of an interconnected power system is its ability to return to its normal or stable operation, after having been subjected to some form of disturbances.

Power system stability problems are classified into three basic types - steady state, dynamic and transient.

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually.

Dynamic stability is more probable than steady state stability. Small disturbances are continually occuring in a power system, which are small enough not to cause the system to lose synchronism, but do excite the system into the state of natural oscillations. The system is said to be dynamically stable, if the oscillations do not acquire more than certain amplitude and die out quickly.

In a dynamically unstable system, the oscillation amplitude is large and these persists for a long time. (ie the system is underdamped). Dynamic stability can be significantly improved through the use of power system stabilizers.

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes, whose magnitudes are depended upon the severity of disturbance. For a large disturbance, changes in angular differences may be so large as to cause the machine to fall out of step. This type of Instability is called transient Instability and is a fast phenomenon usually occuring within 1 s for a generator close to the cause of disturbance.

Whether the system is stable on occurance of a fault depends not only on the system itself, but also on the type of fault, location of fault, rapidity of clearing and method of clearing. The transient stability limit is almost always lower than the steady state limit, but unlikely it may exhibit different values depending on the nature, location and magnitude of disturbance.

Modern power systems have many interconnected generating stations, each with several generators and many loads. The machines located at any one point in a system normally act in unison. It is therefore a common practice in stability analysis to conside all the machines in one point as one large machine. Also machines which are not separated by lines of high reactance are lumped together and considered as one equivalent machine.

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## Dynamics of Synchronous Machines.

Let ;

$J$ = rotor moment of Inertia in $kg.m^2$

$\omega_{sm}$ = angular velocity of rotor in (mech) rad/s.

$\therefore$ Kinetic Energy of the rotor at synchronous machine is

$$K.E = \frac{1}{2} J \omega_{sm}^2 \text{ Joule}$$

$$K.E = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \text{ MJ}$$

If $\omega_s$ = velocity of rotor in (elect) rad/s

$P$ = no: of poles of machine;

$$\therefore \quad \omega_s = \left(\frac{P}{2}\right) . \omega_{sm}$$

$$\therefore \quad KE = \frac{1}{2} . J . \left(\frac{2}{P}\right)^2 . \omega_s^2 \times 10^{-6} \text{ MJ}$$

$$= \frac{1}{2} \underbrace{\left[J . \left(\frac{2}{P}\right)^2 . \omega_s \times 10^{-6}\right]}_{M} \omega_s \text{ MJ}.$$

where $M$ = Moment of Inertia in MJ-s/elect rad.

$$\therefore \quad KE = \frac{1}{2} M \omega_s . \text{ MJ}.$$

Define Inertia Constant $H = \dfrac{\text{stored K.E in MJ at synchr speed}}{\text{Machine rating in MVA}}$.

$$H = \frac{\frac{1}{2} M \omega_s}{G}$$

$G \rightarrow$ Machine base MVA.

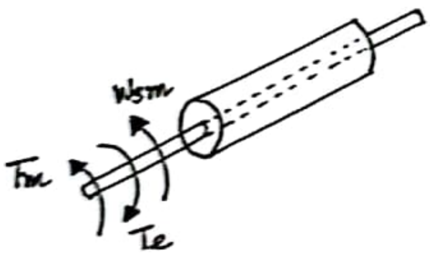
$$M = \frac{2GH}{\omega_s}$$ $f \rightarrow$ frequency in Hz.

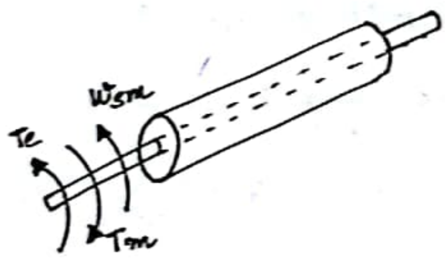
$$= \frac{2GH}{2\pi f} \quad MJ\text{-}s/\text{elect rad.}$$

$$= \frac{GH}{180f} \quad MJ\text{-}s/\text{elect degree}$$

$$\boxed{\frac{M}{G} = \frac{H}{180f} \quad \text{ie} \quad M_{p.u} = \frac{H}{180f}}$$

## Swing Equation.




The rotor of a synchronous machine is subjected to two torques, Te and Tm, which are acting in opposite direction.

where $T_e$ = Net electrical torque in N-m

$T_m$ = Mechanical or shaft torque in N-m.

Under steady state condition, $T_e = T_m$ and the machine runs at constant speed called synchronous speed. If there is any difference between the two, then rotor will have an accelerating or deaccelerating torque, denoted as $T_a$.

$$\therefore \quad T_a = T_m - T_e$$

Let $\theta_m$ = angular displacement of rotor w.r.t stationary reference frame.

$\delta_m$ = angular displacement of rotor w.r.t synchronously rotating reference axis.

By Newton's Second Law; $F = m.a$

$$\therefore \quad T_a = J.\frac{d^2\theta_m}{dt^2}$$

ie $$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad \{ \text{Since } T_a = T_m - T_e \}$$

Angular displacement $\theta_m$ and $\delta_m$ are related by the expression,

$$\theta_m = \omega_{sm}.t + \delta_m$$

$$\therefore \quad \frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$\therefore \quad J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Let $P_m$ = mechanical power (shaft power) neglecting losses (MW)

$P_e$ = Electrical power developed in rotor (MW)

$$P = \frac{2\pi NT}{60} \qquad \therefore \quad P_m = \omega_{sm}.T_m$$

$$= \omega T \qquad\qquad P_e = \omega_{sm}.T_e$$

$$\therefore \quad J \cdot \frac{d^2\delta_m}{dt^2} = \frac{P_m}{\omega_{sm}} - \frac{P_e}{\omega_{sm}}$$

$$\therefore \quad J \cdot \omega_{sm} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e \longrightarrow (1)$$

Now; the Inertia constant $H = \dfrac{\text{stored K.E}}{\text{Base MVA } (G)}$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{G}$$

$$\therefore \quad J\omega_{sm} = \frac{2GH}{\omega_{sm}} \quad \therefore \quad \frac{2GH}{\omega_{sm}} \cdot \frac{d^2\delta_m}{dt^2} = P_m - P_e$$

$$\left\{ \delta_m = 2/p \cdot \delta \quad \omega_{sm} = 2/p\ \omega_s \right\}$$

$$\therefore \quad \frac{2GH}{2\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\therefore \quad \frac{GH}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\therefore \quad \boxed{\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_{m\,(p.u)} - P_{e\,(p.u)}}$$

This Equation is called swing equation, which governs the dynamics of synchronous machine rotor. It is a non-linear second order differential equation.

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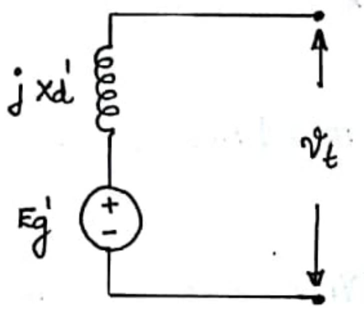
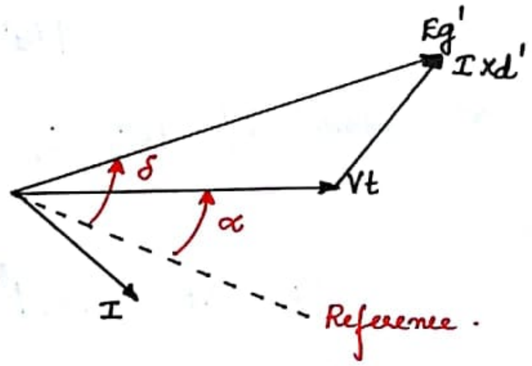
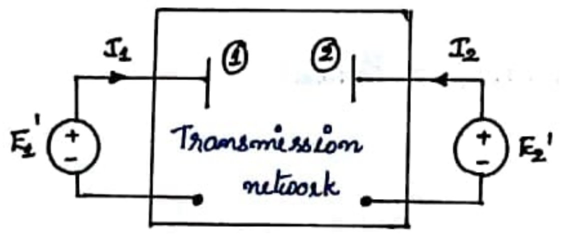
## Power Angle Equation.

The equation relating the electrical power generated in a synchronous machine to the angular displacement of the rotor ($\delta$) is called power angle equation. Power angle equation can be derived using the transient model of the generator.




Let a single generator supplies power through a transmission s/m to a load or a large system at other end. Such a system can be represented by a 2 bus network as shown below.




$E_1' =$ Transient internal voltage of generator at bus 1

$E_2' =$ Voltage at the receiving end.

We know that $I_{bus} = Y_{bus} \cdot V_{bus}$

for the network shown;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix}$$

$$\therefore \quad I_1 = Y_{11} E_1' + Y_{12} E_2'$$

Complex power at bus 1 $= P_1 + jQ_1$

$$= vI^*.$$

$$\therefore \quad P_1 + jQ_1 = E_1'(I_1)^*$$

$$= E_1'\left[E_1' Y_{11} + Y_{12}.E_2'\right]^*$$

$$= E_1'\left[E_1'^* Y_{11}^* + Y_{12}^* E_2'^*\right]$$

$$= |E_1'|^2 . Y_{11}^* + E_1' E_2'^* Y_{12}^*$$

We know; $E_1' = |E_1'| \angle \delta_1$ ; $E_2' = |E_2'| \angle \delta_2$ ; $Y_{11} = |Y_{11}| \angle \theta_{11}$ ; $Y_{12} = |Y_{12}| \angle \theta_{12}$

$$\therefore \quad P_1 + jQ_1 = |E_1'|^2 Y_{11} \angle -\theta_{11} + |E_1'||E_2| \, |Y_{12}| \angle \delta_1 - \delta_2 - \theta_{12}.$$

On separating real and imaginary parts.

$$P_1 = |E_1'|^2 |Y_{11}| \cos(\theta_{11}) + |E_1'| \, |E_2'| \, |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$P_1 = |E_1'|^2 . G_{11} + |E_1'| \, |E_2'| \, |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

Let $|E_1'|^2 . G_{11} = P_c$ ; $|E_1'| \, |E_2'| \, |Y_{12}| := P_{max}$

$$\delta_1 - \delta_2 = \delta$$

$$\theta_{12} = \pi/2 + \gamma$$

$$\therefore \quad P_1 = P_c + P_{max} \cos(\delta - \pi/2 + \gamma)$$

$$P_1 = P_c + P_{max} \sin(\delta - \gamma)$$

ie $$\boxed{P_e = P_c + P_{max} \sin(\delta - \gamma)}$$

Power angle equation

where

$P_c$ = power loss in s/m

$P_{max}$ = max real power that can be transfered

For a purely reactive network $P_c = 0$ since $G_{11} = 0$ ; $\theta_{12} = 90^\circ$
$\therefore \gamma = 0^\circ$

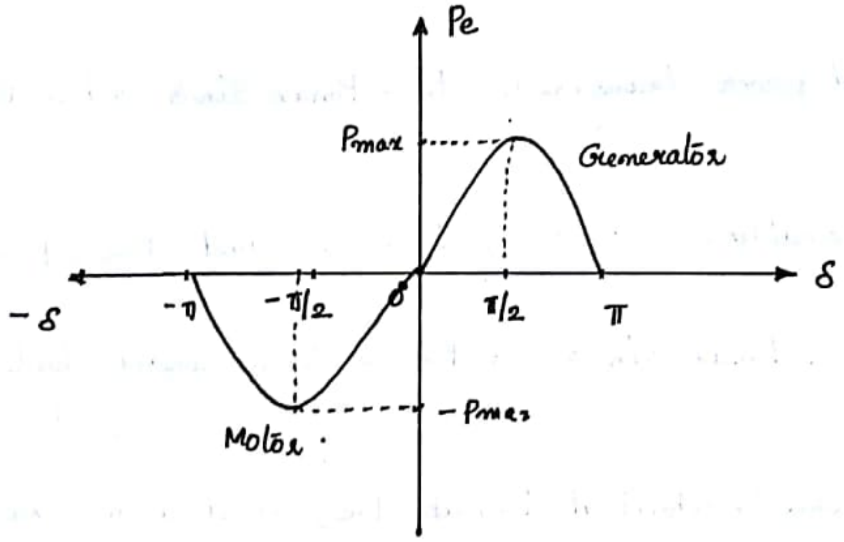
$$\therefore \quad P_e = P_{max} \sin \delta \longrightarrow (1)$$

where $P_{max} = |E_1'||E_2'|.|Y_{12}|$

$$= \frac{|E_1'||E_2'|}{X_{12}}$$

This Equation (1) is called simplified power angle equation.

The plot or graph of Pe as a function of '$\delta$' is called power angle curve




We have Swing Equation $\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$ ; On substituting Pe from power angle equation; we get

$$\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_{max} \sin \delta$$

## Steady state stability.

In steady state, every synchronous machine has a limit for power transfer to a receiving system. Steady state limit of a machine or a transmitting system is the maximum power that can be transmitted to a receiving system, without loss in synchronism.

Let $|E|$ = magnitude of steady state internal Emf of the machine

$|V|$ = magnitude of voltage at receiving end.

$X$ = Transfer reactance between synchronous machine and receiving system.

Then the real power transfered $P_e = P_{max} \sin \delta$ where $P_{max} = \frac{|E||V|}{X}$

Let;

Under ideal condition $\delta = \delta_0$ ; $P_e = P_{e0}$ and $P_m = P_{e0}$.

$$\therefore \quad P_{e_0} = P_{max} \sin \delta_0 = P_m \left\{ \text{Since under stable cond}^n \; P_m = P_e \right\}$$

Let with the same mechanical input $P_m$, load angle changed to $\Delta\delta$

$\therefore$ Electrical power generated changes by $\Delta P$.

$$\therefore \quad P_e = P_{e_0} + \Delta P$$

now $P_e = P_{max} \sin(\delta_0 + \Delta\delta)$

$$= P_{max} \left[ \sin \delta_0 \cos \Delta\delta + \cos \delta_0 \sin \Delta\delta \right]$$

Since $\Delta\delta$ is very small $\sin \Delta\delta \simeq \Delta\delta$ and $\cos \Delta\delta \simeq 1$

$$\therefore \quad P_e = P_{max} \cdot \sin\delta_0 \cdot 1 + P_{max} \cos \delta_0 \, \Delta\delta$$

i.e $P_{e_0} + \Delta P = \underbrace{P_{max} \sin \delta_0}_{P_{e_0}} . + P_{max} \cos \delta_0 \, \Delta\delta$

$$\therefore \quad \Delta P = P_{max} \cos \delta_0 . \Delta\delta$$

Now; as per the swing equation; $\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$

or

$$M . \frac{d^2\delta}{dt^2} = P_m - P_e$$

for a change in load angle $\Delta\delta$;

$$M \frac{d^2}{dt^2}(\delta_0 + \Delta\delta) = P_m - (P_{e_0} + \Delta P) \qquad \because \text{ Since } P_m = P_{e_0}$$

$$= -\Delta P$$

i.e $M . \frac{d^2}{dt^2}(\delta_0 + \Delta\delta) = - P_{max} \cos \delta_0 . \Delta\delta$ ; $\delta_0 \rightarrow$ constant (since initial value).

$$\therefore \quad M \frac{d^2}{dt^2} . \Delta\delta = -(P_{max} \cos \delta_0) . \Delta\delta$$

$$\underbrace{M \frac{d^2}{dt^2}}_{x^2} \Delta\delta + \underbrace{P_{max} \cos \delta_0}_{C} . \Delta\delta = 0$$

$\therefore \quad (M x^2 + C)\, \Delta\delta = 0$ ; Since $\Delta\delta \neq 0$ $\quad M x^2 + C = 0$

$$x = \sqrt{-C/M}$$

where $x$ = roots of the characteristic equation.

## Case 1: when c is positive (ie $P_{max} \cos \delta_0 > 0$)

when c is +ve; the roots of the equation are purely imaginary. System behaviour is purely oscillatory. In this analysis resistances were neglected. If they are also included, the roots will be complex conjugate and the s/m will be stable.

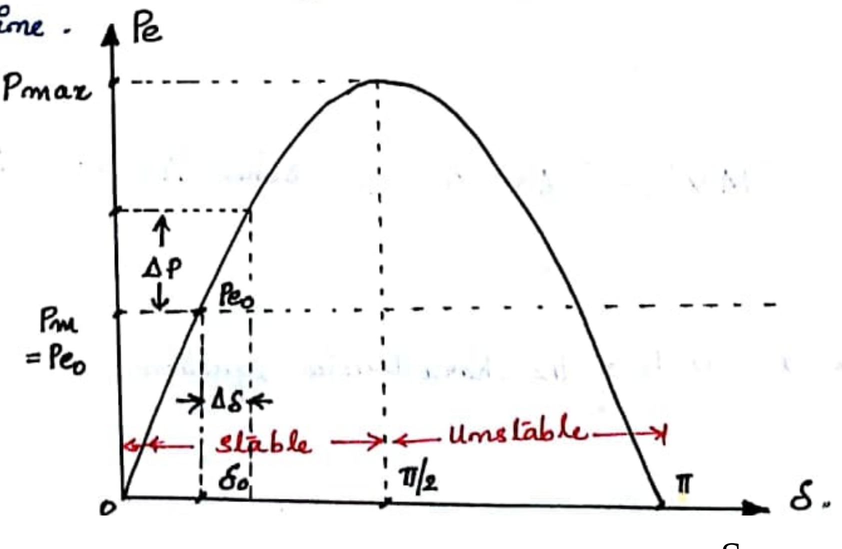
∴ The practical system is stable for small increment in power; provided $P_{max} \cos \delta_0 > 0$.

## Case 2: when c is negative ($P_{max} \cos \delta_0 < 0$)

when c is -ve, the roots of the equation are real and equal in magnitude. Due to the +ve root, the torque angle increases without bound and finally loss synchronism.

## Steady state limit.

The term $P_{max} \cos \delta_0$ denotes the steady state stability of the system. Hence it is called synchronising coefficient or stiffness of synchr. machine.



when $0 < \delta_0 < \pi/2$ $\cos\delta_0 > 0$

ie $P_{max}\cos\delta_0 > 0$ ie; the system is stable.

when $\delta_0 = \pi/2$

$P_{max}\cos\delta_0 = 0$ and $P_e = P_{max}$

when $\pi/2 < \delta_0 < \pi$ $\cos\delta_0 < 0$

$\therefore$ $P_{max}\cos\delta_0 < 0$ ; ie system unstable.

$\therefore$ The machine will operate in stable operating condition for the load angle or torque angle $0 < \delta < \pi/2$. Practically the system has to be operated below the steady state stability limit.

## TRANSIENT STABILITY

The transient stability of a system is concerned with the study of s/m behaviour for large disturbances. The short circuits and switching heavy loads can be treated for this case. The dynamics of the system under transient state is governed by the non-linear swing equation $\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$.

The transient stability of a single machine connected to an infinite bus can be determined using a simple criteria called equal area criteria.

## EQUAL AREA CRITERIA.

The transient stability analysis of a simple system can be performed by using equal area criteria.

During transient state of a power system, there are two situations for change in $\delta$ (torque angle or load angle) with respect to time.

(i) The '$\delta$' may increase to a maximum value and then decrease to a stable value. Then the system is considered as stable.

(ii) The '$\delta$' may keep on increasing indefinitely. In this case, system is unstable.

ie; for s/m to be stable $\frac{d\delta}{dt} = 0$ at some time instant

s/m is unstable if $\frac{d\delta}{dt} > 0$ for long time.

Consider the swing equation of generator connected to infinite bus bar.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

$P_m - P_e = P_a$ Accelerating or decelerating power.

$$\therefore \frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a$$

or

$$M. \frac{d^2\delta}{dt^2} = P_a$$

$\left\{ \text{where } M = \frac{H}{\pi f} = \text{moment of Inertia} \right\}$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

On Multiplying both sides with $2 \cdot \frac{d\delta}{dt}$

$$\therefore 2\frac{d\delta}{dt} \cdot \frac{d^2\delta}{dt^2} = 2 \cdot \frac{d\delta}{dt} \cdot \frac{P_a}{M}$$

$$2 \cdot \frac{d\delta}{dt} \frac{d^2\delta}{dt^2} = \frac{2}{M} \cdot \frac{d\delta}{dt} \cdot P_a$$

$$2 \cdot \frac{d\delta}{dt}\left(\frac{d^2\delta}{dt^2}\right) = \frac{2 \cdot P_a}{M} \frac{d\delta}{dt}$$

On Integrating both sides w.r.t $d\delta$

$$2\int \frac{d^2\delta}{dt^2} \cdot d\delta = \frac{2}{M}\int P_a \cdot d\delta$$

$$= \left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a \cdot d\delta$$

$$\text{or} \quad \frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a \cdot d\delta}$$

for the s/m to be stable $\frac{d\delta}{dt} = 0$ ie $\int_{\delta_0}^{\delta} P_a \cdot d\delta = 0$

$$\text{or} \quad \int_{\delta_0}^{\delta} (P_m - P_e) \cdot d\delta = 0$$

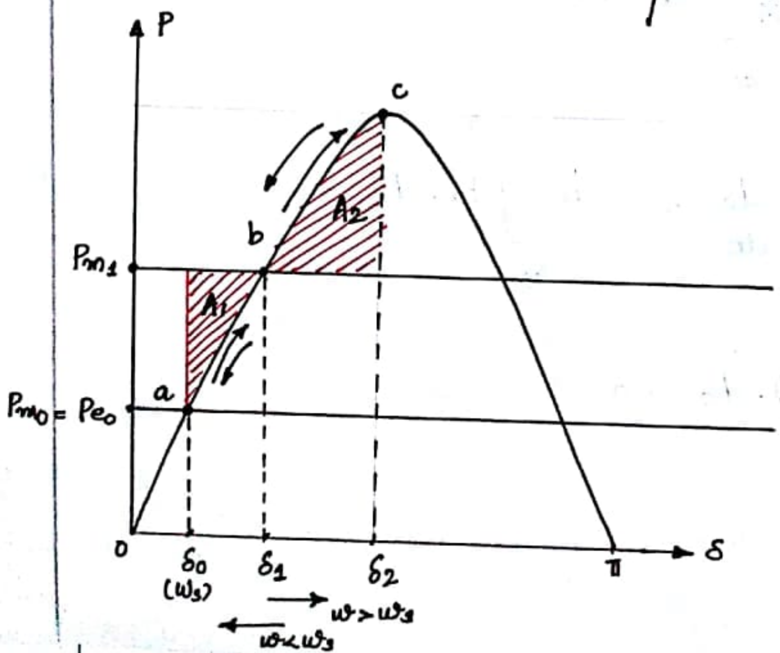
The condition for stability can therefore be stated as: The system is stable if, the area under $P_a - \delta$ curve reduces to zero at some value of $\delta$. In other words, the +ve (accelerating) area under $P_a - \delta$ curve must equal the -ve (decelerating) area under $P_a - \delta$ curve and hence the name equal area criterion of stability.

## Sudden change in Mechanical Input.

Figure shows the transient model of a single machine tied to infinite bus bar. The electrical power transmitted is given by


$$P_e = P_{max} \sin \delta \quad ; \quad P_{max} = \frac{|E'||v|}{x'_d + x_e}$$

Under steady operating condition, $P_{m_0} = P_{e_0} = P_{max} \sin \delta$. This is indicated by point 'a' in the P-δ curve (power angle curve).




Let the mechanical input to the rotor is suddenly increased from $P_{mo}$ to $P_{m1}$. Now the power $P_a = P_{m1} - P_e$ is +ve and causes the rotor to accelerate and so does the rotor angle. As the angle increases from $\delta_o$, the operating region also shifts from point 'a'. At angle $\delta_1$, $P_{m1} = P_e$. ie $P_a = 0$, a stable state is reached, but the rotor angle continue to increase due to the inertia of the machine. Hence the operating region again shifts from 'b' along the P-δ curve. $P_a$ now becomes negative, and the rotor starts decelerating and let at $\delta_2$, at point 'c', the accelerating area $A_1$ equals deaccelrating area $A_2$. Due to deceleration, the rotor speed decreases and so does the rotor angle and the operating region travels back in the P-δ curve and finally settle down at new steady state $\delta_1$, where $P_{m1} = P_e$.

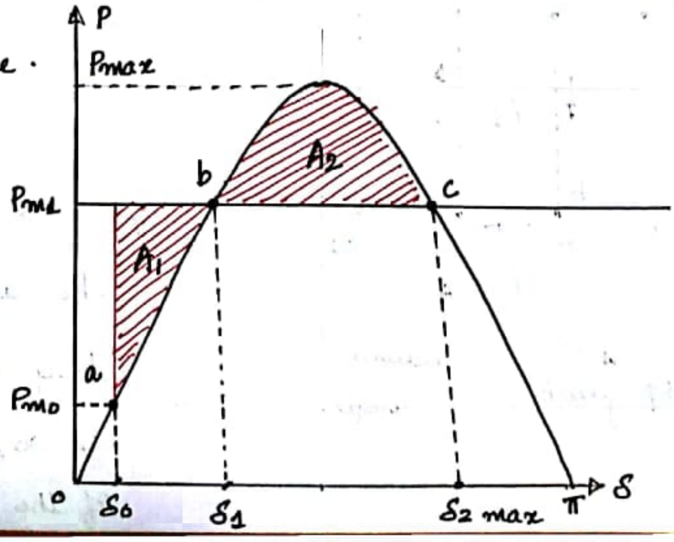
From the figure, $$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e).d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1}).d\delta.$$

For the system to be stable, it should be possible to find the value $\delta_2$, such that $A_1 = A_2$. As $P_m$ is increased, a limiting condition is finally reached, where $A_1$ equals the area $A_2$ as shown. Under this condition, $\delta_2$ acquires the maximum value.

$$\therefore \delta_2 = \delta_{max} = \pi - \delta_1$$

$$= \pi - \sin^{-1}\left(\frac{P_{m1}}{P_{max}}\right)$$

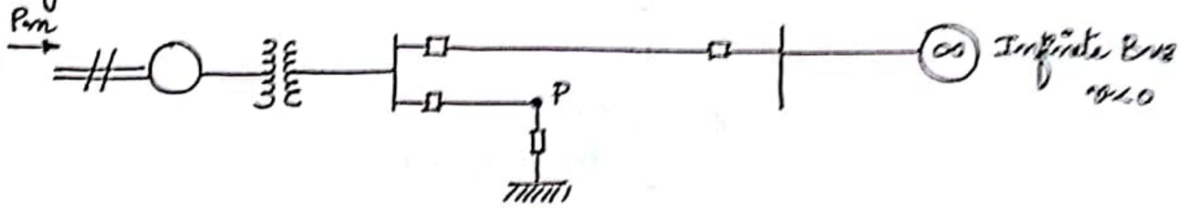


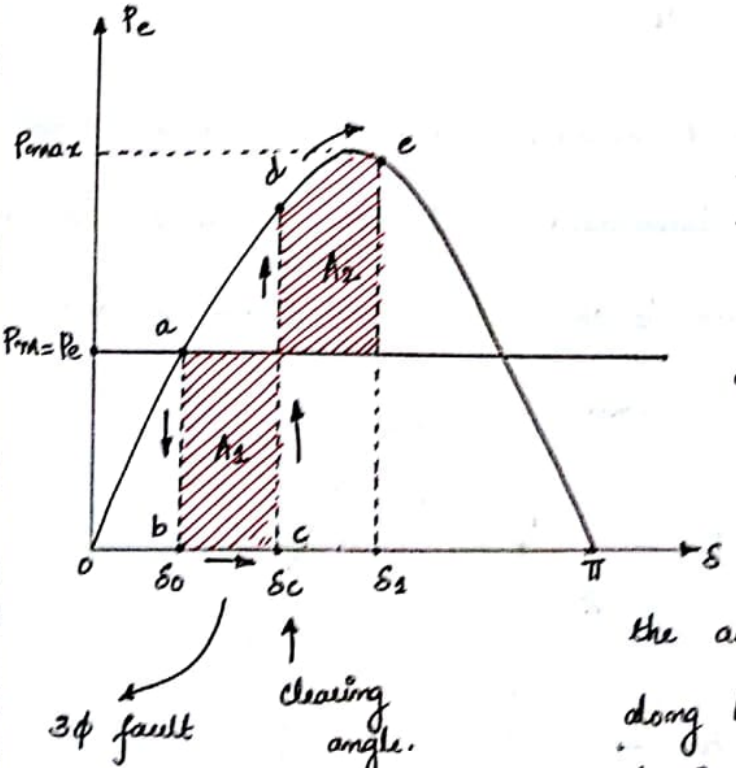
Any further increase in $P_m$, will reduce the area $A_2$ and which violates the equal area criterion of transient stability.

It may be also noted that; even when the rotor angle is increased beyond $\delta = 90^\circ$, the system can be transiently stable, as long as equal area criteria is met. The condition $\delta = 90^\circ$ is meant for steady state stability and does not apply to transient stability case.

## Effect of clearing time on stability.

Let the system shown in figure be operating with mechanical input $P_m$ at a steady angle of $\delta_0$ ($P_m = P_e$) as shown by point 'a' on the $P_e$-$\delta$ diagram.




If a 3$\phi$ fault occurs at the point P of the outgoing radial line, the electrical output of the machine will reduce to zero. ie $P_e = 0$, hence the operating region shifts from 'a' to 'b'. Now $P_a = P_m - P_e$ is +ve and the rotor accelerates and $\delta$ increases. ie, the accelerating area $A_1$ starts increasing along bc. Let at time $t = t_c$ corresponding to $\delta_c$, the fault is cleared, by the opening of the circuit breaker.

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The system once again becomes healthy and transfer power $P_e = P_{max} \sin\delta$. Hence at '$\delta_c$' the operating point shifts from 'c' to 'd' along the P-δ curve. Now $P_a = P_m - P_e$ is negative and the rotor decelerates and the decelerating area $A_2$ starts along 'de'. If at an angle $\delta_1$, $A_1 = A_2$, the system is found to be stable. The system finally settles at the steady operating point 'a' in an oscillatory manner due to inherent damping.

The values of $t_c$ and $\delta_c$ are called clearing time and clearing angle respectively.

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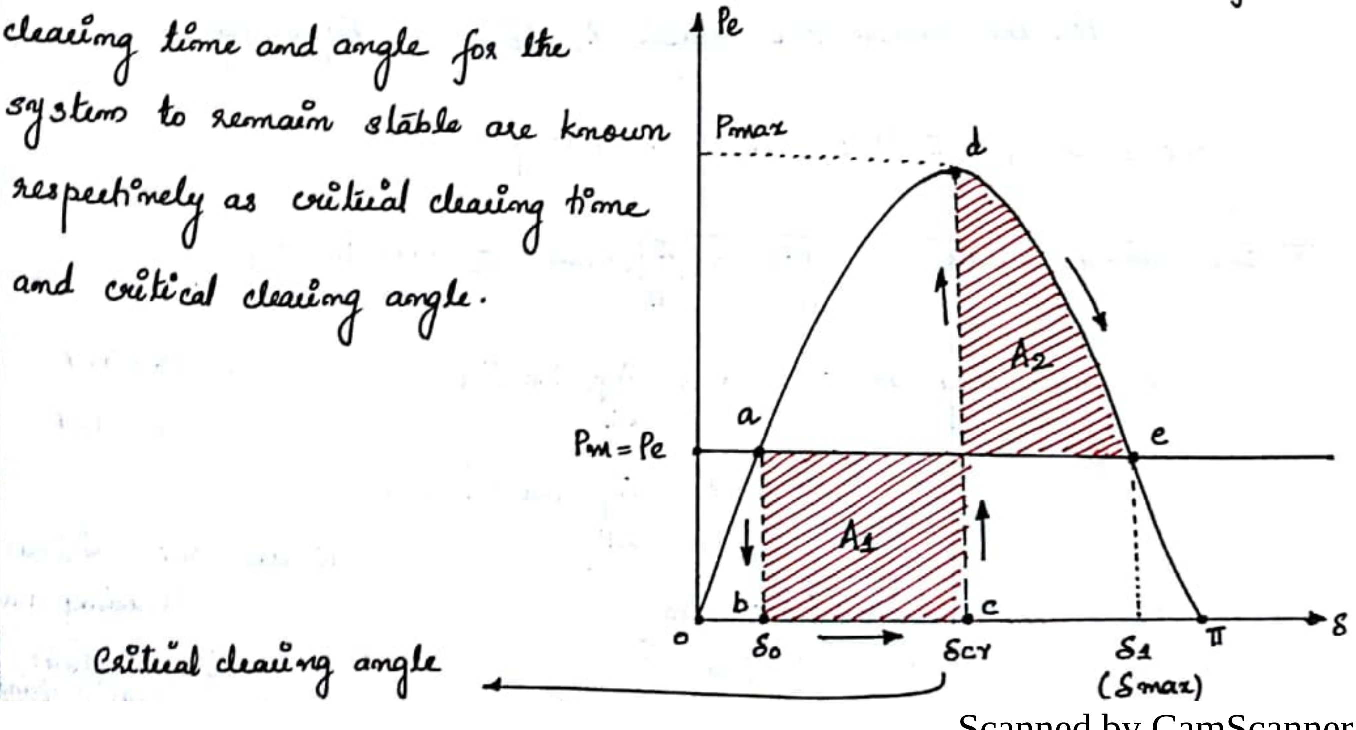
## Critical clearing angle and Critical clearing time.

In the above system mentioned, As the clearing of fault is delayed, the area $A_1$ goes on increases and hence $\delta_1$ also increases. $\delta_1$ can be increased only upto $\delta_{max}$. For the system to be stable, $A_1 = A_2$. Let in the power angle curve shown below, a clearing angle $\delta_{cr}$ is mentioned, beyond which $A_2 < A_1$. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as critical clearing time and critical clearing angle.



For the case mentioned; ie during fault, if $P_e = 0$, the relation of critical clearing angle and time is derived here. All angles are in radians.

(note: For other cases, the equations may be different depending on fault).

Refering to the power angle curve:-

$\delta_{max} = \pi - \delta_0$ ; $P_e = P_{max} \sin\delta$ ; at $\delta_0$; $P_e = P_m$ ; $\therefore P_m = P_{max} \sin\delta_0$

$\therefore \delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max}}\right)$ ie; $\delta_{max} = \pi - \sin^{-1}\left(P_m / P_{max}\right)$.

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_e).\,d\delta \qquad \{\because P_e = 0\}$$

$$= \int_{\delta_0}^{\delta_{cr}} P_m.\,d\delta = P_m\,\delta\Big\}_{\delta_0}^{\delta_{cr}}$$

$$= P_m \{\delta_{cr} - \delta_0\}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_e - P_m).\,d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max} \sin\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m.\,d\delta$$

$$= -P_{max}\cos\delta\Big\}_{\delta_{cr}}^{\delta_{max}} - P_m\,\delta\Big\}_{\delta_{cr}}^{\delta_{max}}$$

$$= -P_{max}\{\cos\delta_{max} - \cos\delta_{cr}\} - P_m\{\delta_{max} - \delta_{cr}\}$$

For the system to be stable $A_1 = A_2$; On Equating two areas,

$$\boxed{\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0).\sin\delta_0 - \cos\delta_0\right]}$$

As per swing equation; $\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}.P_m$ {Since $P_e = 0$}.

On Integrating twice; $\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$ if $\delta = \delta_{cr}$, $t = t_{cr}$

ie $\delta_{cr} = \frac{\pi f}{2H} P_m t_{cr}^2 + \delta_0$

$$\boxed{\text{or} \quad t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}}$$

where $\delta_{cr}$ = critical clearing angle

$t_{cr}$ = critical clearing time.

## Factors affecting Transient stability and Methods for Improving the same.

Transient stability mainly depends on the type and location of the fault.

As $M \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$ {from swing equation}

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

An increase in the moment of inertia M reduces the angle through which rotor swings in a given time interval. Hence stability can be improved by increasing M. But it cannot be practical due to economic reasons. Also increasing M will have an undesirable effect of slowing down the response of speed governor loop.

### Methods of Improving transient stability limit : (May 2016 – 4 marks)

1. Increase of system voltages, use of AVR (Automatic Voltage Regulators)
2. Use of high speed excitation systems.
3. Reduction in system transfer reactance
4. Use of high speed reclosing breakers.

During fault, the reduction in system voltage can be automatically sensed by AVR which helps to restore the generator voltage.

The transfer reactance can be reduced to improve the stability limit. This can be done by (i) reducing conductor spacing or

(ii) by increasing conductor diameter. Compensation for line reactance by series capacitor is an effective and economic method of increasing stability limit specially for transmission distance of more than 350 km. Transfer reactance can also be reduced by increasing the number of parallel line between transmission points. Rapid switching and isolation of unhealthy lines followed by reclosing also improves stability margins.

Recent methods of Improving stability are:-

HVDC Links: Increased use of HVDC links employing thyristors would deviate stability problem. There is no risk of a fault in one system causing loss of stability in the other system.

Breaking Resistors: For improving stability where clearing is delayed or large load is suddenly lost, a resistive load called a breaking resistor is connected at or near the generator bus. This load compensates for some of the reduction of load on generators.

Bypass Valving: In this method, the stability of a unit is improved by decreasing the mechanical input power to the turbine.

Full load Rejection Technique: Fast valving combined with high-speed clearing time will be sufficient to maintain stability in most cases. A full load rejection technique could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large steam bypass system. The main disadvantage is the extra cost of large bypass system.

Multimachine stability : Refer Modern PSA : Kothari.
Chapter Power system stability : Pg no: 455

## TUTORIALS

1. A 50 Hz, four pole turbogenerator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA.

(a) Find the stored energy in the rotor at synchronous speed.

(b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration, neglecting mechanical and electrical losses.

(c) If the acceleration calculated in part b is maintained for 10 cycles, find change in torque angle and rotor speed in revolutions/minute at the end of this period.

(MAY 2015 - 7 marks).

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Soln:

given,

$f = 50\,Hz$, $P = 4$, $H = 8\ MJ/MVA$, $G = 100\ MVA$

(a) Stored Energy = K.E of rotor

$\therefore$ Stored Energy $= H.G$

$= 8 \times 100$

$= \underline{\underline{800\ MJ}}$

$\left\{ \text{we have } H = \dfrac{\text{Stored Energy (KE)}}{\text{Rated MVA}} \right.$

(b) $P_m = 80\,MW$ $P_e = 50\,MW$, $\dfrac{d^2\delta}{dt^2} = ?$ (Rotor acceleration)

we have

$$M.\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{(from swing equation)}$$

$$M(p.u) = \frac{H}{\pi f} = \frac{H}{180 f} \quad ; \quad M = \frac{GH}{180 f} \quad \left\{ \begin{array}{l} \text{where } M \text{ is not in p.u} \\ f \text{ in Hz} \end{array} \right\}.$$

$$\therefore \quad M = \frac{100 \times 8}{180 \times 50} = 0.0888 \text{ MJ-s/elect degree}$$

$$\therefore \quad \frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

$$= \frac{80 - 50}{0.0888} = \underline{\underline{337.83}} \text{ elect degree/s}^2$$

(c) $10 \text{ cycles} = 0.2\,s \quad \{ \text{ since } f = 50\,Hz \text{ ; for one cycle} \rightarrow 20\,ms$

we have $\frac{d^2\delta}{dt^2} = 337.83$.

$$d^2\delta = 337.83 \times (dt)^2$$

Change in torque angle $\left.\right\}$ $\therefore \; d\delta = \frac{1}{2} \cdot 337.83 \times (0.2)^2$

$$= \underline{\underline{6.75 \text{ elect degrees}}}.$$

2. A 50 Hz, 4 pole turbogenerator rated 40 MVA, 11 kV has an inertia constant of 15 kW-s per kVA. Determine the KE stored in the rotor at synchronous speed. Determine the acceleration and accelerating torque, if the shaft input less the rotational losses is 20 MW and electrical power developed is 15 MW. (MAY 2016 - 8 marks)

soln:

given,

$f = 50\ Hz,\ P = 4,\ G = 40\ MVA,\ H = 15\ MJ/MVA$

KE stored in the rotor $= G \cdot H$

$= 40 \times 15$

$= \underline{\underline{600\ MJ}}$

To find acceleration ; $\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$

$P_m = 20\ MW,\ P_e = 15\ MW$

$M = \frac{GH}{180 \cdot f} = \frac{40 \times 15}{180 \times 50} = 0.0666$ MJs/elect degree.

$\therefore \frac{d^2\delta}{dt^2} = \frac{20 - 15}{0.0666} = \underline{\underline{75.07 \text{ elect degree}/s^2}}$

Accelerating torque $= \frac{\text{Accelerating Power}}{\omega_{sm}}$

$= \frac{P_m - P_e}{2\pi f \cdot (2/P)} = \frac{20-15}{2\pi \cdot 50 \cdot 2/4}$

$= \underline{\underline{0.55\ N \cdot m}}$

3. A generator operating at 50 Hz delivers 1 p.u power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the max power transferable to 0.5 p.u whereas before the fault, this power was 2.0 pu and after the clearance of the fault, it is 1.5 p.u. By the use of equal area criteria, determine the critical clearing angle. (November 2015).

Let case I → Pre fault

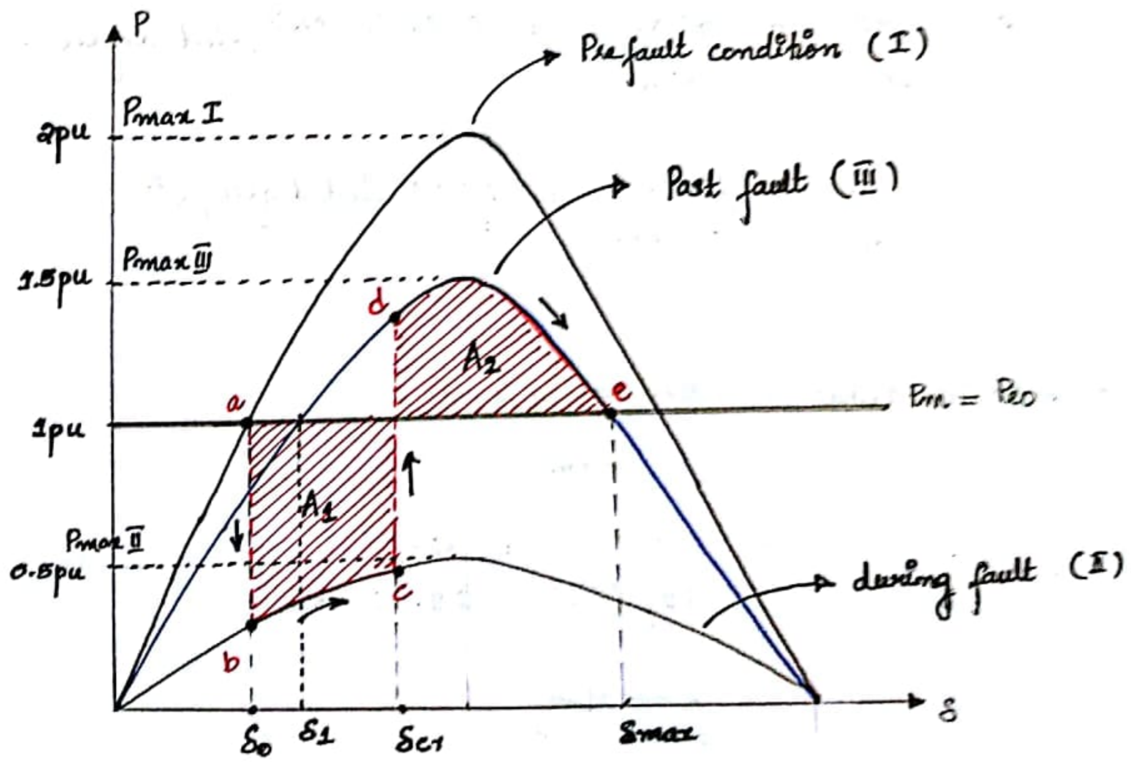
II → During fault

III → Post fault (after fault cleared).

∴ given, $f = 50\ Hz$ ; $P_{e_0} = 1.p.u$ ie $P_m = 1 p.u$

$P_{max\ I} = 2.0\ pu$ , $P_{max\ II} = 0.5\ pu$ and $P_{max\ III} = 1.5\ pu$

The power angle curves for the above conditions can be drawn as;



Let the s/m was operating under stable condition at point 'a'; where $P_m = P_e = 1$ p.u in the I curve. Suddenly a fault occurs and the operating region falls from 'a' to 'b' in the II curve. Let at $\delta_{cr}$, the fault is cleared at point 'c' and operating region shifts to d in curve III.

For the clearing angle to be critical, Area $A_1$ = Area $A_2$.

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e_{\underline{II}}}) \cdot d\delta \qquad A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e_{\underline{III}}} - P_m) \cdot d\delta .$$

Since $P_e = P_{max} \sin \delta$ (general expression)

$P_{e_{\underline{II}}} = P_{max_{\underline{II}}} \sin \delta$ and $P_{e_{\underline{III}}} = P_{max_{\underline{III}}} \sin \delta$.

$\delta_{max} = \pi - \delta_1$ {from the power angle curve}.

$$\therefore \quad P_{max_{\underline{II}}} \sin \delta_1 = 1 \text{ p.u}$$

$$\therefore \quad \delta_1 = \sin^{-1}\left(1/P_{max_{\underline{III}}}\right) = \sin^{-1}\left(1/1.5\right)$$

$$= \underline{\underline{41.8^\circ}} = \underline{\underline{0.729 \text{ rad}}}$$

$$\therefore \quad \delta_{max} = \pi - \delta_1$$

$$= 180 - 41.8 = \underline{\underline{138.18^\circ}} = \underline{\underline{2.411 \text{ rad}}}$$

$P_{max_I} \sin \delta_0 = 1$

$\delta_0 = \sin^{-1}\left(1/P_{max_I}\right)$

$= \sin^{-1}\left(1/2\right)$

$= \underline{\underline{30^\circ}} = \underline{\underline{0.523 \text{ rad}}}$

$$\therefore \text{Area } A_1 = \int_{\delta_o}^{\delta_{cr}} (P_m - P_{e\,II})\, d\delta$$

$$= \int_{\delta_o}^{\delta_{cr}} P_m - \int_{\delta_o}^{\delta_{cr}} P_{max\,II} \sin\delta$$

$$= P_m \cdot \delta \Big\}_{\delta_o}^{\delta_{cr}} + P_{max\,II} \cos\delta \Big\}_{\delta_o}^{\delta_{cr}}$$

$$= \{\delta_{cr} - \delta_o\} + 0.5\{\cos\delta_{cr} - \cos\delta_o\}$$

$$= \{\delta_{cr} - 0.523\} + 0.5\{\cos\delta_{cr} - 0.866\}$$

$$= \delta_{cr} - 0.523 + 0.5\cos\delta_{cr} - 0.433$$

$$= \underline{\underline{\delta_{cr} + 0.5\cos\delta_{cr} - 0.956}}$$

$$\text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max\,III} \sin\delta\, d\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m\, d\delta$$

$$= -1.5\cos\delta \Big\}_{\delta_{cr}}^{\delta_{max}} - \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

$$= -1.5\{\cos\delta_{max} - \cos\delta_{cr}\} - \{\delta_{max} - \delta_{cr}\}$$

$$= -1.5\{-0.745 - \cos\delta_{cr}\} - \{2.41 - \delta_{cr}\}$$

$$= 1.117 + 1.5\cos\delta_{cr} - 2.41 + \delta_{cr}$$

$$= \underline{\underline{\delta_{cr} + 1.5\cos\delta_{cr} - 1.293}}$$

On Equating both Areas;

$$\cancel{\delta_{cr}} + 0.5\cos\delta_{cr} - 0.956 = \cancel{\delta_{cr}} + 1.5\cos\delta_{cr} - 1.293$$

$$\therefore \; -\cos\delta_{cr} = -0.337$$

$$\therefore \delta_{cr} = \cos^{-1}(0.337) = \underline{\underline{70.3^\circ}} \text{ or } 1.22 \text{ radians.}$$

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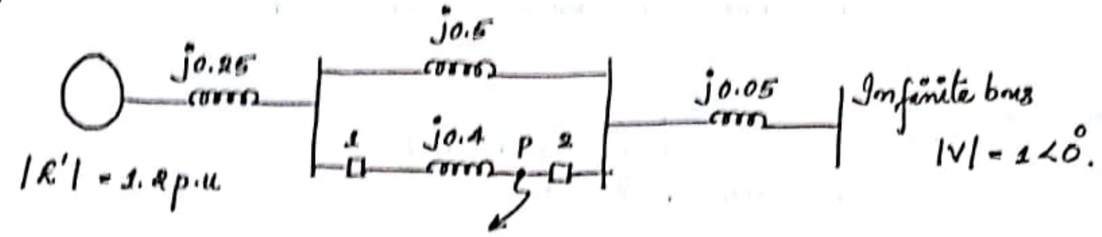
4. Consider the system shown in figure, where a three phase fault is applied at the point P as shown.



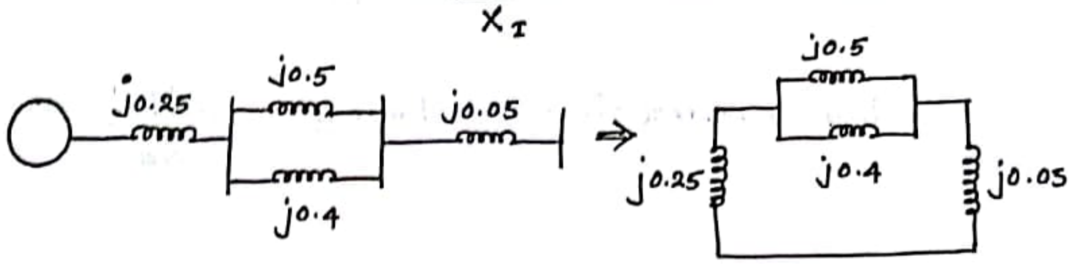

Find the critical angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 p.u power at the instant preceding the fault.

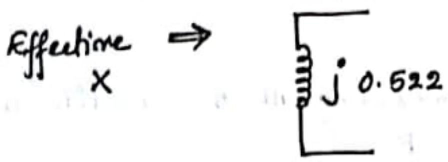
sol:

We need to analyse 3 condition :- Prefault, during fault, Post fault.

I. Normal operation or Pre-fault condition:

$Pe_I = P_{max\,I} \sin\delta$ ; $P_{max\,I} = \dfrac{|E||V|}{X_I}$




$\therefore \quad P_{max\,I} = \dfrac{1.2 \times 1}{0.522}$

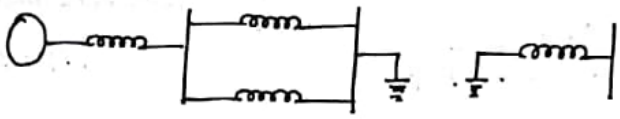
$= \underline{2.29\ p.u}$

$\therefore \quad Pe_I = 2.29 \sin\delta$

## II During fault.

During fault, no power is transfered.

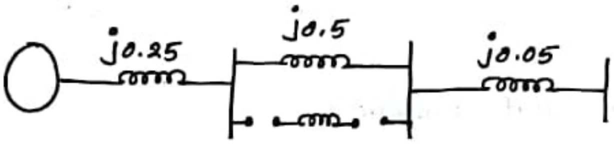
ie



ie $Pe_{II} = 0$

## III Post fault condition (After the fault is cleared).

Here the fault is cleared by opening the circuit breakers 1 and 2 simultaneously. Hence the equivalent circuit will be



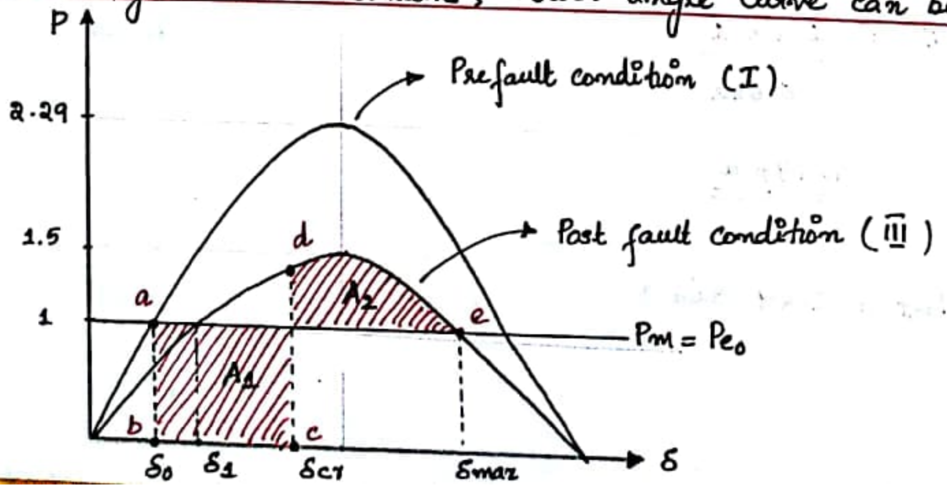

The effective $X = j0.25 + j0.5 + j0.05 = 0.8j$.

$Pe_{III} = P_{max\,III} \sin\delta$ ; $P_{max\,III} = \frac{|E||V|}{X_{III}} = \frac{1.2 \times 1}{0.8}$

$= \underline{\underline{1.5\ p.u}}$

$\therefore \quad Pe_{III} = 1.5 \sin\delta$.

On combining all the conditions, Power angle curve can be drawn as:



Let the system was operating under stable condition at point 'a'; where $P_m = P_e = 1$ p.u in the I curve. Suddenly a fault occurs and the operating region shifts from 'a' to 'b'. The load angle continue to increase and let at $\delta_{cr}$, the fault is cleared and operating region shifts from 'c' to 'd' in curve III.

For the fault clearing angle to be critical; the Accelerating area = deaccelerating area

ie Area $A_1$ = Area $A_2$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e\,II}) \cdot d\delta \quad \text{and} \quad \text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m) \cdot d\delta .$$

$P_{e\,II} = 0$

$P_{max\,I} \sin \delta_0 = 1$

$$\therefore \delta_0 = \sin^{-1}(1/P_{max\,I})$$

$$= \sin^{-1}(1/2.29)$$

$$= \underline{\underline{0.451 \text{ rad}}}$$

$P_{e\,III} = P_{max\,III} \sin \delta$

$= 1.5 \sin \delta$

$$\delta_{max} = \pi - \delta_1$$

$$\delta_1 = \sin^{-1}(1/P_{max\,III})$$

$$= \sin^{-1}(1/1.5)$$

$$= 0.729 \text{ rad}$$

$$\therefore \delta_{max} = \underline{\underline{2.411 \text{ rad}}}$$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{e\,II}) \cdot d\delta$$

$$= \int_{\delta_0}^{\delta_{cr}} P_m \cdot d\delta$$

$$= P_m \{ \delta_{cr} - \delta_0 \}$$

$$= 1 \{ \delta_{cr} - 0.451 \}$$

$$\text{Area } A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e\,III} - P_m) \cdot d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{e\,III}\, d\delta - \int_{\delta_{cr}}^{\delta_{max}} P_m \cdot d\delta$$

$$= \int_{\delta_{cr}}^{\delta_{max}} P_{max\,III} \sin\delta \, d\delta - P_m \cdot \delta \Big\}_{\delta_{cr}}^{\delta_{max}}$$

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$= \delta_{cr} - 0.451$

$\therefore A_1 = \delta_{cr} - 0.451$

$= -P_{max II} \cos\delta \Big|_{\delta_{cr}}^{\delta_{max}} - P_m \cdot \delta \Big|_{\delta_{cr}}^{\delta_{max}}$

$= -1.5 \{\cos\delta_{max} - \cos\delta_{cr}\} - \{\delta_{max} - \delta_{cr}\}$

$= -1.5 \{\cos 2.411 - \cos\delta_{cr}\} - \{2.411 - \delta_{cr}\}$

$A_2 = 1.117 + 1.5\cos\delta_{cr} - 2.411 + \delta_{cr}$

On Equating both Areas.

$\cancel{\delta_{cr}} - 0.451 = 1.117 + 1.5\cos\delta_{cr} - 2.411 + \cancel{\delta_{cr}}$

$1.5\cos\delta_{cr} = 0.843$

$\delta_{cr} = \cos^{-1}\left(\frac{0.843}{1.5}\right)$

$= \underline{\underline{55.8^\circ}} = 0.\underline{\underline{562 \text{ radians}}}$

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